Mechanism Design By Creditability

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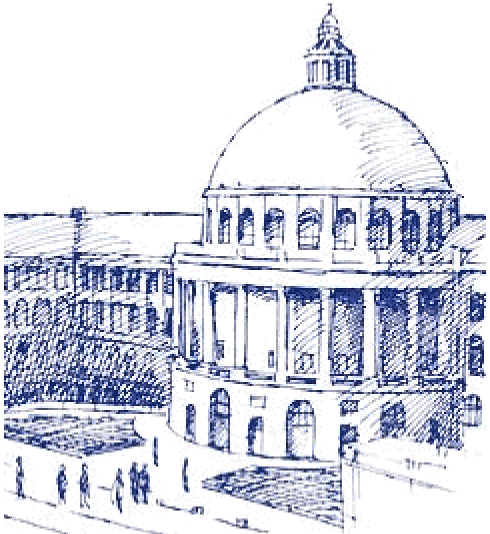
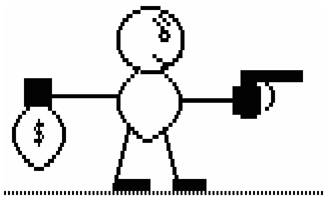


Conference on Combinatorial Optimization and Applications (COCOA)

Xi'an Jiaotong University, Xi'an, Shaanxi, China, August 2007

The Two Bank Robbers: Prisoners’ Dilemma



bank robber 2

		s	t
bank robber 1	s	3 3	0 4
	t	4 0	1 1

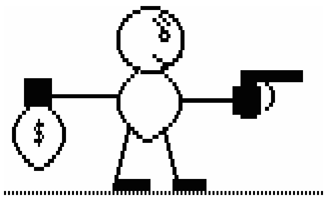
Utility: years saved

Strategies:
- s : be silent
- t : testify

dominant strategy profile

Nash equilibrium

The Two Bank Robbers: Al Capone



	s	t
s	3 3	0 4
t	4 0	1 1

Strategies:
- s : be silent
- t : testify

Al Capone pays

Assumption:

1 year in prison = 1 million ¥

	s	t
s	1 1	2 0
t	0 2	

Paying 2 million ¥ saves 4 years in prison!

Resulting game:

	s	t
s	4 4	2 4
t	4 2	1 1

Outline

- Game Theory Background
- Previous Work: k-Implementations
  [Monderer,Tennenholtz, EC 2003]
- Our Results: 0-Implementations,
  polynomial-time implementation algorithms,
  simulation, variations
- Conclusions

Game Theory Background

Formal modeling of social situations and analysis of rational behavior

Game $G = \langle N,X,U \rangle$

- Set of players: N
- Strategies: $X = X_1 \times X_2 \times \dots X_{|N|}$
- Utility functions: $U = (U_1, U_2, \dots, U_{|N|})$

  $$U_i = X \rightarrow \mathbf{R}$$
- Players are rational and select any *non-dominated* strategy

> x_i *dominates* y_i iff $U_i(x_i,x_{-i}) \geq U(y_i,x_{-i}) \quad \forall\, x_{-i} \in X_{-i}$
> and strict inequality holds for at least one x_{-i}.

- Set of non-dominated strategy profiles: X^*

Previous Work: k-Implementation Model

[Monderer,Tennenholtz, EC 2003]

Goal: Investigate influence of an interested third party in strategic games

How: offering payments to players depending on the game's outcome

- Game $G=<N,X,U>$
- Payments by third party : $V = (V_1, V_2, \ldots, V_{|N|})$

  $V_i = X \rightarrow \mathbf{R}^+$
- Resulting game: $G(V)=<N,X,[U+V]>$
- Target Set: $O \subseteq 2^{X_1} \times 2^{X_2} \times \ldots \times 2^{X_{|N|}}$

worst-case implementation cost

V *k-implements* O if $\emptyset \subset X^*(V) \subseteq O$ and $\max_{x \in X^*} \sum_{i \in N} V_i(x) \leq k$

V *k-implements* O *exactly* if additionally $X^*(V) = O$

Aim of 3rd party: Given O, minimize k

Previous Work: Related Results

[Monderer,Tennenholtz, EC 2003]

- Thm: Every strategy profile singleton o has an optimal implementation V with cost

  $k(o) = \sum_{i \in N} \max_{x_i \in X_i} (U_i(x_i, o_{-i}) - U_i(o_i, o_{-i}))$.

  Moreover, o is a Nash equilibrium iff o has a 0-implementation.

Generalization

- Thm: Computing optimal non-exact implementations NP-complete
- Thm: Computing optimal exact implementations in P
- Conjecture: Both problems NP-complete

Proof wrong

Algorithm wrong

Best Response Graphs

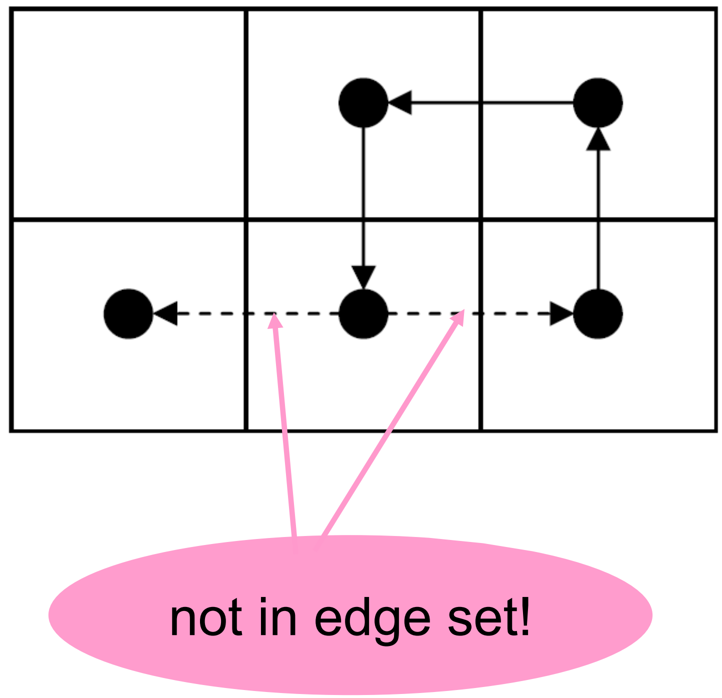
Relations between strategy profiles:

vertices v_x for $x \in X$ if x a best response for $\geq$ 1 player

directed edge $e = (v_x, v_y)$ if $\exists\, i \in N$ s.t. $x_{-i}=y_{-i}$ and

y_i the only best response for y_{-i}

5 / 4	5 / 5	5 / 1
10 / 10	10 / 0	0 / 10



0-implementations: Bankrupt Third Party

- Thm: If a O has an *exact* 0-implementation, the best response graph contains no edges out of O.

  For *non-exact* 0-implementations a subgraph without outgoing edges is required.

If |O|=1 and no outgoing edges then O is a Nash equilibrium.

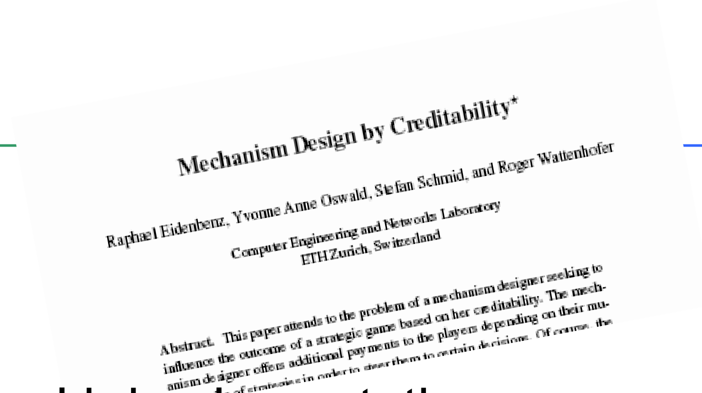
Generalization of singleton result, only a necessary condition

(unfortunately not sufficient)

- Algorithm for exact 0-implemenations

  Runtime $O(|N|\ |X|^2)$

More Results...


Mechanism Design by Creditability*

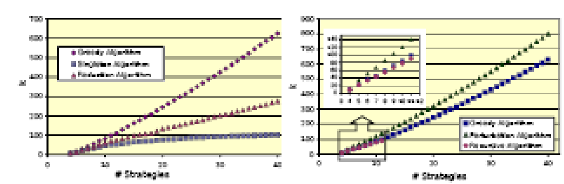
Raphael Eidenbenz, Yvonne Anne Oswald, Stefan Schmid, and Roger Wattenhofer

Computer Engineering and Networks Laboratory
ETH Zurich, Switzerland

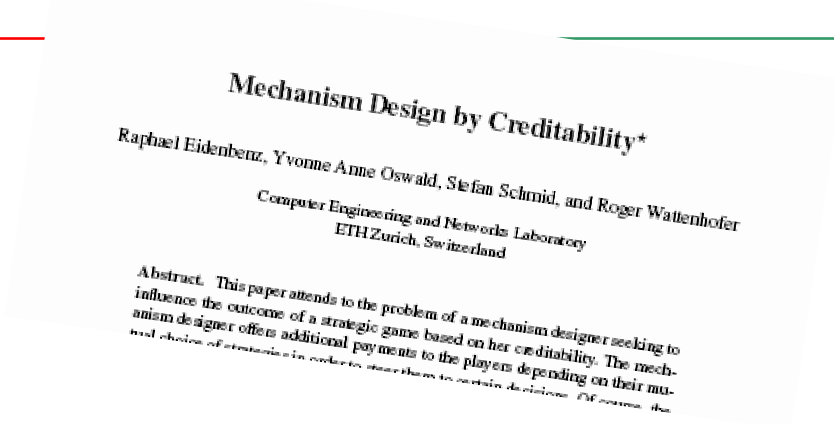
Abstract. This paper attends to the problem of a mechanism designer seeking to influence the outcome of a strategic game based on her creditability. The mechanism designer offers additional payments to the players depending on their mu-

- Correct algorithms for optimal k-implementations
  - Exact and non-exact case
  - Runtime exact: $O(|N||X|^2 + |N||O|\max_{i\in N}|O_i|^{|N|\max\{i\in N\}|X_i^*|})$
  - Runtime non-exact: even larger…

- Polynomial-time heuristics computing cheap implementations
  - Greedy algorithm
  - Greedy reduction algorithm
  - Simulations with random 2-player games



Even More Results...



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- Variations
  - Average payoff model

    every O 0-implementable
  - Risk-averse players

    polynomial-time optimal algorithms
  - Round-based mechanism

    every O 0-implementable in 2 rounds

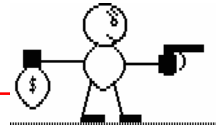
Players select strategy with maximal average payoffs

Players play strategy where the minimal gain is maximized

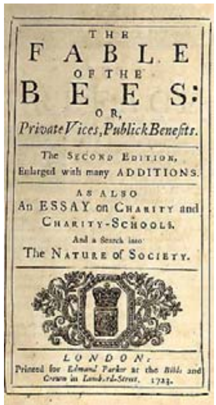
Players can change strategies in every round

3rd party offers payments in every round

Conclusions



Pay attention, conclusion in LNCS is wrong!



THE
FABLE
OF THE
BEES:
OR,
Private Vices, Publick Benefits.

The SECOND EDITION,
Enlarged with many ADDITIONS.

AS ALSO
An ESSAY on CHARITY and
CHARITY-SCHOOLS.
And a Search into
The NATURE of SOCIETY.

LONDON:

- Third parties can influence outcome of games with monetary incentives, sometimes even by mere creditablility

"Private Vices by the dextrous Management of a skilful Politician may be turned into Publick Benefits."
[Mandeville, Fable of the Bees, *1714*]

- Open questions:

Lower bound for time needed to compute optimal k-implementation?

To what extent can the outcome be manipulated?

Classes of games where optimal implementations can be determined efficiently?

That's it…

THANKS!

Questions?