Worst-Case Optimal and Average-Case Efficient Geometric Ad-Hoc Routing

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Geometric Routing
Greedy Routing

- Each node forwards message to “best” neighbor
Greedy Routing

- Each node forwards message to “best” neighbor

- But greedy routing may fail: message may get stuck in a “dead end”
- Needed: Correct geometric routing algorithm
What is Geometric Routing?

• A.k.a. location-based, position-based, geographic, etc.

• Each node knows its own position and position of neighbors
• Source knows the position of the destination
• No routing tables stored in nodes!

• Geometric routing is important:
  – GPS/Galileo, local positioning algorithm,
    overlay P2P network, Geocasting
  – Most importantly: Learn about general ad-hoc routing
## Related Work in Geometric Routing

<table>
<thead>
<tr>
<th>Name</th>
<th>Venue</th>
<th>Algorithm</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kleinrock et al.</td>
<td>Various 1975ff</td>
<td>MFR et al.</td>
<td>Geometric Routing proposed</td>
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<tr>
<td>Kranakis, Singh, Urrutia</td>
<td>CCCG 1999</td>
<td>Face Routing</td>
<td>First <strong>correct</strong> algorithm</td>
</tr>
<tr>
<td>Bose, Morin, Stojmenovic, Urrutia</td>
<td>DialM 1999</td>
<td>GFG</td>
<td>First average-case <strong>efficient</strong> algorithm (simulation but no proof)</td>
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<tr>
<td>Karp, Kung</td>
<td>MobiCom 2000</td>
<td>GPSR</td>
<td>A <strong>new name</strong> for GFG</td>
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<tr>
<td>Kuhn, Wattenhofer, Zollinger</td>
<td>DialM 2002</td>
<td>AFR</td>
<td>First <strong>worst-case</strong> analysis. Tight $\Omega(c^2)$ bound.</td>
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<tr>
<td>Kuhn, Wattenhofer, Zollinger</td>
<td>MobiHoc 2003</td>
<td>GOAFR</td>
<td>Worst-case optimal <strong>and</strong> average-case efficient, percolation theory</td>
</tr>
</tbody>
</table>
Overview

• Introduction
  – What is Geometric Routing?
  – Greedy Routing

• Correct Geometric Routing: Face Routing

• Efficient Geometric Routing
  – Adaptively Bound Searchable Area
  – Lower Bound, Worst-Case Optimality
  – Average-Case Efficiency
  – Critical Density
  – GOAFR

• Conclusions
Face Routing

- Based on ideas by [Kranakis, Singh, Urrutia CCG 1999]
- Here simplified (and actually improved)
Face Routing

- Remark: Planar graph can easily (and locally!) be computed with the Gabriel Graph, for example

Planarity is NOT an assumption
Face Routing
Face Routing
Face Routing
Face Routing
Face Routing
Face Routing
Face Routing
Face Routing Properties

• All necessary information is stored in the message
  – Source and destination positions
  – Point of transition to next face

• Completely local:
  – Knowledge about direct neighbors’ positions sufficient
  – Faces are implicit

• Planarity of graph is computed locally (not an assumption)
  – Computation for instance with Gabriel Graph

“Right Hand Rule”
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• Conclusions
Face Routing

- Theorem: Face Routing reaches destination in $O(n)$ steps
- But: Can be very bad compared to the optimal route
Bounding Searchable Area
Adaptively Bound Searchable Area

What is the correct size of the bounding area?

- Start with a small searchable area
- Grow area each time you cannot reach the destination
- In other words, adapt area size whenever it is too small

→ Adaptive Face Routing AFR

Theorem: AFR Algorithm finds destination after $O(c^2)$ steps, where $c$ is the cost of the optimal path from source to destination.

Theorem: AFR Algorithm is asymptotically worst-case optimal.

[Kuhn, Wattenhofer, Zollinger DIALM 2002]
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• Conclusions
GOAFR – Greedy Other Adaptive Face Routing

- AFR Algorithm is not very efficient (especially in dense graphs)
- Combine Greedy and (Other Adaptive) Face Routing
  - Route greedily as long as possible
  - Overcome “dead ends” by use of face routing
  - Then route greedily again
- Similar as GFG/GPSR, but adaptive
Early Fallback to Greedy Routing?

- We could fall back to greedy routing as soon as we are closer to $t$ than the local minimum.
- But:

  - “Maze” with $\Omega(c^2)$ edges is traversed $\Omega(c)$ times $\rightarrow \Omega(c^3)$ steps.
GOAFR Is Worst-Case Optimal

• GOAFR traverses complete face boundary:

Theorem: GOAFR is asymptotically worst-case optimal.

• Remark: GFG/GPSR is not
  – Searchable area not bounded
  – Immediate fallback to greedy routing

• GOAFR’s average-case efficiency?
Average Case

- Not interesting when graph not dense enough
- Not interesting when graph is too dense
- **Critical density range** ("percolation")
  - Shortest path is significantly longer than Euclidean distance
Critical Density: Shortest Path vs. Euclidean Distance

- Shortest path is significantly longer than Euclidean distance

\[ \frac{|p^*|}{|st|} \]

- Critical density range mandatory for the simulation of any routing algorithm (not only geometric)
Randomly Generated Graphs: Critical Density Range

Network Density [nodes per unit disk] vs. Shortest Path Span

Connectivity
Greedy success
Shortest Path Span

\[ \frac{|p^*|}{|st|} \]

Critical density range

MobiHoc 2003
Average-Case Performance: Face vs. Greedy/Face

Network Density [nodes per unit disk] vs. Frequency

- Face Routing
- Greedy/Face Routing

Performance: better → worse

Critical point at 5 nodes per unit disk
Simulation on Randomly Generated Graphs

Network Density [nodes per unit disk]

Performance

GFG/GPSR

GOAFR+

better

worse

Critical

Frequency
## Conclusion

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<tr>
<th>Algorithm</th>
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<th>Avg-Case Efficient</th>
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<tr>
<td>Greedy Routing (MFR)</td>
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Questions?
Comments?
Demo?