Exercise 5: Multi-Objective Optimization

Rehan Ahmed
02.11.2016
Let us suppose, we would like to select a typewriting device. Criteria are

- mobility (related to weight)
- comfort (related to keyboard size and performance)

<table>
<thead>
<tr>
<th>Icon</th>
<th>Device</th>
<th>weight (kg)</th>
<th>comfort rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>🏬</td>
<td>PC of 2009</td>
<td>20.00</td>
<td>10</td>
</tr>
<tr>
<td>🏬</td>
<td>PC of 1984</td>
<td>7.50</td>
<td>7</td>
</tr>
<tr>
<td>🏬</td>
<td>Laptop</td>
<td>3.00</td>
<td>9</td>
</tr>
<tr>
<td>📖</td>
<td>Typewriter</td>
<td>9.00</td>
<td>5</td>
</tr>
<tr>
<td>📱</td>
<td>Touchscreen Smartphone</td>
<td>0.11</td>
<td>2</td>
</tr>
<tr>
<td>📱</td>
<td>PDA with large keyboard</td>
<td>0.09</td>
<td>3</td>
</tr>
<tr>
<td>📱</td>
<td>PDA with small keyboard</td>
<td>0.11</td>
<td>4</td>
</tr>
<tr>
<td>📱</td>
<td>Organizer with tiny keyboard</td>
<td>0.08</td>
<td>1</td>
</tr>
</tbody>
</table>
Evolutionary Algorithm Cycle

- **representation**
- **mating selection**
- **crossover operator**

- **initial/parent set**

- **children set**

- **environmental selection**

- **mutation operator**
The Hypervolume Indicator

- **Environmental selection**: Select subset of solutions that maximizes hypervolume indicator.

- Example: select optimal subset of 4 solutions from the 8 solutions in the set.

**hypervolume indicator**: hypervolume of the dominated subspace.
Why does the hypervolume indicator lead to diversity and optimality?

**Diversity:** It appears that the indicator well covers the intuitive notion of diversity in objective space.
Problem 5.1

Multi-objective Problem

\[
\begin{align*}
\min f_1(x) &= x^2 \\
\min f_2(x) &= (x - 5)^2 \\
x &\in [-10, 10]
\end{align*}
\]

Solution Sets

\[
S_1 = \{9, 3, 8\} \\
S_2 = \{1, 2, 7\}
\]

a) Construct preordering/dominance imposed by the two objectives on each solution set. Which set has higher number of pareto-dominated solutions?
b) Propose a single reference point for computing hypervolume indicator
c) Compute hypervolume indicator for each set. Which set has higher hypervolume
d) Which set is a better approximation of pareto set?
Problem 5.2

Solve Problem 5.1 using an evolutionary algorithm

Algorithm 1: My First Evolutionary Algorithm

Data: Solution set $S$

Result: Optimized solution set $S'$

repeat

Delete from $S$ all pareto-dominated solutions;

$S' \leftarrow \text{set of numbers by averaging each pair of values in } S$;

$S'' \leftarrow S \cup S'$;

Delete from $S''$ all pareto-dominated solutions;

if $F_H(S'') \geq F_H(S)$ then

$S \leftarrow S''$;

end

until number of iterations $\leq 10$;

a) Apply one iteration of the algorithm to solution sets $S_1$ and $S_2$.
b) Algorithm 1 will not find the Pareto-optimal solution set for either $S_1$ or $S_2$. Explain Why.
c) How would you modify Algorithm 1 to find Pareto-optimal solution. Can you identify Pareto-optimal solutions
Problem 5.3

Task partitioning problem

We are given a set of tasks and a set of cores. Tasks have different computation times and power dissipations depending on the core they are executed on. The problem has the following objectives:

Energy Consumption

\[
\text{Total\_Energy} = \sum_{j \in M} \sum_{i \in T_j} \text{Computation}(i, j) \times \text{Power}(j)
\]

Makespan

\[
\text{Makespan} = \max_{j \in M} \left( \sum_{i \in T_j} \text{Computation}(i, j) \right) \quad \forall j \in M
\]

We have to find:

a) Encoding, b) Constraints, c) crossover/mutation, d) environmental selection
Solution Problem 5.1

Multi-objective Problem

\[
\min f_1(x) = x^2 \\
\min f_2(x) = (x - 5)^2 \\
x \in [-10, 10]
\]

Solution Sets

\[S_1 = \{9, 3, 8\}\]
\[S_2 = \{1, 2, 7\}\]

a) Construct preordering/dominance imposed by the two objectives on each solution set. Which set has higher number of pareto-dominated solutions?

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Obj 1: (x^2)</th>
<th>Obj 2: ((x-5)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>81</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>4</td>
</tr>
</tbody>
</table>

3<8<9: 2 Pareto-dominated solutions

No pareto-dominated solutions
b) Propose a single reference point for computing hypervolume indicator
We can choose the highest value (worst possible solution) as a reference point: (100, 225)

c) Compute the hypervolume of each set

\[
\text{For } S_1: \text{ area } = (100 - 9) \times (225 - 4) = 20,111.
\]
\[
\text{For } S_2: \text{ area } = (100 - 49) \times (225 - 4) + (49 - 4) \times (225 - 9) + (4 - 1) \times (225 - 16) = 21,618
\]

d) Which set is better?
\( S_2 \) is better due to higher hypervolume
Solution Problem 5.2

Solve Problem 5.1 using an evolutionary algorithm

Algorithm 1: My First Evolutionary Algorithm

<table>
<thead>
<tr>
<th>Data: Solution set $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result: Optimized solution set $S$</td>
</tr>
<tr>
<td>repeat</td>
</tr>
<tr>
<td>Delete from $S$ all pareto-dominated solutions;</td>
</tr>
<tr>
<td>$S' \leftarrow$ set of numbers by averaging each pair of values in $S$;</td>
</tr>
<tr>
<td>$S'' \leftarrow S' \cup S'$;</td>
</tr>
<tr>
<td>Delete from $S''$ all pareto-dominated solutions;</td>
</tr>
<tr>
<td>if $F_H(S'') \geq F_H(S)$ then</td>
</tr>
<tr>
<td>$S \leftarrow S''$;</td>
</tr>
<tr>
<td>end</td>
</tr>
<tr>
<td>until number of iterations $\leq 10$;</td>
</tr>
</tbody>
</table>

\[ S_1 = \{9, 3, 8\} \]
\[ S_2 = \{1, 2, 7\} \]

(a) Apply one iteration of the algorithm to solution sets $S_1$ and $S_2$.

$S_1$: 9 and 8 are removed since they are Pareto-dominated. Thus $S_1' = \emptyset$

$S_2$: $S_2' = \{1.5, 4.5, 4\}$  $S_2'' = \{1, 1.5, 2, 4, 4.5, 7\}$
7 is now dominated by 4 and 4.5. therefore 7 is removed.

We now compute hypervolume for $S_2''$

$S_2 = \{1, 1.5, 2, 4, 4.5\}$ at the end of iteration
b) Algorithm 1 will not find the Pareto-optimal solution for either S1 and S2. Explain Why.

Neighborhood operator does not cover the entire optimization space
S₁: 3 is the solution
S₂: Solution is upper bounded by 7 and lower bounded by 1
Will never find 0 which is a pareto optimal solution

c) How would you modify Algorithm 1 to find Pareto-optimal solution. Can you identify Pareto-optimal solutions?

Add a mutation operator. e.g. adding a random number to the set. All solutions from [0, 5] are Pareto-optimal
Task partitioning problem

We are given a set of tasks and a set of cores. Tasks have different computation times and power dissipations depending on the core they are executed on. The problem has the following objectives:

**Energy Consumption**

\[
\text{Total Energy} = \sum_{j \in M} \sum_{i \in T_j} \text{Computation}(i, j) \times \text{Power}(j)
\]

**Makespan**

\[
\text{Makespan} = \max_{j \in M} \left( \sum_{i \in T_j} \text{Computation}(i, j) \right) \quad \forall j \in M
\]

a) **Encoding:**

Assume 5 tasks and 2 cores

Core on which task 1 is executed

b) **Constraints:**

No constraints necessary for this encoding
Problem 5.3

c) Neighbourhood operations:

Crossover:

Mutation: Change the assignment of a random task

d) Environmental selection:

Any standard approach can be used - for instance based on the hypervolume indicator as discussed in the lecture.