Time-Optimal Information Exchange on Multiple Channels

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Problem: 

\[ n := \# \text{ nodes} \]
Problem:

$n := \# \text{nodes}

k := \# \text{information}

Have information

Disseminate to all!
Problem:

Disseminate to all!
Problem:

Disseminate to all!
Easy: $O(n)$
Faster?
Problem:

\[ n := \# \text{nodes} \]

Unique IDs 1…n
I can:

- send / receive
- reach each node

Time-Optimal Information Exchange on Multiple Channels
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- send / receive
- reach each node

Time-Optimal Information Exchange on Multiple Channels
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I can:

- send / receive
- reach each node
- no collision detection
I can:

- send / receive
- reach each node
- no collision detection
- switch channels

switch channels:
- 101 Mhz
- 117 Mhz
- 132 Mhz
- ...
I can:

- send / receive
- reach each node
- switch channels
- no collision detection

Complexity:
- computation: free
- radio: time 1

Synchronous
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)

One Information / \( \log n \) bits per message

\( \Rightarrow \Omega(k) \)
Time-Optimal Information Exchange on Multiple Channels

\[ n = \text{# nodes} \]
\[ k = \text{# information} \]

\[ \Theta(k) \]

\[ \Omega(k + \log n) \]

Multi channel

One channel

[Kushilevitz, Mansour SIAM JComp 1998]
Time-Optimal Information Exchange on Multiple Channels

$n$: # nodes
$k$: # information

What can I do?
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)

Communicate in parallel on different channels
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Communicate in parallel on different channels
Time-Optimal Information Exchange on Multiple Channels

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Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)

\( k := \# \text{ information} \)

\[ \text{TREE} \]
Time-Optimal Information Exchange on Multiple Channels

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Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)

\( O(k + \log n) \)
**Time-Optimal Information Exchange on Multiple Channels**

n:= # nodes
k:= # information

What if k < log n?

\[ O(k) \text{ if } k > \log n \]
Time-Optimal Information Exchange on Multiple Channels

$n$: number of nodes
$k$: number of information transfers

What if $k < \log n$?
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{nodes} \]
\[ k := \# \text{information} \]

Assume: \( k \) known
Time-Optimal Information Exchange on Multiple Channels

\(n:=\# \text{ nodes}\)
\(k:=\# \text{ information}\)

Assume: \(k\) known
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{ nodes} \]
\[ k := \# \text{ information} \]
Assume: \( k \) known

\[ \text{ID} = 1 \ldots 2^k \]
n:= # nodes
k:= # information
Assume: k known

ID=1…2^k

Balls into Bins
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information
Assume: k known

ID=1...2^k

○ Listens on channel 1
○ Listens on channel 2
○ Listens on channel 3
○ Listens on channel 4

Send on random channel 1...2^k
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{ nodes} \]
\[ k := \# \text{ information} \]
Assume: \( k \) known

\[ \text{ID}=1\ldots2^k \]
- Listens on channel 1
- Listens on channel 2
- Listens on channel 3
- Listens on channel 4

Send on random channel 1\ldots2^k
**Time-Optimal Information Exchange on Multiple Channels**

\( n := \# \text{ nodes} \)  
\( k := \# \text{ information} \)  
Assume: \( k \) known

\[ \text{ID} = 1 \ldots 2^k \]

\[ \Pr \text{ no collision} \]

**Balls into Bins**
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information
Assume: k known

ID=1…2^k

Pr no collision

Balls into Bins
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{ nodes} \]

\[ k := \# \text{ information} \]

Assume: \( k \) known

\[ \text{ID} = 1 \ldots 2^k \]

Pr no collision

Balls into Bins
Time-Optimal Information Exchange on Multiple Channels

$n := \# \text{nodes}$

$k := \# \text{information}$

Assume: $k$ known

$ID = 1 \ldots 2^k$

Repeat $k$ times

$Pr \text{ no collision}$
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)
Assume: \( k \) known

ID = 1 \ldots 2^k

Repeat \( k \) times

Pr [no collision] > 1 - \( \frac{1}{n-c} \)
If \( k > \sqrt{\log n} \)

2^k channels

\( O( k ) \) if \( k > \sqrt{\log n} \)
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information
Assume: k known

Size: $\sqrt{n \log n}$

Time: $O(k)$

What if $k < \sqrt{\log n}$?
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{nodes} \]

\[ k := \# \text{information} \]

Assume: \( k \) known

Size: \( \sqrt{n \log n} \)
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)
Assume: \( k \) known

\[ \text{Size: } \sqrt{n \log n} \]
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information
Assume: k known

Size: $\sqrt{n \log n}$

Send on random channel $\in \{1, \ldots, 2^{\frac{\log n}{2k}}\}$. 
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{ nodes} \]
\[ k := \# \text{ information} \]
\text{Assume: } k \text{ known}

\[
\text{Size: } \sqrt{n \log n}
\]

Send on random channel \( \in \{1, \ldots, 2^{\frac{\log n}{2k}}\} \).
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information
Assume: k known

Size: $\sqrt{n \log n}$

map: $\{1, \ldots, \sqrt{n \log n}\}$ $\rightarrow$ subsets of $\left\{1, \ldots, 2^{\frac{\log n}{2k}}\right\}$
of size at most k

Pr[at most half messages collide] $> 1 - \frac{1}{n-c}$

Send on random channel $\in \{1, \ldots, 2^{\frac{\log n}{2k}}\}$.

Unique Subset

Not too big …
Not too small …
Just right!
Time-Optimal Information Exchange on Multiple Channels

$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: $k$ known

Example: 3 channels

- Channel 1
  - Unique Subset
  - Subsets: $\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}$

- Channel 3

Unique Subset
Time-Optimal Information Exchange on Multiple Channels

$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: $k$ known

Example: 3 channels

- Send $k$ times
- Channel 1
- Channel 3
- Unique Subset

$\{1\}$
$\{2\}$
$\{3\}$
$\{1,2\}$
$\{1,3\}$
$\{2,3\}$
Time-Optimal Information Exchange on Multiple Channels

$n$: # nodes
$k$: # information
Assume: $k$ known

Example: 3 channels

Send $k$ times

Channel 1

Channel 3

Unique Subset
Time-Optimal Information Exchange on Multiple Channels

\[ n := \# \text{nodes} \]
\[ k := \# \text{information} \]
Assume: \( k \) known

Example: 3 channels

Send \( k \) times

Unique Subset
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
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Assume: \( k \) known

Example: 3 channels

Send \( k \) times

Channel 1

Channel 3

Unique Subset
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
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Assume: \( k \) known

Example: 3 channels

Send \( k \) times

Unique Subset
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\[ n := \# \text{nodes} \]
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Example: 3 channels

Send \( k \) times

Unique Subset
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\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)
Assume: \( k \) known

Example: 3 channels

Send \( k \) times

Channel 1

Channel 3

Unique Subset
Time-Optimal Information Exchange on Multiple Channels

n:= # nodes
k:= # information
Assume: k known

Example: 3 channels

\[ \Pr[\text{at most half messages collide}] > 1 - \frac{1}{n^{-c}} \]

\[ \mathcal{O}(k) \]
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)
Assume: \( k \) known

**Example: 3 channels**

\[ \Pr[\text{at most half messages collide}] > 1 - \frac{1}{n^c} \]

\[ O( k + \frac{k}{2} + \frac{k}{4} + \ldots ) \]
Time-Optimal Information Exchange on Multiple Channels

\( n := \# \text{ nodes} \)
\( k := \# \text{ information} \)
Assume: \( k \) known

Example: 3 channels

\[
\Pr[\text{at most half messages collide}] > 1 - \frac{1}{n^{-c}}
\]

\[
2^{\frac{\log n}{2k}} = \sqrt{n} \text{ channels}
\]

\( O(k) \quad \Pr[\text{this works}] > 1 - \frac{1}{n^{-c}} \)
Time-Optimal Information Exchange on Multiple Channels

$n := \# \text{nodes}$

$k := \# \text{information}$

Assume: $k$ known

$\sqrt{\log n} \leq k < \log n$

$\log n \leq k$
Time-Optimal Information Exchange on Multiple Channels

\[ n := \text{\# nodes} \]
\[ k := \text{\# information} \]
Assume: \( k \) known unknown

- \( \sqrt{n} \) channels
- \( 2^k \) channels
- \( n \) channels
- \( n \) channels

**Unique Subset**

**Balls into Bins**

**TREE**

\[ k < \sqrt{\log n} \]
\[ \sqrt{\log n} \leq k < \log n \]
\[ \log n \leq k \]

Future directions: deterministic less channels lower bounds
Time-Optimal Information Exchange on Multiple Channels

in Summary ...

Detect / Disseminate Information!

\[ \Theta(k) \]

101 Mhz
117 Mhz
132 Mhz
...

\{1,3\}
Thank You!
Questions & Comments?

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