Networks Cannot Compute Their Diameter in Sublinear Time

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Distributed network
Graph $G$ of $n$ nodes
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Local information only
Distributed network
Graph $G$ of $n$ nodes

Slide inspired by Danupon Nanongkai
Distributed network
Graph $G$ of $n$ nodes

Local information only

?
Distributed network

Graph $G$ of $n$ nodes

Limited bandwidth

Local information only
Distributed network
Graph \( G \) of \( n \) nodes

Limited bandwidth

Synchronized

Internal computations negligible

Time complexity: number of communication rounds

Local information only
Distributed algorithms: a simple example
Count the nodes!
Count the nodes!

1. Compute BFS-Tree
Count the nodes!

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Count the nodes!

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Count the nodes!

1. Compute BFS-Tree
2. Count nodes in subtrees

Runtime: Diameter
Diameter of a network

- **Distance** between two nodes = Number of hops of shortest path
Diameter of a network

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Diameter of a network

- **Distance** between two nodes = Number of hops of shortest path
- **Diameter** of network = Maximum distance, between any two nodes
Diameter of a network

- Distance between two nodes = Number of hops of shortest path
- Diameter of network = Maximum distance, between any two nodes

Diameter of this network?
Fundamental graph-problems

- **Spanning Tree** – Broadcasting, Aggregation, etc.
- **Minimum Spanning Tree** – Efficient broadcasting, etc.
- **Shortest path** – Routing, etc.
- **Steiner tree** – Multicasting, etc.
- Many other graph problems.

Thanks for slide to Danupon Nanongkai
Fundamental graph-problems

- **Spanning Tree** – Broadcasting, Aggregation, etc.
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- Many other graph problems.

- Global problems: $\Omega(D)$
Fundamental graph-problems

- Maximal Independent Set
- Coloring
- Matching
Fundamental graph-problems

• Maximal Independent Set
• Coloring
• Matching

• Local problems:

runtime independent of / smaller than D
  e.g. $O(\log n)$
• Diameter appears frequently in distributed computing
Complexity of computing D?

Known:

- \( \Omega(D) \)
- \( \approx \Omega(1) \)

This talk:

- Even if \( D = 3 \)
- \( \Omega(n) \)
Networks cannot compute their diameter in sublinear time!
Diameter of a network

Diameter of this network?

- **Distance** between two nodes = Number of hops of shortest path
- **Diameter** of network = Maximum distance, between any two nodes
Networks cannot compute their diameter in sublinear time!
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Base graph has diameter 3
Networks cannot compute their diameter in sublinear time!
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

has diameter 3

2?
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

Has diameter 3 or 2?
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

**Now: slightly more formal**
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

Label potential edges
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

Label potential edges
Networks cannot compute their diameter in sublinear time!

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Label potential edges
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?
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Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

\( \Theta(n) \) edges
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

Same as “A and B not disjoint?”

$\Theta(n)$ edges
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

Same as “A and B not disjoint?”

\( A \subseteq [n^2] \)

\( B \subseteq [n^2] \)

\( \Theta(n) \) edges
Networks cannot compute their diameter in sublinear time!

“A and B not disjoint?”

A ⊆ \([n^2]\)

B ⊆ \([n^2]\)
Networks cannot compute their diameter in sublinear time!

Pair of nodes not connected on both sides?

Same as “A and B not disjoint?”

Communication Complexity
randomized: $\Omega(n^2)$ bits

$A \subseteq [n^2]$

$B \subseteq [n^2]$

$\Theta(n)$ edges

$\Omega(n)$ time

$\Omega(n)$ time
Diameter Approximation
3/2-ε approximating the diameter takes $\Omega(n^{1/2})$.

Extend

2 vs. 3

D = 9 base graph
$3/2 - \epsilon$ approximating the diameter takes $\Omega(n^{1/2})$

Extend

2 vs. 3

D = 9 base graph
3/2-ε approximating the diameter takes $\Omega(n^{1/2})$

D = 9 base graph
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Extend

2 vs. 3

$D = 9$ base graph
$3/2 - \varepsilon$ approximating the diameter takes $\Omega(n^{1/2})$

Extend

2 vs. 3

$D = 9$ base graph
3/2-ε approximating the diameter takes $\Omega(n^{1/2})$

Extend
2 vs. 3

D = 9 base graph
D = 7 good graph

Factor: 3/2
$3/2 - \varepsilon$ approximating the diameter takes $\Omega(n^{1/2})$

Extend

2 vs. 3

D = 9 base graph
D = 7 good graph

Factor: $3/2 - \varepsilon$
Technique is general
Technique is general
Technique is general

\[ f(G) \]
Technique is general

\[ f(G, ) \]
Technique is general
Technique is general

\[ f(G_a, G_b) \]
\[ f'(G_a, G_b) \]

\[ f(G) \]
Alice \quad Bob

\[ G_a \quad G_b \]

\[ \text{cut} \]

\[ f'(G_a, G_b) \]

\[ G_c \]
Alice

\[ a \]

\[ G_a \]

Bob

\[ b \]

\[ G_b \]

\[ f'(G_a, G_b) \]

\[ f(G_c) \]
Alice

\[ a \]

\[ \mathcal{G}_a \]

cut

Bob

\[ b \]

\[ \mathcal{G}_b \]

\[ g(a, b) \]

\[ f'( \mathcal{G}_a, \mathcal{G}_b ) \]

\[ f(\mathcal{G}) \]
Alice

Bob

\[ a \]

\[ b \]

\[ G_a \]

\[ G_b \]

\[ g(a,b) \]

\[ f'(G_a,G_b) \]

\[ \text{cut} \]

\[ \text{Time}(g) \]

\[ \text{Time}(f) \geq \frac{|\text{cut}|}{|cut|} \]
Summary

Diameter $\Omega(n)$

3/2-eps approximation takes $\Omega(n^{1/2})$

general technique
Thanks!