Embedded Systems FS 2017

Exercise 8: Integer Linear Programming & Iterative Algorithms

Discussion Date: 31.05.2017

Task 1: Integer Linear Programming

Given the sequence graph $G_S = (V_S, E_S)$ in Fig. 1.

For the execution times of the operations assume: A multiplication operation (MULT) takes 2 time units and all other (ALU) operations take 1 time unit each. Two units of the resource type $r_1$ (multiplier) and two units of the resource type $r_2$ (ALU) are allocated.

a) Apply the ASAP and ALAP algorithms to compute the earliest ($l_i$) and the latest ($h_i$) starting time of all operations $v_i \in V_s, i \in \{1, \ldots, 11\}$. For ALAP, assume the maximum latency $\bar{L} = 7$. Fill in the starting times in Table 1.
Table 1: Earliest and latest starting times (Task 1a)

<table>
<thead>
<tr>
<th>v_1</th>
<th>v_2</th>
<th>v_3</th>
<th>v_4</th>
<th>v_5</th>
<th>v_6</th>
<th>v_7</th>
<th>v_8</th>
<th>v_9</th>
<th>v_{10}</th>
<th>v_{11}</th>
</tr>
</thead>
<tbody>
<tr>
<td>l_1 (ASAP)</td>
<td>h_1 (ALAP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Formulate the problem of latency minimization with restricted resources as an integer linear program (ILP). For this, you should introduce the binary variables $x_{i,t} \in \{0,1\}$ $\forall v_i \in V_S$ and $\forall t \in \{t \in \mathbb{Z} \mid l_i \leq t \leq h_i\}$. $\tau(v_i)$ is used to denote the starting time of operation $v_i \in V_S$ and $\alpha(r_i)$ with $r_i \in V_R = \{\text{MULT, ALU}\}$ denotes the number of allocated resource instances. Given the above notations, write down the following equations/inequations without using the $\sum$ symbol.

i) Express the objective function of the ILP

ii) Define $\tau(v_i)$ $\forall i \in \{1, \ldots, 11\}$ as a function of $x_{i,t}$, where $l_1 \leq t \leq h_1$

iii) Express all data dependencies

iv) Express all resource limitations

c) In an analogous manner try to formulate an ILP that solves the problem of cost minimization with latency limitation. Hint: We assume that the cost of a realization is the sum of the costs $c$ of the multipliers with $c(r_1) = 2$ per allocated unit, and of the ALUs with $c(r_2) = 1$ per allocated unit. For the latency bound, we choose $L = 6$.

Task 2: Iterative Algorithms

Please answer the following questions considering the given video codec application specified as a marked graph in Figure 2.

a) Formulate all existing dependencies in Figure 2 from $v_i$ to $v_j$ in the form of

$$\tau(v_j) - \tau(v_i) \geq w(v_i) - d_{ij} \cdot P,$$

where $P$ is the minimum iteration interval. The execution time of each function is listed in Table 2.

b) Assuming unlimited resources and only one token on the edge between $v_5$ and $v_1$, determine the minimum iteration interval $P$ and the latency $L$. To justify your answer, draw the scheduling on the timeline given in Figure 3 with the dependency from $v_5$ to $v_1$ highlighted.

c) The motion estimation function ($v_1$) uses the result of the previous frame (See the dependency between $v_1$ and $v_5$). Let us now suppose that any arbitrary number of tokens can be inserted to reduce $P$ using functional pipelining. Then, determine the minimum number of tokens that should be added on the edge $v_5 \rightarrow v_1$ to achieve $P = 10$? To justify your answer, draw the pipelined scheduling on the timeline given in Figure 4 with the dependency from $v_5$ to $v_1$ highlighted and calculate the latency $L$ of the schedule.
Figure 2: Video codec marked graph representation

Table 2: Execution time of each function

<table>
<thead>
<tr>
<th>( v_i )</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w(\nu_i) )</td>
<td>( \nu_1 )</td>
<td>( \nu_2 )</td>
<td>( \nu_3 )</td>
<td>( \nu_4 )</td>
<td>( \nu_5 )</td>
</tr>
</tbody>
</table>

Figure 3: Scheduling result of the video codec

Figure 4: Pipelined scheduling result of the video codec