Question 1

• Q: Consider the following taskset:

\[ \tau_1 : T_1 = 4, \ D_1 = 3, \ C_1 = 2, \ \Phi_1 = 0 \]

\[ \tau_2 : T_2 = 6, \ D_2 = 3, \ C_2 = ?, \ \Phi_2 = 0 \]

What is the largest value of \( C_2 \) such that a feasible schedule exists (there exists a schedule that meets all task deadlines).

• A: Both tasks have to finish execution by time 3. Since \( C_1 = 2 \), we have \( C_2 \leq 1 \).
Question 2

• **Q:** Consider the following taskset:

\[ \tau_1 : T_1 = 4, \quad D_1 = 3, \quad C_1 = 2 \]

\[ \tau_2 : T_2 = 5, \quad D_2 = 4, \quad C_2 = 2 \]

Following cyclic-executive is used to schedule the taskset: \( f=1, \quad P=20 \).

• **A:** Violated: Tasks start and finish within a single frame.
Question 3

• **Q:** Consider the following taskset:

\[\tau_1 : T_1 = 4, \quad D_1 = 3, \quad C_1 = 2\]

\[\tau_2 : T_2 = 5, \quad D_2 = 4, \quad C_2 = 2\]

Following cyclic-executive is used to schedule the taskset: f=3, P=20.

• **A:** Violated: P is a multiple of f. Between release time and deadline of any task, there is at least one full frame.
Question 4

• **Q:** Consider the following taskset:

\[ \tau_1 : \ T_1 = 4, \ D_1 = 3, \ C_1 = 2 \]

\[ \tau_2 : \ T_2 = 5, \ D_2 = 4, \ C_2 = 2 \]

Following cyclic-executive is used to schedule the taskset: \( f=4, P=20 \).

• **A:** Violated: Between release time and deadline of any task, there is at least one full frame.
Question 5

• Q: Consider the following taskset:

\[ \tau_1 : T_1 = 4, \quad D_1 = 3, \quad C_1 = 2 \]

\[ \tau_2 : T_2 = 5, \quad D_2 = 4, \quad C_2 = 2 \]

Following cyclic-executive is used to schedule the taskset: \( f=2, \quad P=20 \).

• A: All conditions are satisfied!
Real Time Systems
Cyclic-executive Scheduling

Stefan Drašković, stefan.draskovic@tik.ee.ethz.ch
Today’s Exercise

- Introduction to Cyclic-executive Scheduling
- You solve Task 1
- Discussion about Task 1
- Repeat for Task 2
Definitions

• $\Gamma$: denotes the set of all periodic tasks
• $\tau_i$: denotes a periodic task
• $\tau_{i,j}$: denotes the $j$th instance (job) of task $i$
• $r_{i,j}, d_{i,j}$: denote the release time and absolute deadline of the $j$th instance of task $i$
• $\Phi_i$: phase of task $i$, release time of first instance
• $D_i$: relative deadline of task $i$
Time-triggered Cyclic-executive Scheduling

- Tasks are periodic, but may have different periods. Instances of a task are regularly activated with a period $T_i$.

  $$r_{i,j} = \Phi_i + (j - 1) T_i$$

- All instances have same worst case execution time $C_i$.
- All instances have same relative deadline $D_i$. The absolute deadlines are:

  $$d_{i,j} = \Phi_i + (j - 1) T_i + D_i$$
Time-triggered Cyclic-executive Scheduling

- The period $P$ of the system is divided into frames $f$
- Timer interrupts regularly every frame start
- Schedule computed *off-line*
- Deterministic behaviour at runtime
Example

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$T_i$</th>
<th>$\Phi_i$</th>
<th>$D_i$</th>
<th>$C_i$</th>
<th>frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>12</td>
<td>2</td>
<td>8</td>
<td>2.8</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>12</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1, 2, 3</td>
</tr>
</tbody>
</table>

$P = 12, f = 4$
Example

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$T_i$</th>
<th>$\Phi_i$</th>
<th>$D_i$</th>
<th>$C_i$</th>
<th>frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>12</td>
<td>2</td>
<td>8</td>
<td>2.8</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>12</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1, 2, 3</td>
</tr>
</tbody>
</table>

$P = 12, f = 4$
Example

<table>
<thead>
<tr>
<th>Γ</th>
<th>$T_i$</th>
<th>$\Phi_i$</th>
<th>$D_i$</th>
<th>$C_i$</th>
<th>frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>12</td>
<td>2</td>
<td>8</td>
<td>2.8</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>12</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1, 2, 3</td>
</tr>
</tbody>
</table>

$P = 12, \ f = 4$
Example

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$T_i$</th>
<th>$\Phi_i$</th>
<th>$D_i$</th>
<th>$C_i$</th>
<th>frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>12</td>
<td>2</td>
<td>8</td>
<td>2.8</td>
<td>2</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>12</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1, 2, 3</td>
</tr>
</tbody>
</table>

$P = 12, f = 4$
Conditions for $P$ and $f$

- A task executes at most once within frame:

\[ f \leq T_i \quad \forall \tau_i \]

- $P$ is a multiple of $f$

- Tasks start and complete within a single frame:

\[ f \geq C_i \quad \forall \tau_i \]

- Between the release time and deadline of every task there is at least one full frame:

\[ 2f - \gcd(T_i, f) \leq D_i \quad \forall \tau_i \]
Correctness of Schedule

- $f_{ij}$ notes that instance $j$ of task $\tau_i$ executes in frame $f_{ij}$
- Is $P$ a common multiple of all periods $T_i$? Is $P$ a multiple of $f$?
- Is the frame sufficiently long?

$$\sum_{\{i|f_{ij}=k\}} C_i \leq f \quad \forall 1 \leq k \leq \frac{P}{f}$$

- Determine offsets such that instances start after release time:

$$\Phi_i = \min_{1 \leq j \leq \frac{P}{T_i}} \{(f_{ij} - 1)f - (j - 1)T_i\} \quad \forall \tau_i$$

- Are deadlines respected?

$$(j - 1)T_i + \Phi_i + D_i \geq f_{ij}f \quad \forall \tau_i, \ 1 \leq j \leq \frac{P}{T_i}$$
Task 1: Check Schedule Correctness!

- $f_{ij}$ notes that instance $j$ of task $\tau_i$ executes in frame $f_{ij}$
- Is $P$ a common multiple of all periods $T_i$? Is $P$ a multiple of $f$?
- Is the frame sufficiently long?

$$\sum_{\{i|f_{ij}=k\}} C_i \leq f \quad \forall 1 \leq k \leq \frac{P}{f}$$

- Determine offsets such that instances start after release time:

$$\Phi_i = \min_{1 \leq j \leq \frac{P}{T_i}} \left\{ (f_{ij} - 1)f - (j - 1)T_i \right\} \quad \forall \tau_i$$

- Are deadlines respected?

$$(j - 1)T_i + \Phi_i + D_i \geq f_{ij}f \quad \forall \tau_i, \ 1 \leq j \leq \frac{P}{T_i}$$
Task 1: Solution

- Is $P$ a common multiple of all periods $T_i$? Is $P$ a multiple of $f$?
  Yes!

- Is the frame sufficiently long?
  Yes!
Task 1: Solution

- Determine offsets such that instances start after release time.

\[
\Phi_1 = \min \left\{ \begin{array}{l}
(2 - 1)4 - (1 - 1)15 \\
(5 - 1)4 - (2 - 1)15 \\
(9 - 1)4 - (3 - 1)15 \\
(12 - 1)4 - (4 - 1)15
\end{array} \right. = \begin{array}{l}
4 \\
1 \\
2 \\
-1
\end{array}
\]

\[
\Phi_2 = 0 \quad \Phi_3 = -2 \quad \Phi_4 = 2
\]
Task 1: Solution

• Are deadlines respected?
  Yes, for example for $\tau_1$:

\[
\begin{align*}
(1 - 1)15 - 1 + 9 &= 8 \geq 8 = 2 \cdot 4 \\
(2 - 1)15 - 1 + 9 &= 23 \geq 20 = 5 \cdot 4 \\
(3 - 1)15 - 1 + 9 &= 38 \geq 36 = 9 \cdot 4 \\
(4 - 1)15 - 1 + 9 &= 53 \geq 48 = 12 \cdot 4
\end{align*}
\]
Task 2: Find Schedule

<table>
<thead>
<tr>
<th>Task</th>
<th>Period</th>
<th>Deadline</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
Task 2: Possible Solution