Distributed Medium Access Control with Dynamic Altruism *

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Abstract. In this paper, we consider medium access control of local area networks (LANs) under limited-information conditions as befits a distributed system. Rather than assuming "by rule" conformance to a protocol designed to regulate packet-flow rates (as in, *e.g.*, CSMA windowing), we begin with a non-cooperative game framework and build a dynamic altruism term into the net utility. Our objective is to define a utility model that captures more closely the expected behavior of users, which according to recent results from behavioral and experimental economics should include a conditionally altruistic dimension. The effects of our proposed dynamic altruism are analyzed at Nash equilibrium in the quasi-stationary (fictitious play) regime. We consider either power or throughput based costs, and the cases of identical or heterogeneous (independent) users/players.

Key words: MAC, game theory, altruism

1 Introduction

Flow and congestion control are fundamental networking problems due to the distributed, information-limited nature of the decision making process in many popular access technologies. Various distributed mechanisms have been implemented to cooperatively desynchronize demand, *e.g.*, TCP, ALOHA, CSMA. Typically, when congestion is detected, all end-devices are expected to slow down their transmission rates and then increase again slowly hoping to find a fair and efficient equilibrium.

The fact that this process is not incentive compatible (a user/player could selfishly benefit by not following the prescribed protocol) raises an important issue. Since users could have access to alternative implementations of the prescribed ("by rule") protocols, *e.g.*, ones that slow down less than they should, the result could be an unfair allocation or even congestion collapse (see *e.g.*, [11,38]).

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Note that experience with TCP has shown that developers do create versions of the protocol that depart from the standard, cooperative (by-rule) congestion-avoidance algorithm, like Turbo TCP. The fact that non-cooperative devices at the MAC level have not been as widespread is perhaps due to the increased difficulty of modifying lower-level networking drivers by users or third parties, but such modifications are possible. As more network “players” behave selfishly and thereby more significantly reduce the performance of the rest, the other players are increasingly incentivized to adopt selfish strategies themselves, potentially leading to deadlock.

To address this threat, there is a steadily growing literature that analyzes the equilibria of different distributed network resource allocation *games* [2,11,13,21–23,27,29,30,34,43]. Such models provide useful insights on the expected equilibria when users do have the option to choose alternative implementations of the MAC protocol and constitute a framework for devising and analyzing incentive mechanisms to encourage the behavior that would lead to the most desirable equilibria. For example, in a Markovian setting without fictitious play[2], [30] introduces a cooperation parameter (a probability to stop transmitting), and then follows a detection and punishment methodology regarding selfish behavior.

In addition, even when users do follow the prescribed protocol, game theoretical models could be used as analytical frameworks that enable more informed choices in the implementation of the corresponding flow and congestion control protocols (*e.g.*, by associating a utility function to end-devices, which can then be the basis of actions by rationally selfish players). To this end, for a random-access LAN, several authors have recently considered the problem of distributed optimization of a global objective (total throughput, social welfare) subject to a fairness constraint. For example, in [13], a utility function design problem is studied considering estimation errors of the network state.

Our work falls into the first category of game theoretic models, but is different than the typical approach in addressing potentially selfish behavior at the MAC layer. Our objective is to formulate a more realistic utility model that captures the dynamics of altruistic motivations, which do play an important role in shaping human behavior as demonstrated by research in the fields of behavioral and experimental economics. Our analysis could then lead to advanced incentive schemes, which instead of attempting to punish selfish behavior, it will aim to encourage altruistic behavior under certain conditions. For example, a possible realistic outcome could be the design of a high-level user interface which will allow users to set the urgency of their communication, and which will encourage them to assign lower priority to their traffic when there is evidence that other users generally are doing the same. If successful, such a mechanism will not only improve performance at any given moment but it will also allow certain users to increase their own throughput only when they really need it, improving this way also the efficiency of the system over time without the need for complex and unattractive pricing schemes (e.g., [21]).

[2] *i.e.*, without steady-state estimates of certain quantities.

As our main contribution, we formulate and analyze a novel CSMA medium access control game with conditionally altruistic players. Altruistic tactics in evolutionary/mean-field games have long been considered, see [19] as a recent reference. In networking, altruism has been modeled as a user's *statically* personalized weight on the utility of others in games of: network formation [3], packet forwarding in delay tolerant networks [20], routing [5, 42], and medium access control by us in [25].

However, as we argue in detail in the next section, such static altruistic models, although theoretically interesting, fail to capture important realistic attributes of altruistic behavior studied in the behavioral and experimental economics literature. In this paper, we formulate a fictitious play model where altruism by one user is based on perceived mean throughput of the other players modulated (*i.e.*, made "dynamic") by factoring the estimated mean total channel idle time. Unlike Heusee et al. [17], who propose a window-update algorithm that tries to directly minimize the average idle time of the channel, in our model users will use less than their "fair" share when they do not really need it, but under the constraint that others do the same. For example, large idle time may be a signal that competing devices are also behaving in a socially sensitive manner, expressing a cooperative "social norm." In this case, excessive altruism would result in an underused channel.

Note also that our system with heterogeneous users will respond similarly to a selfish user with low throughput demand and a more altruistic one with high throughput demand. Interestingly, as shown in previous studies for the same problem [25] and in more general settings [32] the existence of altruistic motivations among users does not always result in a better outcome. But we should stress that our objective is not to optimize the overall throughput of the system but to study the stable equilibria that altruistic devices could reach. Finally, we do not assume that the users share information and act in a coordinated fashion, *i.e.*, so as to form a player coalition.

This paper is outlined as follows. In Section 2, we give a brief background on altruistic behavior. A fictitious-play model with dynamic altruism for a slotted-ALOHA LAN is given in Section 3. In Section 4, variations of the LAN model are considered. Numerical studies are given in Section 5. Finally, in Section 6, we conclude with a summary and discussion of future work.

2 Background on altruistic behavior

Economists are often criticized for the common assumption that humans are "rational" (*i.e.*, purely self-interested), which leads to a pessimistic view of the outcome of various formulated game-theoretic models. In reality, many people act "altruistically", defined as an "unselfish concern for or devotion to the welfare of others" [3]. In fact, despite this selfishness stereotype certain branches of economics, such as behavioral and experimental economics, do incorporate

[3] See http://dictionary.reference.com/browse/altruism

social, cognitive, and psychological factors in their models of human behavior (see [12] for a historical overview), in a way not typically captured in cooperative game-theoretic frameworks.

Two common scenarios in which altruistic behavior consistently appears in experiments with real users is in ultimatum bargaining and public-good contribution games [26, 28]. Ultimatum bargaining reveals the altruistic instincts of humans in a resource-sharing problem in which one player decides how to share a fixed amount of money and the other decides whether to accept or reject sharing: here rejection leaves both with zero profit. Experiments show that people altruistically sacrifice their own profit to punish unfair decisions by others. Analysis of more traditional public-good experiments, where players determine their individual contribution toward the construction of a pure public good, similarly challenges the assumption that free riding is always the dominant strategy.

An important lesson of experimental economics is that altruism does not seem to be a static and hardwired characteristic of humans but depends on many aspects of the environment. In other words, the level of altruism of an individual is *dynamic* and could change over time depending on the context and the behavior of the group. Indeed, the cooperation rate in many experiments has been proven to be much higher if subjects know that there is a possibility of meeting the same partners again in future periods [15], when their perception on the overall level of altruism in their group is high [9], or even just by a positive framing of the experiment [14].

From these and many other contextual factors that can affect the cooperation levels in a group, social norms is perhaps the most influential (see [8, 36]) but complex to incorporate in a simple economic model. To this end, Fehr and Schmidt [16] have proposed a utility function to model the altruistic behavior of people in ultimatum experiments, which incorporates a measure of fairness (or "inequity aversion") in a static way, *i.e.*, its main parameters are indifferent to the dynamics of the system. As a more realistic but less tractable alternative, H. Margolis argues in favor of a more dynamic and complex model, called "neither selfish nor exploited" [31], which proposes a dual utility model which takes into account the history of one's actions, the current overall behavior, the effect of altruistic action, and the developed norms in a society.

When altruism can bring future concrete benefits, one could also see altruistic behavior as a long-term net utility optimization. A characteristic example is the notion of direct and indirect reciprocity and the related work in evolutionary game theory that tries to explain the source of cooperative behavior in nature [4, 35]. We leave to future work the study of such evolutionary extensions of the LAN systems we formulate below.

3 Slotted-ALOHA random-access LAN with dynamic altruism

3.1 Altruistic framework with power based cost and concave utility of throughput

In our scenario, the high complexity of human nature and the surrounding social environment plays a less important role since the cooperation game that we study is limited in time, the identity of the players are hidden, the stakes are relatively low, and the decisions of users are mediated through a programmed device. So, we propose to incorporate in a simple utility function the effect of the external manifestation of altruistic behavior, that is a *statistical norm* as termed in [31], or simply "what others do" [9]. To perceive this, the availability of reliable information about the group's statistical behavior is critical. Our use of the mean idle time per active player to determine the level of altruism in the system is realistic in terms of information availability since it can be easily measured by the different users, though, again, low demand could be mistakenly taken for altruistic behavior and vice versa.

Consider a slotted ALOHA random-access LAN wherein the $N \geq 2$ participating nodes control their access probability parameter, q. A basic assumption is that nodes' control actions are based on observations in steady-state, *i.e.*, "fictitious play" [10], resulting in a quasi-stationary dynamical system [21,43] based on the mean throughputs:

$$\gamma_i(\underline{q}) = q_i \prod_{j \neq i}(1 - q_j).$$

Another basic assumption in the following is that the source of a successful transmission is evident to all other participating nodes. We further assume that the degree of altruism α_i of each node i depends on the activity of the other users as:

$$\begin{aligned} \alpha_i(\underline{q}_{-i}) = \prod_{j \neq i}(1 - q_j) \; &= \; \frac{\gamma_i(\underline{q})}{q_i} \\ &= \gamma_i(\underline{q}) + \prod_{j}(1 - q_j), \end{aligned}$$

where the second term is just the mean idle time of the channel; thus, every node can easily estimate its (dynamic) altruism. By using its control action (strategy), q_i, each i seeks to maximize its *net* utility:

$$V_i(\underline{q}) = C_i \log(\gamma_i(\underline{q})) + A_i \alpha_i(\underline{q}_{-i}) \overline{\gamma}_{-i}(\underline{q}) - M_i q_i \tag{1}$$

where: the dynamic altruism factor α modulates the contribution of the mean service of all other players to the net utility of player i,

$$\overline{\gamma}_{-i}(\underline{q}) = \frac{1}{N-1} \sum_{j \neq i} \gamma_j(\underline{q}); \tag{2}$$

the utility derived by one's own throughput is modulated by a concave function [21, 22] as modeled here in the form of a logarithm (for tractability); and we have assumed a power based cost[4] Mq. Note that because we assume that the source of each successfully transmitted packet is evident to all nodes, each node i can easily estimate $\overline{\gamma}_{-i}$. Again, though each player i optimizes V_i in a non-cooperative fashion, the game is called altruistic to reflect the second term in (1).

In the following, we consider an *iterated* game where players pursue mixed strategies based on observations of throughput γ_i observed in steady-state. Note that if we further assume that nodes are aware of the C, M parameters of other nodes, then we can replace $\overline{\gamma}$ with the net utility of the other players as in [25] (particularly for throughput based costs $M\gamma$).

Proposition 1. *If the game is synchronous-play and all users i have the same (normalized) parameters*

$$c := C_i/M_i \ < \ 1 \text{ and } a := A_i/M_i,$$

then there is a symmetric Nash equilibrium $\underline{q}^* = q^*\underline{1}$, where $0 < q^* < 1$ is a solution to

$$f(q) \ := \ aq^2(1-q)^{2N-3} + q - c = 0. \tag{3}$$

Proof. When $q_i = q$ for all i, the first-order necessary conditions of a symmetric Nash equilibrium,

$$0 = \frac{\partial V_i}{\partial q_i}(q\underline{1}) \ = \ -\frac{M}{q} f(q),$$

i.e., equivalent to (3). Note that $f(0) = -c < 0$ and $f(1) = 1 - c > 0$, the latter by hypothesis. So, by the continuity of f and the intermediate value theorem, a root of f exists in $(0, 1)$.

All such solutions $q^*\underline{1}$ correspond to Nash equilibria because $\partial^2 V_i(\underline{q})/\partial q_i^2 = -C_i/q_i^2 < 0$ for all $i, \underline{q}$.

The following corollary is immediate.

Corollary 1. *There is a unique symmetric Nash equilibrium point (NEP) if* $\min_{q\in(0,1)} f'(q) > 0$ *(*i.e.*, f is strictly increasing), a condition on parameters N and a.*

Note that there may be non-symmetric Nash equilibria in these games, even for the case of homogeneous users, *e.g.*, [24]. Also, it is well known that Nash equilibria of iterative games are not necessarily asymptotically stable, *e.g.*, [1,40, 44]. In [21] for a slotted-ALOHA game with throughput based costs $M\gamma$, using a Lyapunov function for arbitrary $N \geq 2$ players, a non-cooperative two-player

[4] Power based costs are borne whether or not the transmission is successful.

ALOHA was shown to have two different interior[5] Nash equilibria only one of which was locally asymptotically stable (see also [33]).

For the stability analysis of our altruistic game, consider the discrete-time (n), synchronous-play gradient-ascent dynamics,

$$q_i(n) = \arg\max_{q_i} V_i(q_i; \underline{q}_{-i}(n-1)) \quad \forall i. \tag{4}$$

In a distributed system[6], the corresponding continuous-time Jacobi iteration approximation is:

$$\dot{q}_i(t) = \frac{\partial V_i}{\partial q_i}(\underline{q}(t)) \quad \forall i, \tag{5}$$

and is motivated when players take small steps toward their currently optimal response, *i.e.*, better-response dynamics [41]. That is, for positive step-size $\varepsilon \ll 1$ (5) approximates the discrete-time better-response dynamics,

$$q_i(n) = q_i(n-1) + \varepsilon \frac{\partial V_i}{\partial q_i}(\underline{q}(n)) \quad \forall i, \tag{6}$$

which is a kind of distributed gradient ascent. The Jacobi iteration is also motivated by the desire to take small steps to avoid regions of attraction of undesirable boundary NEPs, particularly those corresponding to the capture strategy ($q_i = 1$ for some i). Note that when more than one player selects this strategy, the result is a deadlocked "tragedy of the commons." Additionally the players avoid the opt-out strategy ($q_i = 0$ for some i). In summary, (6) represents a repeated game in which players adjust their transmission parameters q_i to (locally) maximize their net utility V_i.

To find conditions on the parameters of net utilities (1) for local stability of the equilibria, we can apply the Hartman-Grobman theorem [37] to (5), *i.e.*, check that the Jacobian is negative definite. The following proposition uses the conditions of [39] for stability (and uniqueness) for a special case.

Proposition 2. *In the case where players have the same parameters C and A, the symmetric NEP $q^*\underline{1}$ is locally asymptotically stable under the dynamics in (5) when the normalized parameters satisfy*

$$C > 2(N-1)A. \tag{7}$$

Proof. By [39], the result follows if the symmetric $N \times N$ matrix $H(\underline{q})$ is negative definite, where

$$H_{ij} = \frac{\partial^2 V_i}{\partial q_i \partial q_j} + \frac{\partial^2 V_j}{\partial q_j \partial q_i}.$$

First note that, for all i,

[5] *i.e.*, not including the stable boundary deadlock equilibrium at $\underline{q} = \underline{1}$.

[6] *cf.* Section 4.3 for a discussion of asynchronous play.

$$H_{ii}(\underline{q}) = -\frac{C}{q_i^2} \;<\; -C.$$

For $l \neq i$,

$$\begin{aligned}\frac{\partial^2 V_i}{\partial q_i \partial q_l} &= \frac{\partial}{\partial q_l}\left(\frac{C}{q_i} - A\alpha(\underline{q}_{-i})\frac{1}{N-1}\sum_{j\neq i} q_j \prod_{k\neq i,j}(1-q_k)\right) \\ &= A\prod_{j\neq i,l}(1-q_j)\frac{1}{N-1}\sum_{j\neq i} q_j \prod_{k\neq i,j}(1-q_k) \\ &\quad + A\alpha(\underline{q}_{-i})\frac{1}{N-1}\left(\sum_{j\neq i,l} q_j \prod_{k\neq i,j,l}(1-q_k) - \prod_{k\neq i,l}(1-q_k)\right).\end{aligned}$$

Now because $0 < q_i < 1$ for all i and the triangle inequality,

$$|H_{ij}(\underline{q})| \leq 2A \quad \forall j \neq i.$$

So, by the Gershgorin circle (disc) theorem (see p. 344 of [18]), all of $H(\underline{q})$'s eigenvalues are less than $-C+(N-1)2A$. So, if (7) holds, then all the eigenvalues of $H(\underline{q})$ are negative, and so $H(\underline{q})$ is negative definite.

3.2 The marginal effect of altruism

In this section, we will write q^* (of the symmetric NEP $q^*\underline{1}$ in symmetric users case) as a function of the normalized altruism parameter $a := A/M$, $q^*(a)$. Note that $q^*(0) = c := C/M$.

Recall that the total throughput for slotted ALOHA, $Nc(1-c)^{N-1}$, is maximal when $c = 1/N$. The maximum total throughput is $(1-1/N)^{N-1} \approx \mathrm{e}^{-1}$ for large N, *i.e.*, the maximum throughput per player is $1/(N\mathrm{e})$ in this *cooperative* setting *without* networking costs accounted for.

So, if $c > 1/N$, *i.e.*, total throughput is less than e^{-1} because of excessive demand (overloaded system), then a marginal increase in altruism from zero ($0 < a \ll 1$) will cause a marginal decrease in $q^* \downarrow 1/N$, resulting in an increase in throughput per user $\gamma \uparrow 1/(N\mathrm{e})$. Also, if $c < 1/N$, *i.e.*, total throughput is less than e^{-1} because of low demand (underloaded system), then a marginal increase in altruism from zero will again cause a marginal decrease in q^*, but here resulting in a decrease in throughput γ (further away from the optimum e^{-1}).

4 Model variations

In this section, we discuss model variations, which we subsequently analyze.

4.1 Throughput based costs

In [25] we considered throughput based costs with a static altruism parameter and with utility proportional to throughput. Instead of (1), for throughput based costs with dynamic altruism and utility being a concave (log) function of throughput, we can model the net utility as:

$$\tilde{V}_i(\underline{q}) = C_i \log(\gamma_i(\underline{q})) + A_i \alpha_i(\underline{q}_{-i})\overline{\gamma}_{-i}(\underline{q}) - M_i \gamma_i(\underline{q}). \tag{8}$$

Proposition 1 can easily be adapted for power based costs. Instead of (3), the first-order necessary condition for a symmetric Nash equilibrium $q\underline{1}$ under throughput based cost is

$$\tilde{f}(q) \ := aq^2(1-q)^{2N-3} + q(1-q)^{N-1} - c = 0. \tag{9}$$

All solutions q for (9) correspond to NEPs $q\underline{1}$ because $\partial^2 \tilde{V}_i(\underline{q})/\partial q_i^2 = -C_i/q_i^2 < 0$ for all $i, \underline{q}$ (as for power based cost). Note that $\tilde{f}(0) = \tilde{f}(1) = -c < 0$, so we cannot simply use the intermediate value theorem as we did for Proposition 1 to establish existence of a symmetric Nash equilibrium when $c < 1$. Here, existence requires

$$\max_{0<q<1} \tilde{f}(q) \geq 0, \tag{10}$$

a condition on N, c, a. Note that if the inequality in (10) strictly holds then there will be an even number of symmetric NEPs, again by the intermediate value theorem. If the maximum equals zero then there may be a unique symmetric NEP.

4.2 Proportional throughput utility

Suppose that utility is simply proportional to throughput and cost is power based, *i.e.*, the net utility is

$$\hat{V}_i(\underline{q}) = C_i \gamma_i(\underline{q}) + A_i \alpha_i(\underline{q}_{-i})\overline{\gamma}_{-i}(\underline{q}) - M_i q_i. \tag{11}$$

Note that the net utility $\hat{V}_i$ is *linear* in q_i (this would also be the case if throughput based costs were involved). This normally leads to candidate "bang-bang" Nash equilibrium play-actions, $q_i \in \{0,1\}$ for all players i; *i.e.*, the players are either out of the game ($q_i = 0$ if $\partial \hat{V}_i/\partial q_i < 0$) or are *all in* ($q_i = 1$ if $\partial \hat{V}_i/\partial q_i > 0$). Note that the latter play action, potentially leading to the deadlock of "tragedy of the commons", is *not* an equilibrium here because if $q_j = 1$ then $\partial \hat{V}_i/\partial q_i = -M < 0$ for all $i \neq j$.

It turns out that for our scenario, there is a symmetric interior equilibrium $q\underline{1}$ for the identical players case with $0 < q < 1$, *i.e.*, where

$$\begin{aligned} \hat{f}(q) &:= \tfrac{\partial \hat{V}_i}{\partial q_i}(q\underline{1}) \\ &= c(1-q)^{N-1} - aq(1-q)^{2N-3} - 1 \ = \ 0. \end{aligned} \tag{12}$$

If $c > 1$, $\hat{f}(0) = c - 1 > 0$ and $\hat{f}(1) = -1 < 0$ and so there is a solution to $\hat{f}(q) = 0$ for $0 < q < 1$ by the intermediate value theorem. It should be noted, however, that such an interior Nash equilibrium $q\underline{1}$ is not stable, *i.e.*, it's a saddle point in the domain $[0,1]^N$.

4.3 Asynchronous/Multirate Players

Asynchronous players were considered previously in [22] using the ideas from [6,7]. A very similar approach can be used to extend the results herein to account for the effects of asynchronous play. Numerical results for this case are given in Section 5.3 below.

5 Numerical studies

We performed numerical experiments for scenarios with power based costs, log-utility of throughput, and normalized utility parameter $c = 0.5$.

5.1 Comparing altruism and non-cooperation under identical users

We compare here the Nash equilibria under altruistic player action with equilibria in purely non-cooperative scenarios. For the purely non-cooperative scenario, *i.e.*, $a = 0$, the symmetric Nash equilibrium $q = c = 0.5$ is simply obtained by solving (3). For the scenarios with altruism, the normalized altruism parameter was taken to be $a = 20$. Recall that for static altruism, $\alpha \equiv 1$. At Nash equilibrium $\underline{q}^* = q^*\underline{1}$, the throughput ($\gamma^* = q^*(1-q^*)^{N-1}$) and utility (1) performance per user is given in the following table, in decreasing order of throughput.

Scenario	N	q^*	γ^*	V^*/M
Dynamic Altruism	4	.22	.1044	-0.36
Static Altruism	4	.16	.0935	0.53
Non-cooperative	4	.50	.0625	-1.89
Static Altruism	8	.28	.0277	-1.52
Dynamic Altruism	8	.50	.0039	-3.27
Non-cooperative	8	.50	.0039	-3.27

An interesting observation from this simple numerical study is that our dynamic altruism model does not perform well when contention is high, as it is the case for $N = 8$, under the assumed parameters. This is so because high contention is perceived as non-cooperation and is punished.

To improve the estimation of the altruism levels in the system, we could include in our model a measure on the expected congestion levels based on the number of users sharing the same channel and then estimate the current altruism level of the system as the deviation of this expected average idle time. Finally, if the level of altruism is too high, under either the static or dynamics mechanisms, the channel may be underused; in this case, the altruism parameters could be adjusted via an "evolutionary" process to avoid the waste of resources.

5.2 Players with different altruism parameters

Consider the game with power based costs. In this section, we consider players with different normalized altruism parameters a for $N = 3$ otherwise identical players with normalized parameter $c = 0.5$ associated with power-based cost. Specifically, the first player has a_1 ranging from 30 to 70, while the other two players both have $a = 50$. Note that changing a in this manner will result in changes in the NEP q^* and corresponding throughputs γ^* (and utilities V^* per user, as shown in the following table):

a_1	$q_1^*, q_2^* = q_3^*$	$\gamma_1^*, \gamma_2^* = \gamma_3^*$	$V_1^*, V_2^* = V_3^*$
30	$0.15, 0.10$	$0.13, 0.074$	$0.754, 2.37$
40	$0.12, 0.10$	$0.10, 0.080$	$1.40, 2.24$
50	$0.10, 0.10$	$0.083, 0.083$	$2.10, 2.10$
60	$0.091, 0.11$	$0.073, 0.087$	$2.79, 1.83$
70	$0.079, 0.11$	$0.063, 0.090$	$3.56, 1.82$

Following intuition, increased altruism, $a_1 > 50$, by player 1 resulted in lower throughput for him and higher throughput for the other two players. Similarly, decreased altruism by player 1, $a_1 < 50$, resulted in higher throughput for him and lower throughput for the other players.

5.3 Sizes of regions of attractions under different play-rates

In this section, we study how the volume of the regions of attractions of different equilibria are sensitive to players adopting different play-rates, while retaining our assumption of fictitious/quasi-stationary play. Consider the case of $N = 3$ players two of whom have the same play rate while the other adopts a play rate that is a multiple, r, of the other two. We consider the case of throughput based costs, *i.e.*,

$$\dot{\underline{q}}_i(t) = r_i \frac{\partial \tilde{V}_i}{\partial q_i}(\underline{q}(t)) \quad \forall i, \tag{13}$$

where $r_i = r$ for player $i = 1$, otherwise $r_i = 1$ and $\tilde{V}$ is given in Section 4.1. Numerically simulating (13) from different initial points chosen from a grid in the hypercube $[0, 1]^3$, we counted the number of initial points converging to a given NEP so as to estimate the volume of its region of attraction. Note that the introduction of such play-rate parameters r_i does not change the position of the NEPs. Using normalized parameters $a = 50$ and $c = 0.5$, the function $\tilde{f}$ whose roots are the NEPs is depicted in Figure 1. As can be seen from the following table, the region of attraction is very sensitive to r in the range 0.1 to 10.

Volume	NEP = $(0.1)\underline{1}$	NEP = $(0.75)\underline{1}$
$r = 0.1$	0.502	0.498
$r = 0.25$	0.507	0.493
$r = 1$	0.556	0.444
$r = 4$	0.839	0.161
$r = 10$	0.841	0.159

The results are again intuitive: a lower r effectively corresponds to a reluctance to be altruistic and thereby results in a smaller domain of attraction for the more altruistic Pareto equilibrium $(0.1)\underline{1}$ (corresponding to throughputs of $\gamma = 0.081$ and utilities $V = 1.94$, respectively compared to $\gamma = 0.047$ and $V = -1.43$ corresponding to $(0.75)\underline{1}$).

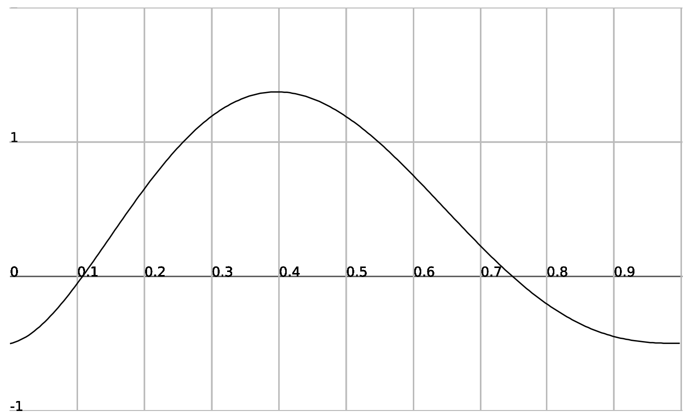

Fig. 1. Power based costs with $N = 3$, $a = 50$ and $c = 0.5$

6 Summary

In this paper, we extended a non-cooperative game framework for information-limited MAC of a LAN by adding an altruism term that depended on both the mean throughput of the other players and the mean channel idle time. The cases of heterogeneous or homogeneous users, and of power or throughput based costs, were considered for a quasi-stationary model of the game.

Our numerical studies produce intuitive results which means that our model is self-consistent and could form the basis for more sophisticated extensions. To this end, we plan to improve its realism, for example by departing from ideal quasi-stationary/fictitious-play dynamics and consider the effects of measurement error (as in [13, 33]), and study mechanisms that could motivate the users to adjust their behavior over time and reach better equilibria in terms of performance and/or efficiency. For example, the altruism parameter of a given user could be allowed to be configured according to the current mode of operation (*e.g.*, urgent versus low priority communication); then, an interesting evolutionary version of our game could be considered.

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