How Does Randomized Beamforming Improve the Connectivity of Ad Hoc Networks?

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***Abstract*— This paper analyzes the impact of beamforming antennas on the topological connectivity of multihop wireless networks. As a metric for the connectivity of the network, we use the percentage $P(\text{path})$ of nodes that are connected via a multihop path. We show that simple randomized beamforming — i.e., each node adjusts its main beam into a randomly chosen direction for transmission and reception — significantly improves $P(\text{path})$ compared to networks with omnidirectional antennas employing the same power and sensitivity. The study is performed using accurate, analytical antenna models for uniform linear and circular antenna arrays. Already small arrays with four antenna elements give high gains of $P(\text{path})$. These gains are achieved although the nodes' average number of neighbors does not necessarily increase.**

***Index Terms*— Wireless multihop networks, beamforming, adaptive antennas, antenna arrays, connectivity, path probability.**

I. Introduction

THERE is an increasing interest in the application of beamforming antennas in wireless multihop ("ad hoc") networks. Most publications in this area, however, focus on MAC-layer issues. For example, it was shown that the distributed coordination function of IEEE 802.11 does not work properly, if beamforming antennas are employed [1]. Hence, several extensions to 802.11 have been proposed [1–11]. Further recent work addressed the design of neighbor discovery [12], routing [9], broadcasting [13], energy-efficient multicasting [14], and analytical work on throughput [15] and capacity [16, 17].

Surprisingly, only very little attention has been paid to another very fundamental issue: What is the impact of beamforming antennas on the *connectivity* of the resulting multihop network topology? There exist several papers on the connectivity of ad hoc networks, but they are all specific to omnidirectional antennas (e.g., [18, 19]). This is our motivation to analyze in detail whether, how, and why the application of beamforming antennas increases or decreases the level of connectivity of the resulting ad hoc network. Since ad hoc networks must operate completely decentralized and self-organizing, we believe that simple solutions are often promising. We are thus investigating a very simple form of beamforming, namely a randomized beamforming, where each node forms its main beam into a randomly chosen direction without any coordination with other nodes. We then study the level of connectivity resulting from this scheme and compare it to networks with usual isotropic antennas which radiate into all directions with the same power.

The remainder of this paper is structured as follows: Section II describes the used antenna model. Here, we apply two realistic and precise physical models for two well-known antenna types. Section III defines our model to create wireless links between nodes. Based on these modeling assumptions, Section IV represents the main part of this paper: it investigates in detail the connectivity properties of random topologies. We analyze the percentage of connected node pairs in a variety of scenarios and show that beamforming has the potential to make the network significantly better connected. Finally, Section V summarizes the main results and gives ideas for future research.

II. Antenna Model

A. General Definitions

Let us consider an antenna located in the origin of a spherical coordinate system. The angle from the x-axis in the xy-plane is $\phi \in [0, 2\pi[$; the one from the z-axis is $\theta \in [0, \pi[$. The following paragraphs give some definitions from antenna theory [20, 21].

The *radiation intensity* $u(\theta, \phi)$ of the antenna in a given direction (θ, ϕ) is defined as the radiated power per unit solid angle. The overall radiated power is given by the surface integral

$$p_t = \int_0^{2\pi} \int_0^{\pi} u(\theta, \phi) \, \sin\theta \, d\theta \, d\phi \,. \qquad (1)$$

An isotropic antenna has a constant radiation intensity $u = u_0 \forall(\theta, \phi)$, hence a radiated power of $p_t = 4\pi \, u_0$.

The *gain* of an antenna in the direction (θ, ϕ) is defined by

$$g(\theta, \phi) = \eta \, \frac{u(\theta, \phi)}{u_0} \,. \qquad (2)$$

It is the ratio between the radiation intensity in a given direction and the radiation intensity that would be obtained if the same power was radiated isotropically. The term η is a measure for the efficiency of the antenna. We assume lossless antennas and thus set $\eta = 1$. Combining these expressions yields

$$g(\theta, \phi) = \frac{u(\theta, \phi)}{\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} u(\theta, \phi) \, \sin\theta \, d\theta \, d\phi} \,. \qquad (3)$$

In this paper, we consider a network in two dimensions. Without loss of generality, we regard the yz-plane ($\phi = \pm\frac{\pi}{2}$).

Most publications on ad hoc networking with beamforming antennas use rather simple models to describe the gain pattern (see, e.g., [10]). It is typically assumed that the antenna forms a beam of width θ_w in the direction θ_b with a constant gain, i.e.,

$$g\left(\theta, \frac{\pi}{2}\right) = \begin{cases} \text{const} & \text{for } \theta_b - \frac{\theta_w}{2} \leq \theta < \theta_b + \frac{\theta_w}{2} \\ 0 & \text{otherwise} \end{cases} \,. \qquad (4)$$

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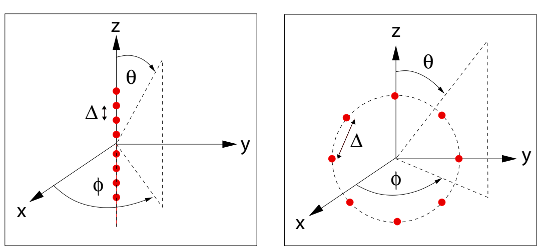
Such ideally sectorized patterns are useful in analytical studies and ease the implementation into simulation tools, but no real-world antenna can provide them. To overcome this drawback, we employ a more realistic model.

The beamforming antenna used in this paper is an antenna array consisting of m antenna elements. The antenna elements are assumed to be identical isotropic radiators, which transmit at a wavelength $\lambda = \frac{c}{f}$ with $c = 3 \cdot 10^8$ m/s and carrier frequency f. Beamforming is achieved by choosing the transmit power and phase of each antenna element. In this way, the radiation fields from the elements superimpose constructively (add) in some directions and superimpose destructively (cancel out) in other directions. We apply a pure phase shift beamforming: Each antenna element transmits with the same power p_t/m, and the beams are formed by choosing the phase shift between the elements, such that the resulting beam pattern achieves its maximum gain toward the intended direction. The value of $u(\theta, \phi)$ in a given direction depends on the geometric arrangement of the antenna elements.

B. *Uniform Linear Array*

A straightforward and commonly used arrangement is the placement of the elements along a line, with a distance Δ between two neighboring elements. This array type is called *uniform linear array* (ULA) and is illustrated in Figure 1(a). Each element i $(i = 1, \dots, m)$ has a progressive phase shift γ relative to the preceding element, i.e., $\gamma_i = \gamma_{i-1} + \gamma$.



(a) ULA (b) UCA

Fig. 1. Illustration of ULA and UCA. Definition of angles ϕ and θ.

Given this constellation, the radiation intensity fulfills [21]

$$u(\theta, \phi) \propto \left(\frac{1}{m} \frac{\sin(m\psi)}{\sin(\psi)} \right)^2 \tag{5}$$

with the auxiliary variable $\psi = \frac{1}{2} \left(\frac{2\pi\Delta}{\lambda} \cos\theta + \gamma \right)$. Due to the rotational symmetry of the setup, the intensity is independent of ϕ. The directions in which the radiation intensity achieves its maximum are called *boresight directions* θ_b. With the ULA, the maximum radiation intensity is achieved for

$$\theta_b = \pm \arccos \left(-\frac{\gamma\lambda}{2\pi\Delta} \right) . \tag{6}$$

In other words, if we intend to maximize the radiation intensity toward a destination node that is located in a certain direction θ, we must set the phase of the antenna elements to $\gamma = -\frac{2\pi\Delta}{\lambda} \cos\theta$. This insight enables us to express ψ as a function of the boresight direction θ_b, namely

$$\psi = \frac{\pi\Delta}{\lambda} \left(\cos\theta - \cos\theta_b \right) . \tag{7}$$

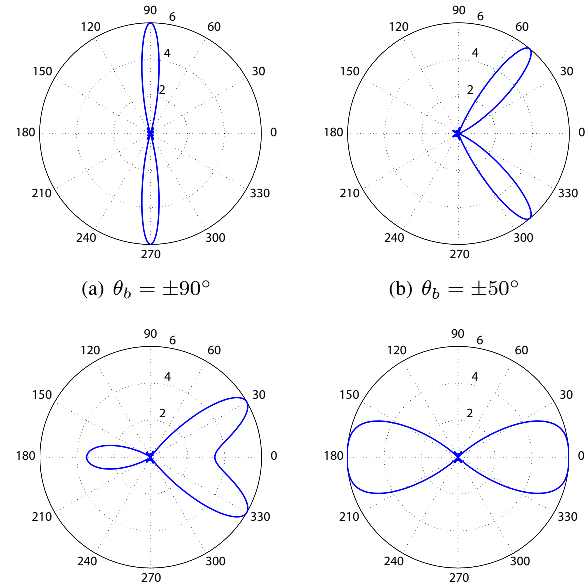
Finally, the antenna gain in the direction θ can be computed by

$$g(\theta) = \frac{\left(\frac{\sin(m\psi)}{\sin(\psi)} \right)^2}{\frac{1}{2} \int_0^{\pi} \left(\frac{\sin(m\psi)}{\sin(\psi)} \right)^2 \sin\theta \, d\theta} . \tag{8}$$

A common choice for the distance between the antenna elements is $\Delta = \lambda/2$. Applying this value in (8), the expression for the gain simplifies to

$$g(\theta) = \frac{1}{m} \left(\frac{\sin(m\psi)}{\sin(\psi)} \right)^2 . \tag{9}$$

This gain pattern is shown in Figure 2 for different values of θ_b. We observe two beams with high gain directed in the boresight directions (main beams) and a number of beams with very low gain (side beams). The gain of the main beams $g(\theta_b)$ is independent of the boresight direction; it is always equal to the number of antenna elements m. This is because $\psi_{|\theta=\theta_b} = 0 \Rightarrow \frac{\sin(m\psi)}{\sin(\psi)} \to m$. If the boresight direction is orthogonal to the linear arrangement of the nodes ($\theta_b = \pm 90°$), the main beams are rather narrow. For decreasing θ_b, the beamwidth increases, until two rather wide beams are obtained for $\theta_b = 0°$.

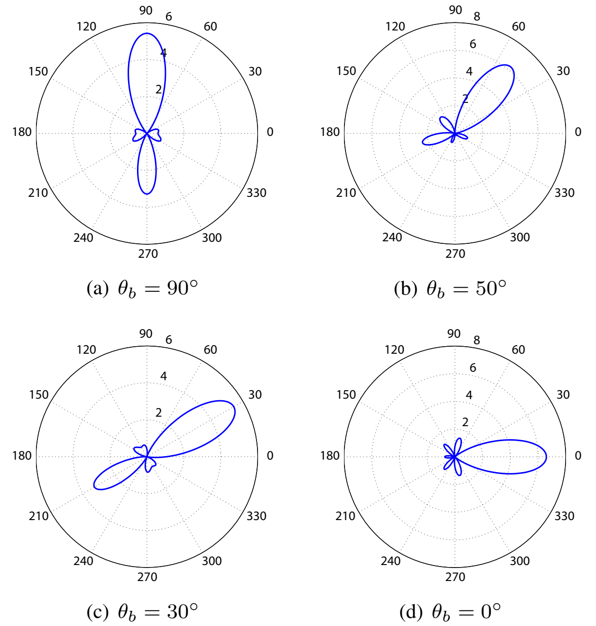


(a) $\theta_b = \pm 90°$ (b) $\theta_b = \pm 50°$ (c) $\theta_b = \pm 30°$ (d) $\theta_b = 0°, 180°$

Fig. 2. Gain patterns of ULA with $m = 6$ elements.

C. *Uniform Circular Array*

Another common array type is the *uniform circular array* (UCA). As shown in Figure 1(b), the m elements are now arranged on a circle, where the spacing between two neighboring elements is again given by Δ.



(a) $\theta_b = 90°$ (b) $\theta_b = 50°$ (c) $\theta_b = 30°$ (d) $\theta_b = 0°$

Fig. 3. Gain patterns of UCA with $m = 6$ elements.

The gain can be computed in a similar manner as for the ULA. Example patterns are shown in Figure 3. As opposed to the ULA, the UCA yields a single main beam, whose width is almost independent of its direction θ_b. The value of the maximum gain $g(\theta_b)$ now depends on θ_b; it can be higher as well as lower than m. The level of the side beams is stronger compared to the ULA; indeed, some side beams achieve a gain above 1, thus being stronger than an isotropic radiator.

III. Wireless Link Model

Given these models for ULA and UCA with their respective gain patterns, let us now define how to decide whether or not there is a wireless link between two given nodes. As illustrated in Figure 4, we assume that one of the nodes transmits a signal with power p_t that is received by the other node with power p_r. The gain of the antenna at the transmitting node in the direction toward the receiver is g_t. The above gain patterns can also be applied for reception. The gain of the receiver's antenna in the corresponding direction toward the transmitter is g_r.

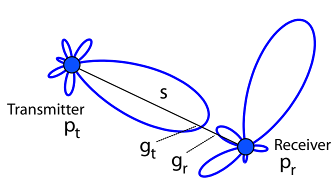


Fig. 4. Illustration of link model. Transmission and reception are directional.

Taking into account the two antenna gains as well as the path loss caused by the distance s between the nodes, we can write

$$\frac{p_r}{p_t} = g_t \, g_r \left(\frac{s}{1\,\text{m}} \right)^{-\alpha} , \tag{10}$$

where α is the *pathloss exponent* of the environment (e.g., $\alpha \approx 2$ in free space, $\alpha \approx 3$ in an urban outdoor environment). If p_r is larger than or equal to a certain threshold power p_{r0}, called receiver sensitivity, the transmitted signal is received properly. If so, the transmitter establishes a *link* to the receiver. In the following, we assume that all nodes have the same p_t and p_{r0}. Thus, all links can be considered as being bidirectional (undirected). Two nodes having a link between them are denoted as *neighbors* in the topology. The number of neighbors of a node is called its *degree* d. A node with degree $d = 0$ is isolated.

Typically, the inverse value of (10) is called *attenuation* a and is expressed in terms of decibel as $a = 10 \log \frac{p_t}{p_r}$ dB. Two nodes establish a link if the attenuation between them is smaller or equal to the threshold attenuation $a_0 = 10 \log \frac{p_t}{p_{r0}}$ dB.

IV. Analysis of Network Connectivity

A. Problem Statement

We are now interested in the question: Does beamforming improve the overall connectivity among nodes? To be more specific: if we compare a network in which all nodes are equipped with ULAs or UCAs, respectively, to a network in which all nodes have a single isotropic antenna with the same transmission power and sensitivity, can nodes in the first scenario find routes to more nodes (on average) than in the isotropic scenario? Clearly, the answer is "yes," if nodes have knowledge about the location of neighbors and can adjust a main beam into the correct direction. This requires, however, additional signaling for directional neighbor discovery and significant signal processing for direction estimation of incoming signals. Swiveling the beam from 0 to 2π would certainly increase the connectivity as well, but in turn, it would increase the interference among nodes and introduce delay. In this paper, we are interested in a much simpler way of beamforming, namely *random direction beamforming*: each node chooses a boresight direction θ_b from a uniform random distribution on $[0, 2\pi[$, completely independent of other nodes. We are then interested in the question: Does *random direction* beamforming improve the overall connectivity among nodes? An answer to this question is by far not obvious. Beamforming enables nodes to transmit to a higher distance, but this increase in transmission range is bounded to a certain direction. Hence, the nodes lose links to nodes located nearby and might end up being isolated if the main beam is too narrow.

As a measure for the level of connectivity we employ the *path probability* $P(\text{path})$. It is defined as the probability that two randomly chosen nodes in a random ad hoc network are connected via a multihop path or a direct link. We analyze this probability in the following scenario. From a set of n nodes, each node is placed randomly using a uniform distribution on a square system area of length l. The physical orientation of each node's antenna, with respect to the z-axis, is randomly chosen from a uniform distribution on the interval $[0, 2\pi[$; afterward, random direction beamforming is applied in each node to create links. Some example topologies are shown in Figure 5.

In the topology with isotropic antennas, each node has a link to all nodes that are located within a certain distance. In the topologies with beamforming antennas, some of these links are

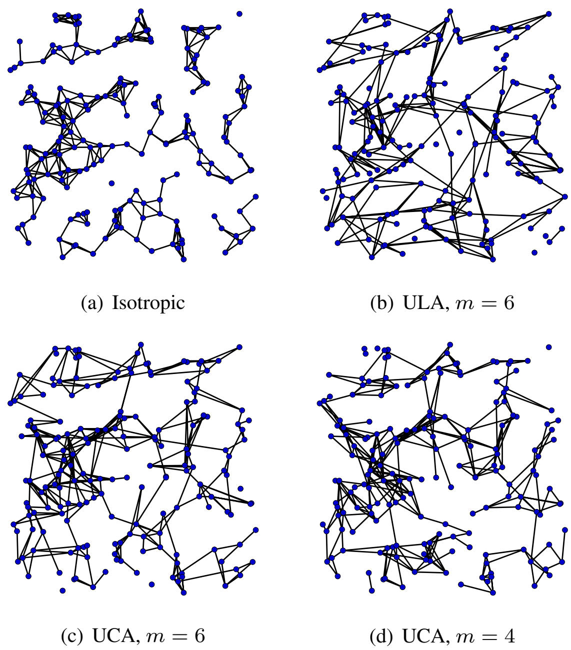


Fig. 5. Random topologies from $n = 180$ nodes distributed uniformly at random on $500 \times 500\,\mathrm{m}^2$ area with path loss exponent $\alpha = 3$ and $a_0 = 50\,\mathrm{dB}$.

taken away, but in turn some links to further away nodes are added. The network with isotropic antennas consists of seven connected components (including two isolated nodes). The networks with ULAs and UCAs contain a much larger connected component that spans almost all nodes of the network. All in all, the path probability in the three random beamforming topologies is higher than that of the isotropic one.

After this example, let us perform a simulation-based study with a large number of random topologies, in order to obtain an empirical value for the path probability $P(\text{path})$ as a function of the antenna type and the parameters n, m, a_0, α, and l. For given values of these parameters, we generate $\Omega = 10000$ topologies, each with independent random node placement, antenna orientation, and beamforming. For each of these topologies, we calculate the percentage of connected node pairs as

$$\frac{\text{\# connected node pairs}}{\text{\# node pairs}} = \frac{\sum_{i=1}^{\nu} \frac{1}{2}\, n_i\,(n_i - 1)}{\frac{1}{2}\, n\,(n-1)}\,, \qquad (11)$$

where ν denotes the number of connected components of the given network, and n_i denotes the number of nodes in the i-th connected component. The value of (11) is 1, if and only if the network is connected in a graph-theoretical sense. It is 0 if all n nodes are isolated. Taking the average of the path percentage over a large number of random topologies gives us the path probability $P(\text{path})$.

B. Results on Path Probability

Figure 6 shows the simulation results for different setups. Let us first interpret Figure 6(a), which compares the path probability of a network in which all nodes have isotropic antennas to one in which all nodes are equipped with a 6-element UCA that radiates to a random boresight direction θ_b. The threshold attenuation in both scenarios is $a_0 = 50\,\mathrm{dB}$ and the pathloss exponent is $\alpha = 3$. For low node density, the path probability is low, and there is no visible difference between the two antenna types. As the node density increases, however, the curve for the UCA achieves a higher $P(\text{path})$ for the same density. For example, using $n = 180$ nodes (as in Fig. 5), more than $85\,\%$ of all node pairs are connected in the UCA scenario, whereas the isotropic scenario yields only $55\,\%$. About 225 isotropic nodes would be needed to achieve $P(\text{path}) = 85\,\%$. This comparison shows that the gain in connectivity achieved by using simple random beamforming with UCAs is significant.

What happens if we decrease the number of antenna elements from $m = 6$ to 4, meaning that the antenna elements are arranged on a square of length $\lambda/2$ and occupy less space than the 6-element UCA? Figure 6(b) shows the result: there is no significant loss in the path probability compared to the 6-element UCA. Figure 6(c) shows another scenario in which the threshold attenuation has been reduced to $40\,\mathrm{dB}$. Also in this case, the network with UCAs achieves a desired path probability at significantly lower node densities. Figure 6(d) shows the result with a reduced pathloss exponent $\alpha = 2$ (free space). In this case, the relative difference between the beamforming and isotropic networks is even higher. For example, $n = 100$ isotropic nodes achieve a path probability of about $15\,\%$. The same number of UCA nodes leads to $85\,\%$ — a gain of over $450\,\%$. To achieve $85\,\%$ with isotropic antennas, we would require about 190 instead of 100 nodes. The fact that $\alpha = 2$ yields a higher relative improvement than $\alpha = 3$ can be made plausible with (10). Assuming the maximum possible distance between two linked nodes in the isotropic scenario is s_0, the maximum possible distance in the beamforming scenario becomes

$$s = s_0 \sqrt[\alpha]{g_t g_r}\,. \qquad (12)$$

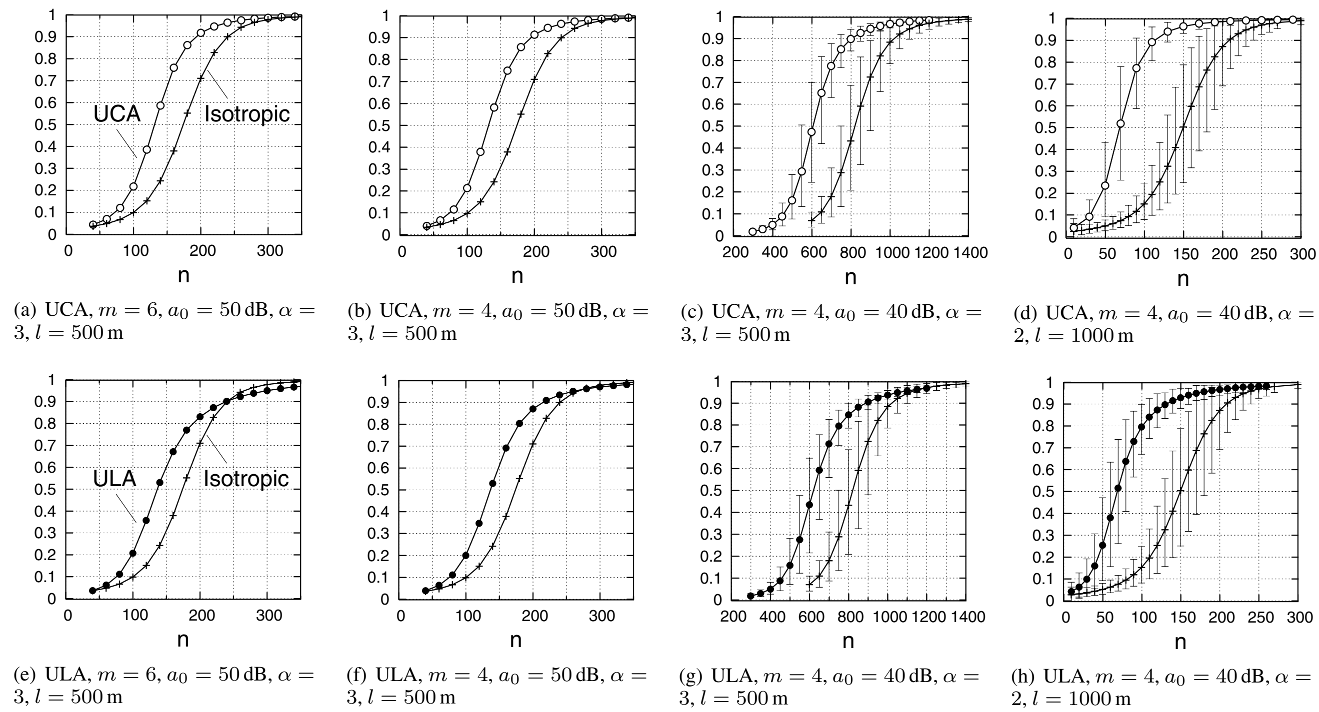
With increasing α, if $g_t g_r > 1$, the contribution of the antenna gains $g_t g_r$ to the achievable link length s decreases.

Figures 6(e)-(h) show the same simulation series for networks with ULAs. In all scenarios, the improvement in $P(\text{path})$ compared to isotropic antennas is slightly lower. In addition, for $P(\text{path})$ close to one, a ULA network performs slightly worse than one with isotropic antennas. This behavior can be explained with border effects: nodes located at the border of the system area may happen to steer their main beam(s) toward the area outside the system area. Due to the very low levels of the side beams, these nodes usually become isolated and thus contribute in a negative manner to $P(\text{path})$.

In summary, simple randomized beamforming with antenna arrays yields a significant improvement of the multihop connectivity compared to ad hoc networks with omnidirectional antennas. This improvement is already achieved for small arrays (here $m = 4$). In all simulated cases, the relative improvement decreases with increasing pathloss exponent α, and the UCA achieves a better path probability than the ULA.

C. Relation to Node Degree

The increased path probability is for sure a beneficial feature for ad hoc networks, since they can only operate if the connectivity among nodes is reasonably high. However, the following



(a) UCA, $m = 6$, $a_0 = 50\,\text{dB}$, $\alpha = 3$, $l = 500\,\text{m}$
(b) UCA, $m = 4$, $a_0 = 50\,\text{dB}$, $\alpha = 3$, $l = 500\,\text{m}$
(c) UCA, $m = 4$, $a_0 = 40\,\text{dB}$, $\alpha = 3$, $l = 500\,\text{m}$
(d) UCA, $m = 4$, $a_0 = 40\,\text{dB}$, $\alpha = 2$, $l = 1000\,\text{m}$
(e) ULA, $m = 6$, $a_0 = 50\,\text{dB}$, $\alpha = 3$, $l = 500\,\text{m}$
(f) ULA, $m = 4$, $a_0 = 50\,\text{dB}$, $\alpha = 3$, $l = 500\,\text{m}$
(g) ULA, $m = 4$, $a_0 = 40\,\text{dB}$, $\alpha = 3$, $l = 500\,\text{m}$
(h) ULA, $m = 4$, $a_0 = 40\,\text{dB}$, $\alpha = 2$, $l = 1000\,\text{m}$

Fig. 6. Path probability $P(\text{path})$ of n uniformly distributed nodes on $l \times l$ area with pathloss exponent α and threshold attenuation a_0. Comparison of networks with m-element antenna arrays with beamforming to a random boresight direction θ_b to networks with isotropic antennas. The results are based on $\Omega = 10000$ random topologies ($\Omega = 100000$ for (d) and (h)). The intervals show the 10 % and 90 % quantiles, i.e., the percentage of connected node pairs is within the given interval in 80 % of all simulated random topologies.

questions arise now: Is the increased path probability simply achieved because nodes have, on average, more neighbors (i.e., a higher degree d)? If we take the degree of a node as a simple measure for the level of possible interference that it causes, we can also ask: Is the increased path probability bought at the price of higher interference between the nodes?

To study these questions, we analyze the *average degree* $E\{d\}$ of the nodes in our random scenario. We measure the degree d of each individual node in all Ω topologies and finally compute the average over all nodes. Clearly, for large n, the value of $E\{d\}$ increases linearly with n. The simulation results are shown in Table I. They compare $E\{d\}/n$ of networks with isotropic, UCA, and ULA antennas for the same sets of parameters as used in the study of $P(\text{path})$. In the first setup, with pathloss exponent $\alpha = 3$ and $a_0 = 50\,\text{dB}$, the use of random direction beamforming decreases the average degree compared to isotropic antennas. Here, the ULA achieves the best results. The same results hold for $\alpha = 3$ and $a_0 = 40\,\text{dB}$, although the degree reduction is slightly less significant. The inverse behavior can be observed for a pathloss exponent $\alpha = 2$. Here, the gains in path probability were the highest but on the other hand $E\{d\}$ is increased as well.

In summary, we can state that the increased path probability is not necessarily a consequence of a higher node degree. In fact, this result is also apparent in Figure 5. The three beamforming topologies have obviously a higher path probability than the isotropic topology, while at the same time the average node degree appears to be less or at least the same.

Finally, we would like to mention that beamforming has also a negative side effect in our scenarios. As shown in Figure 7, it tends to increase the *percentage of isolated nodes* (i.e., the percentage of nodes with $d = 0$). The difference to isotropic nodes is especially visible for the ULA and in networks with relatively high connectivity. The primary reasons for this result are the above described border effect and the fact that nodes with a ULA have a relatively high isolation probability for boresight directions in which the main beams are very narrow.

TABLE I
AVERAGE NODE DEGREE

α	$a_0/\,\text{dB}$	l/m	antenna	m	$E\{d\}/n$
3	50	500	isotropic	1	0.0250
			UCA	4	0.0227 (−9 %)
			ULA	4	0.0207 (−17 %)
			UCA	6	0.0230 (−8 %)
			ULA	6	0.0185 (−26 %)
3	40	500	isotropic	1	0.00561
			UCA	4	0.00522 (−7 %)
			ULA	4	0.00480 (−14 %)
2	40	1000	isotropic	1	0.028
			UCA	4	0.0375 (+34 %)
			ULA	4	0.0355 (+27 %)

D. Main Reason for Improved Path Probability

So why does beamforming increase $P(\text{path})$, although it sometimes decreases the expected node degree and increases the node isolation probability? Let us again consider Figure 5.

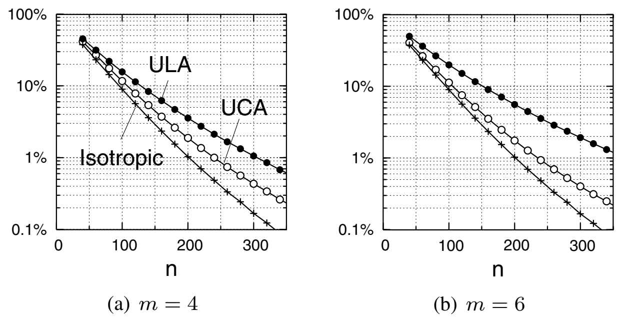


Fig. 7. Average percentage of isolated nodes $P(\text{node iso})$ in a network of n uniformly distributed nodes, $a_0 = 50\,\text{dB}$, $\alpha = 3$, and $l = 500\,\text{m}$.

On the one hand, a beamforming node "loses" links to closely located neighbors that are not within the main beam or a strong side beam. On the other hand, the beamforming creates links to nodes that are further away, i.e., out of the transmission range of an omnidirectional antenna with the power p_t. It is these long links that are important. They have the potential to "build the bridge" between previously isolated subnetworks.

This connectivity phenomenon is related to the ones reported in [22, 23]. In [22], Booth *et al.* realize that using anisotropic radiation patterns makes a multihop network percolate easier than using isotropic patterns. This means that a lower expected node degree is needed to achieve an infinitely large connected component on an infinite area. In [23], Bettstetter and Hartmann show that a high variance of the shadow fading tends to increase the probability that a wireless multihop network is connected in a graph-theoretical sense, i.e., has $P(\text{path}) = 1$.

We can now elaborate in more detail on our observation that a lower α leads to a higher relative improvement of $P(\text{path})$. Equation (12) reveals the following: On the one hand, the value of s/s_0 *increases* with decreasing α for node pairs with $g_t g_r > 1$. On the other hand, s/s_0 *decreases* with decreasing α for pairs with $g_t g_r < 1$. In other words: If α is low, the longest links in the beamforming scenario (links with $g_t g_r \gg 1$) are much longer than the links in the isotropic case. If α is high, however, they are only sightly longer. At the same time, pairs of nodes with $g_t g_r < 1$ suffer (benefit) from a low (high) α, but the links between those pairs are rather short anyway. The impact of the long links seams to overweight the impact of the short links, hence, the relative gain in $P(\text{path})$ is higher for low α.

V. Conclusions

The application of randomized beamforming leads to significant improvements in the multihop connectivity of ad hoc networks. Already a small number of antenna elements is sufficient to increase the path probability. These gains are achieved without any increment of the overall transmission power compared to a single isotropic antenna. In addition, it does not increase the average node degree in some scenarios, thus keeping the interference low. Most important, randomized beamforming does not require coordination among nodes, and it works fine without channel measurements or direction estimation. Hence, it is very practical and has low complexity with respect to protocols and signal processing.

In a further analysis we show to what extend the application of a neighbor discovery protocol for isolated nodes increases the connectivity further. In addition, we define a better metric for the interference among nodes and perform a deeper evaluation of the tradeoff between connectivity and interference.

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