5.1 Properties of Sets

Consider the following constrained multiobjective problem.

\[
\begin{align*}
\min f_1(x) &= x^2 \\
\min f_2(x) &= (x - 5)^2 \\
x &\in [-10, 10]
\end{align*}
\]  

(1)

Consider the following two sets of solutions.

\[S_1 = \{9, 3, 8\}\]
\[S_2 = \{1, 2, 7\}\]

(a) For each set of solutions, construct the preordering imposed by the two objectives. Which set has a smaller number of pareto-dominated solutions?

(b) The hypervolume indicator is computed with respect to point or points in the objective space. For this problem propose a single point you would choose as reference and explain why?

(c) For each set of solutions, compute the hypervolume indicator with respect to the point chosen in the previous question. Which set has a higher hypervolume indicator? You may use the following log plots to assist you in computing hypervolume.

\[\text{Figure 1: Computing hypervolume indicator for } S_1 \text{ and } S_2\]

(d) Which of the two sets is a better approximation of the pareto-set? Why?
5.2 Evolutionary Algorithm

Consider the following algorithm to solve the problem from (1). Here $F_H(\cdot)$ defines the hypervolume indicator of a set with respect to the point computed in the previous question.

<table>
<thead>
<tr>
<th>Algorithm 1: My First Evolutionary Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data:</strong> Solution set $S$</td>
</tr>
<tr>
<td><strong>Result:</strong> Optimized solution set $S$</td>
</tr>
<tr>
<td><strong>repeat</strong></td>
</tr>
<tr>
<td>Delete from $S$ all pareto-dominated solutions;</td>
</tr>
<tr>
<td>$S' \leftarrow$ set of numbers by averaging each pair of values in $S$;</td>
</tr>
<tr>
<td>$S'' \leftarrow S \cup S'$;</td>
</tr>
<tr>
<td>Delete from $S''$ all pareto-dominated solutions;</td>
</tr>
<tr>
<td>if $F_H(S'') \geq F_H(S)$ then</td>
</tr>
<tr>
<td>$S \leftarrow S''$;</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td><strong>until number of iterations ( \leq 10 );</strong></td>
</tr>
</tbody>
</table>

(a) Apply one iteration of the algorithm for the two given solution sets $S_1$ and $S_2$, and compute the resultant solution sets.

(b) The proposed algorithm will not find the Pareto-optimal solution for (1) for either solution set $S_1$ and $S_2$. Explain why?

(c) How would you modify the algorithm to solve the problem in (1)? Can you identify the pareto-optimal set of solutions?

5.3 A Path-Finding Problem

Consider the following problem of planning the path of a robot on a terrain shown in the following figure. The entry and exit points and orientations of the robot are shown. Also shown are checkpoints which are places the robot must visit at least once.

The robot can either move ahead by a step of 1, or turn left or right by 90 degrees. With these atomic operations, we are to plan the path of the robot.

There are two quantities we want to minimize: the number of move operations, and the number of turn operations.

Figure 2: Terrain map for the robot with entry and exit points and orientations and checkpoints (gray boxes).

Propose an evolutionary algorithm to solve this problem. In particular, provide

(a) an appropriate encoding of the decision space,

(b) explicit definition of any constraints,

(c) neighborhood operations of crossover and mutation,

(d) environmental selection operation.

Manually try out a few steps of your proposed algorithm.