Embedded Systems

Exercise 2: Scheduling Real-Time Aperiodic Tasks

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Definitions: RT Tasks

Arrival Time $a$

Execution Time $C$

Deadline $d$

Response Time $R$

Lateness $L$
Overview

Scheduling aperiodic tasks with real-time constraints

Some known algorithms:

<table>
<thead>
<tr>
<th></th>
<th>Equal arrival times non preemptive</th>
<th>Arbitrary arrival times preemptive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent tasks</td>
<td>EDD (Jackson)</td>
<td>EDF (Horn)</td>
</tr>
<tr>
<td>Dependent tasks</td>
<td>LDF (Lawler)</td>
<td>EDF* (Chetto)</td>
</tr>
</tbody>
</table>
Earliest Due Deadline (EDD) (1/2)

Optimization goal:
– minimize the maximum lateness

Assumptions on the task set:
– independent tasks
– synchronous arrival times

EDD is non preemptive
Earliest Due Deadline (EDD) (2/2)

Jackson’s rule:
execute tasks in order of non decreasing deadlines

T1
T2
T3

T1
T2
T3

T2
T1
T3
Consider the following task set:

\[ \tau_1 : T_1 = 2, \quad D_1 = 8 \]
\[ \tau_2 : T_2 = 4, \quad D_2 = 6 \]
\[ \tau_3 : T_3 = 5, \quad D_2 = 11 \]

What is the order of task execution using EDD?

\[ \tau_2 \rightarrow \tau_1 \rightarrow \tau_3 \]
Latest Deadline First (LDF) (1/3)

Optimization goal:
– minimize the maximum lateness

Assumptions on the task set:
– tasks with precedence relations
– synchronous arrival times

LDF is non preemptive

Note: Not covered in lecture!
Latest Deadline First (LDF) (2/3)

Main Idea [Lawler]:
Build a scheduling queue. At run time, execute tasks from that queue

- Building the scheduling queue:
  - proceed from tail to head
  - among the tasks without successors or whose successors have been all scheduled, select the task with the latest deadline to be scheduled last
  - repeat the procedure until all tasks in the set are selected
Latest Deadline First (LDF) (3/3)

Building the scheduling queue (example):

<table>
<thead>
<tr>
<th>deadlines</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>d&lt;sub&gt;i&lt;/sub&gt;</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

DAG

Scheduling Queue: E D C A B

run-time scheduler
Consider the same task set:

\( \tau_1 : T_1 = 2, \ D_1 = 8 \)
\( \tau_2 : T_2 = 4, \ D_2 = 6 \)
\( \tau_3 : T_2 = 5, \ D_2 = 11 \)

But with the given precedence:

\[ T_1 \rightarrow T_2 \]
\[ T_1 \rightarrow T_3 \]
\[ T_2 \rightarrow T_3 \]

What is the order of task execution using LDF?

\( \tau_1 \rightarrow \tau_2 \rightarrow \tau_3 \)
Earliest Deadline First (EDF) (1/3)

Optimization goal:
– minimize the maximum lateness

Assumptions on the task set:
– independent tasks
– arbitrary arrival times (dynamic arrivals)

EDF is preemptive
Earliest Deadline First (EDF) (2/3)

Horn’s rule:
at any instant execute the task with the earliest absolute deadline
Given the following task set with arrival time, completion time and deadline:

\[
\tau_1 : A_1 = 0, \ C_1 = 5, \ D_1 = 7 \\
\tau_2 : A_2 = 2, \ C_2 = 2, \ D_2 = 5 \\
\tau_3 : A_3 = 6, \ C_3 = 4, \ D_3 = 13
\]

Is a EDF schedule feasible? If so which task would be running between time = 2 and time = 3?

Yes, it is feasible and Task 2 is running at that time.
Earliest Deadline First (EDF*) (1/3)

Optimization goal:
- minimize the maximum lateness

Assumptions on the task set:
- tasks with precedence relations
- arbitrary arrival times

EDF* is preemptive
Earliest Deadline First (EDF*) (2/3)

Main idea:
modify deadlines and arrival times to satisfy the precedence constraints
and then use EDF

Modifying arrival times:
✓ Task must start the execution not earlier than its release time.
✓ Task must not start the execution earlier than the minimum finishing time of its predecessors.

\[ r_j^* = \max \left( r_j, \max \left( r_i^* + C_i : T_i \rightarrow T_j \right) \right) \]
Main idea:
modify deadlines and arrival times to satisfy the precedence constraints and then use EDF

Modifying deadlines:
✓ Task must finish the execution within its deadline
✓ Task must not finish the execution later than the maximum start time of its successor

\[ d_i^* = \min\left(d_i, \min\left(d_j^* - C_j : T_i \rightarrow T_j\right)\right) \]
Consider the following task set:

\[ \tau_1 : C_1 = 2 \]
\[ \tau_2 : C_2 = 4 \]
\[ \tau_3 : C_3 = 5 \]

where all the tasks have the same release time of 0 and the same deadline of 12. Taking into consideration the following precedence constraints:

\[ T_1 \rightarrow T_2 \]
\[ T_1 \rightarrow T_3 \]
\[ T_2 \rightarrow T_3 \]

What is the modified release time of the tasks?

\[ R_1 = 0, \quad R_2 = 2, \quad R_3 = 6 \]
Consider the following task set:

\[ \tau_1 : C_1 = 2 \]
\[ \tau_2 : C_2 = 4 \]
\[ \tau_3 : C_3 = 5 \]

where all the tasks have the same release time of 0 and the same deadline of 12. Taking into consideration the following precedence constraints:

\[ T_1 \rightarrow T_2 \]
\[ T_1 \rightarrow T_3 \]
\[ T_2 \rightarrow T_3 \]

What is the modified deadline of the tasks?

\[ D_1 = 3, \quad D_2 = 7, \quad D_3 = 12 \]
Earliest Due Deadline (EDD) (Solution)
Latest Deadline First (LDF) (Solution)

Scheduling Queue: J8, J7

run-time scheduler
Latest Deadline First (LDF) (Solution)

Scheduling Queue: J8 J7 J6

run-time scheduler
Latest Deadline First (LDF) (Solution)

Scheduling Queue: J8 J7 J6 J4 J5

run-time scheduler
Latest Deadline First (LDF) (Solution)

Scheduling Queue: J8, J7, J6, J4, J5, J3

run-time scheduler
Latest Deadline First (LDF) (Solution)

Scheduling Queue: [J8, J7, J6, J4, J5, J3, J2]
Latest Deadline First (LDF) (Solution)

Scheduling Queue: [J8] [J7] [J6] [J4] [J5] [J3] [J2] [J1]
Latest Deadline First (LDF) (Solution)

Scheduling Queue: J8 J7 J6 J4 J5 J3 J2 J1
Earliest Deadline First (EDF) (Solution)
Earliest Deadline First (EDF) (Solution)

At time $t=3$, existing tasks: $J_1$, $J_2$, $J_3$
  • $J_2$ has finished, $J_1$ / $J_3$ are feasible

Schedulability test after arrival of $J_x$:

Put $f_0 = t = 3$

Task $J_3$: $f_1 = f_0 + c_3(3) = 3 + 4 = 7 \leq 8$ (ok)

Task $J_x$: $f_2 = f_1 + c_x(3) = 7 + 2 = 9 \leq 10$ (ok)

Task $J_1$: $f_3 = f_2 + c_1(3) = 9 + 3 = 12 \leq 16$ (ok)

Repeat for $t=8, 13$
Earliest Deadline First (EDF*) (Solution 5(1))

\[ J_1 \rightarrow J_2, J_2 \rightarrow J_3, J_3 \rightarrow J_4, J_5 \rightarrow J_6, J_6 \rightarrow J_7, \]
\[ J_6 \rightarrow J_8, J_2 \rightarrow J_7, J_7 \rightarrow J_4 \text{ and } J_8 \rightarrow J_7. \]
Earliest Deadline First (EDF*) (Solution 5(2))

<table>
<thead>
<tr>
<th></th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
<th>$J_6$</th>
<th>$J_7$</th>
<th>$J_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_i$</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$d_i$</td>
<td>3</td>
<td>8</td>
<td>15</td>
<td>15</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>$C_i$</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
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<th>$J_6$</th>
<th>$J_7$</th>
<th>$J_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_i^*$</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>$d_i^*$</td>
<td>3</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>$C_i$</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Earliest Deadline First (EDF*) (Solution 5(3))

| \( r_i^* \) | 0 | 3 | 6 | 9 | 0 | 2 | 6 | 3 |
| \( d_i^* \) | 3 | 8 | 12 | 15 | 6 | 7 | 10 | 8 |
| \( C_i \) | 1 | 3 | 3 | 3 | 1 | 1 | 2 | 1 |
No. J4 cannot be started earlier than time 9. Therefore, the minimum finish time of the application is 12 (finish time for the dual core platform).