Embedded Systems

3. Real-Time Models

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Software and Programming
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Basic Terms

- **Real-time systems**
  - **Hard**: A real-time task is said to be hard, if missing its deadline may cause catastrophic consequences on the environment under control. Examples are sensory data acquisition, detection of critical conditions, actuator servoing.
  
  - **Soft**: A real-time task is called soft, if meeting its deadline is desirable for performance reasons, but missing its deadline does not cause serious damage to the environment and does not jeopardize correct system behavior. Examples are command interpreter of the user interface, displaying messages on the screen.
Given a set of tasks \( J = \{J_1, J_2, \ldots\} \):

- A **schedule** is an assignment of tasks to the processor, such that each task is executed until completion.
- A **schedule** can be defined as an integer step function \( \sigma : \mathbb{R} \rightarrow \mathbb{N} \) where \( \sigma(t) \) denotes the task which is executed at time \( t \). If \( \sigma(t) = 0 \) then the processor is called **idle**.
- If \( \sigma(t) \) changes its value at some time, then the processor performs a **context switch**.
- Each interval, in which \( \sigma(t) \) is constant is called a **time slice**.
- A **preemptive schedule** is a schedule in which the running task can be arbitrarily suspended at any time, to assign the CPU to another task according to a predefined scheduling policy.
Schedule and Timing

- A schedule is said to be **feasible**, if all task can be completed according to a set of specified constraints.
- A set of tasks is said to be **schedulable**, if there exists at least one algorithm that can produce a feasible schedule.
- **Arrival time** \( a_i \) or **release time** \( r_i \) is the time at which a task becomes ready for execution.
- **Computation time** \( C_i \) is the time necessary to the processor for executing the task without interruption.
- **Deadline** \( d_i \) is the time at which a task should be completed.
- **Start time** \( s_i \) is the time at which a task starts its execution.
- **Finishing time** \( f_i \) is the time at which a task finishes its execution.
Schedule and Timing

- Using the above definitions, we have \( d_i \geq r_i + C_i \)
- **Lateness** \( L_i = f_i - d_i \) represents the delay of a task completion with respect to its deadline; note that if a task completes before the deadline, its lateness is negative.
- **Tardiness or exceeding time** \( E_i = \max(0, L_i) \) is the time a task stays active after its deadline.
- **Laxity or slack time** \( X_i = d_i - a_i - C_i \) is the maximum time a task can be delayed on its activation to complete within its deadline.
Schedule and Timing

- **Periodic task** $\tau_i$ : infinite sequence of identical activities, called instances or jobs, that are regularly activated at a constant rate with period $T_i$. The activation time of the first instance is called phase $\phi_i$. 

![Diagram showing periodic task](image)
Example

Computation times: $C_1 = 9$, $C_2 = 12$
Start times: $s_1 = 0$, $s_2 = 6$
Finishing times: $f_1 = 18$, $f_2 = 28$
Lateness: $L_1 = -4$, $L_2 = 1$
Tardiness: $E_1 = 0$, $E_2 = 1$
Laxity: $X_1 = 13$, $X_2 = 11$
Precedence Constraints

- **Precedence relations** between graphs can be described through an acyclic directed graph $G$ where tasks are represented by nodes and precedence relations by arrows. $G$ induces a partial order on the task set.

There are different interpretations possible:

- All successors of a task are activated (concurrent task execution).
- One successor of a task is activated (non-deterministic choice).
Precedence Constraints

- Example (concurrent activation):

- Image acquisition \textit{acq1} \textit{acq2}
- Low level image processing \textit{edge1} \textit{edge2}
- Feature/contour extraction \textit{shape}
- Pixel disparities \textit{disp}
- Object size \textit{H}
- Object recognition \textit{rec}
Classification of Scheduling Algorithms

- With **preemptive algorithms**, the running task can be interrupted at any time to assign the processor to another active task, according to a predefined scheduling policy.

- With a **non-preemptive algorithm**, a task, once started, is executed by the processor until completion.

- **Static algorithms** are those in which scheduling decisions are based on fixed parameters, assigned to tasks before their activation.

- **Dynamic algorithms** are those in which scheduling decisions are based on dynamic parameters that may change during system execution.
Classification of Scheduling Algorithms

An algorithm is said **optimal** if it minimizes some given cost function defined over the task set.

An algorithm is said to be **heuristic** if it tends toward but does not guarantee to find the optimal schedule.

Domino effect, if acceptance test wrongly accepted a new task.
Metrics

- Average response time:
  \[
  t_r = \frac{1}{n} \sum_{i=1}^{n} (f_i - r_i)
  \]

- Total completion time:
  \[
  t_c = \max_i (f_i) - \min_i (r_i)
  \]

- Weighted sum of response time:
  \[
  t_w = \frac{\sum_{i=1}^{n} w_i (f_i - r_i)}{\sum_{i=1}^{n} w_i}
  \]

- Maximum lateness:
  \[
  L_{max} = \max_i (f_i - d_i)
  \]

- Number of late tasks:
  \[
  N_{late} = \sum_{i=1}^{n} miss(f_i)
  \]

\[
miss(f_i) = \begin{cases} 
0 & \text{if } f_i \leq d_i \\
1 & \text{otherwise}
\end{cases}
\]
Metrics

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- **Number of late tasks:**

  \[ N_{\text{late}} = \sum_{i=1}^{n} \text{miss}(f_i) \]

  \[ \text{miss}(f_i) = \begin{cases} 
  0 & \text{if } f_i \leq d_i \\
  1 & \text{otherwise}
  \end{cases} \]
Metrics Example

Average response time:
Total completion time:
Weighted sum of response times:
Number of late tasks:
Maximum lateness:

\[
\bar{t}_r = \frac{1}{2} (18 + 24) = 21
\]
\[
t_c = 28 - 0 = 28
\]
\[
w_1 = 2, w_2 = 1: \quad t_w = \frac{2 \cdot 18 + 24}{3} = 20
\]
\[
N_{\text{late}} = 1
\]
\[
L_{\text{max}} = 1
\]
Scheduling Example

- In (a), the maximum lateness is minimized, but all tasks miss their deadlines.
- In (b), the maximal lateness is larger, but only one task misses its deadline.