Embedded Systems

Exercise 6: Scheduling and Marked Graphs

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Scheduling – problem

Given:
- Sequence graph
- Operations of different types
- Allocated a limited number of resource units
- Operations possibly mapped to different resource units
- Operations may have different execution times

Problem:
- Starting times of operations?
Scheduling – definitions

sequence graph

\[ \tau(v_{\text{nop1}}) = 0 \]

\[ \tau(v_{\text{nop2}}) - \tau(v_{\text{nop1}}) \]

\[ \tau(v_{\text{nop2}}) = n \]

operations

Start time of execution

\[ \tau(v_i) = t \]
Scheduling – unlimited resources, operation execution time = 1

time 0

X

X

-

time 1

+  

time 2

<  

time 3

nop

latency = 3
Timing Constraints

- data dependencies & computation time

If multiplication takes 2 time units:

\[ \tau(v_2) \geq \tau(v_1) + 2 \quad \text{(Linear constraint)} \]
Question 1

• Given a sequence graph
• One resource type for all operations
• Execution time of each operation = 1

• Formulate a *linear optimization problem* for finding the starting times of the operations that minimize the latency $L$
Linear Optimization Problem

• Find variables \( x \) that:

\[
\begin{align*}
\text{maximize} & \quad c^T x \\
\text{subject to} & \quad A x \leq b \\
\text{and} & \quad x \geq 0
\end{align*}
\]

Objective function
Linear constraints
Non-negative variables

• Example: Find variables \( x_1, x_2 \) that:

\[
\begin{align*}
\text{maximize} & \quad 2 \cdot x_1 - x_2 \\
\text{subject to} & \quad x_1 - x_2 \leq 1 \\
& \quad -2 \cdot x_1 + 2 \cdot x_2 \leq -3 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]
Question 1

• Define non-negative variables $\tau(v_x)$
• Setup the linear inequalities for the starting times
  – Form: $\tau(v_3) \geq \tau(v_1) + 1$
• Determine the objective function
  – Minimization of latency $L$

• Determine the minimum latencies for the graph under the two resource assumptions
  – Only 1 resource unit allocated
  – Unlimited number of resource units allocated
Question 1
Question 1 – Solution

\[
\begin{align*}
\tau(v_3) & \geq \tau(v_1) + 1 \\
\tau(v_3) & \geq \tau(v_2) + 1 \\
\tau(v_4) & \geq \tau(v_3) + 1 \\
\tau(v_5) & \geq \tau(v_4) + 1 \\
\tau(v_5) & \geq \tau(v_7) + 1 \\
\tau(v_7) & \geq \tau(v_6) + 1 \\
\tau(v_9) & \geq \tau(v_8) + 1 \\
\tau(v_{11}) & \geq \tau(v_{10}) + 1 \\
\tau(v_1) & \geq \tau(v_0) \\
\tau(v_2) & \geq \tau(v_0) \\
\tau(v_6) & \geq \tau(v_0) \\
\tau(v_8) & \geq \tau(v_0) \\
\tau(v_{10}) & \geq \tau(v_0) \\
\tau(v_n) & \geq \tau(v_5) + 1 \\
\tau(v_n) & \geq \tau(v_9) + 1 \\
\tau(v_n) & \geq \tau(v_{11}) + 1 \\
\end{align*}
\]

Initial condition:
\[\tau(v_0) \geq 0\]

Objective function:
\[\min \tau(v_n) - \tau(v_0)\]
Question 1 – Solution

- Case: Resource constraints from task 1: \( L_{\text{min}} = 11 \)
- Case: Unlimited resources: \( L_{\text{min}} = 4 \)

1 resource: e.g.

\[
\tau(v_1) = 0; \tau(v_2) = 1; \tau(v_3) = 2; \tau(v_4) = 3; \tau(v_6) = 4; \\
\tau(v_7) = 5; \tau(v_5) = 6; \tau(v_8) = 7; \tau(v_9) = 8; \tau(v_{10}) = 9; \\
\tau(v_{11}) = 10; \tau(v_n) = L = 11
\]

Unlimited resources: e.g.
Question 2

• Determine the optimal design configurations (Pareto points) in terms of *minimum cost* (number of allocated resource units) and *minimum latency*
Multi-criteria optimization / Pareto-points

A dominates B
Multi-criteria optimization / Pareto-points

A and C not comparable
Multi-criteria optimization / Pareto-points

Points dominated by C

Pareto-front

Pareto-Points = Points, which are not dominated by others
Dominance, Pareto Points

• A (design) point $A$ is *dominated* by $B$, iff $B$ is
  – better than $A$ in at least one criterion and
  – not worse than $A$ in all other criteria

• A point is Pareto-optimal or a *Pareto-point* if it is not
dominated.

• The domination relation imposes a partial order on all
design points
  – We are faced with a set of optimal solutions.
Question 2

• Determine the optimal design configurations (Pareto points) in terms of minimum cost (number of allocated resource units) and minimum latency

  – Calculate upper and lower bounds on the cost and upper and lower bounds on the latency

  – **Hint:** For each number of resource units (within the bounds), calculate the minimum latency
Question 2 – Solution

Latency bounds: $4 \leq L \leq 11$
Cost bounds: $1 \leq c \leq 5$
Marked graphs

node (= operation)

token (= data)

FIFO queues

Firing
Question 3

- Describe output of given marked graphs as a function of input, initial tokens, and previous outputs
  \[ b(k) = f(a(k), b(...), s) \]

- Hint: Start from the beginning:
  \[ b(1) = \ldots \]
  \[ b(2) = \ldots \]
  \[ \ldots \]
  \[ b(n) = \ldots \]
Question 3

• Describe output of given marked graphs as a function of input, initial tokens, and previous outputs
  \[ b(k) = f(a(k), b(...), s) \]

\[ b(1) = a(1) + s \]

\[ b(k) = a(k) + b(k - 1) \quad \text{for} \quad k > 1 \]

with \( s \) being the data value of the initial mark
Question 3 – Solution

\[ a) \quad I) \quad b(1) = a(1) + s \]
\[ b(k) = a(k) + b(k - 1) \text{ for } k > 1 \]
\[ \text{with } s \text{ being the data value of the initial mark} \]

\[ II) \quad \text{For } n = 0 \text{ the output sequence is empty.} \]
\[ \text{For } n > 0: \]
\[ b(k) = a(k) + s_k \text{ for } k \leq n \]
\[ b(k) = a(k) + b(k - n) \text{ for } k > n \]
\[ \text{with } s_1, \ldots, s_n \text{ being the data values of the initial marks.} \]
b) \[ v(1) = 2 \cdot s_2 \cdot (u(1) + s_1) \]
\[ v(2) = 2 \cdot v(1) \cdot (u(2) + s_2) \]
\[ v(k) = 2 \cdot v(k - 1) \cdot (u(k) + v(k - 2)) \text{ for } k > 2 \]
Question 4

• Find a marked graph for the Fibonacci sequence starting from \( n = 1 \): 1, 1, 2, 3, 5, ...

\[
Fib(n) = \begin{cases} 
0 & \text{if } n = 0 \\
1 & \text{if } n = 1 \\
Fib(n - 1) + Fib(n - 2) & \text{if } n > 1 
\end{cases}
\]

**Hint:** Need to have at least two edges: one with one initial data token, and one with two initial data tokens
Question 4 – Solution

\[ x_1 + x_2 = F \]

or

\[ 0 + 0 = 0 \]

\[ 1 + 1 = 2 \]
Question 5 – Sequence graphs

Hierarchical arrangement of dependence graphs
Question 5 – Solution

\[ w := a + b \times d; \]
\[ x := a - b \times e; \]
\[ y := w \times c; \]
\[ z := a + b + d; \]
\[ \text{IF} (z < 0) \text{ THEN} \]
\[ z := -z; \]
\[ \text{END IF} \]
\[ \text{IF} (y > 0) \text{ THEN} \]
\[ v := 1 / y; \]
\[ \text{ELSE} \]
\[ v := z \times y; \]
\[ \text{END IF} \]
w := a + b * d;
x := a - b * e;
y := w * c;
z := a + b + d;
IF (z < 0) THEN
  z := -z;
END IF
IF (y > 0) THEN
  v := 1 / y;
ELSE
  v := z * y;
END IF
w := a + b * d;
x := a - b * e;
y := w * c;
z := a + b + d;
IF (z < 0) THEN
  z := -z;
END IF
IF (y > 0) THEN
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  v := z * y;
END IF
w := a + b * d;
x := a - b * e;
y := w * c;
z := a + b + d;

IF (z < 0) THEN
  z := -z;
END IF

IF (y > 0) THEN
  v := 1 / y;
ELSE
  v := z * y;
END IF
Question 5 – Solution

\[ w := a + b \times d; \]
\[ x := a - b \times e; \]
\[ y := w \times c; \]
\[ z := a + b + d; \]

IF \((z < 0)\) THEN
\[ z := -z; \]
END IF

IF \((y > 0)\) THEN
\[ v := 1 / y; \]
ELSE
\[ v := z \times y; \]
END IF
Question 5 – Solution

\[ w := a + b \times d; \]
\[ x := a - b \times e; \]
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w := a + b * d;
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y := w * c;
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END IF
IF (y > 0) THEN
v := 1 / y;
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v := z * y;
END IF
w := a + b * d;
x := a - b * e;
y := w * c;
z := a + b + d;
IF (z < 0) THEN
  z := -z;
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\[ v := z \times y; \]
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