Approximate Control Design for Solar Driven Sensor Nodes

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Abstract. This paper addresses power management of wireless sensor nodes which receive their energy from solar cells. In an outdoor environment, the future available energy is estimated and used as input to a receding horizon controller. We want to maximize the utility of the sensor application given the time-varying amount of solar energy. In order to avoid real-time optimization, we precompute off-line an explicit state feedback solution. However, it is a well-known problem of the optimal feedback solution that the computational complexity grows very quickly, which is particularly unfavourable for sensor nodes. A new method to derive approximate solutions to a multiparametric linear programming problem is presented. The resulting control laws substantially reduce the on-line complexity in terms of computational and storage demand. We show that a sensor node's performance is not necessary decreased due to suboptimality of the control design.

1 Introduction

Wireless sensor networks (WSN) have opened up an exciting field of research that is increasingly becoming popular nowadays. A WSN can be seen as a system of self-powered, wireless sensors which are able to detect and transmit events to a base station. Above all, sensor nodes are anticipated to be small and inexpensive devices which can be unobtrusively embedded in their environment. Thus, a sensor node's hardware is stringently limited in terms of computation, memory, communication as well as storable energy. These resource constraints limit the complexity of the software executed on a sensor node.

Recently, techniques to harvest energy via photovoltaic cells have received increasing attention in the sensor network community. A general approach to optimize the utilization of solar energy has been presented in [1]. The authors apply multiparametric linear programming to obtain a piecewise linear state feedback over a polyhedral partition of the state space. The optimization problem is basically solved off-line and look-up tables are stored and evaluated in the on-line case. For state explosions, which occur already for problems of moderate complexity, the limited storage capabilities of sensor nodes are quickly exceeded. Furthermore, the evaluation of the numerous states will cost considerable time as well as energy. In this paper, a new algorithm for approximate multiparametric linear programming is presented which generates much simpler look-up tables then the optimal solution. Another approximation method has been proposed, e.g., in [2].

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2 System Model

We restrict ourselves to the discussion of an example application which is modeled by the hierarchical control model illustrated in Fig. 1. The same control model has been used in [3]. It is introduced here since it significantly improves the robustness compared to a system with only one estimation and one control unit. The linear programs underlying subcontroller 1 and 2 have to be solved repeatedly, yielding a receding horizon control (RHC) strategy. For a specification of the linear programs and a detailed description of the system dynamics, the reader is referred to [4]. Note that the capability to bypass the storage device is a typical feature of latest prototypes. It offers the opportunity to save substantial energy by using the solar energy directly when available.

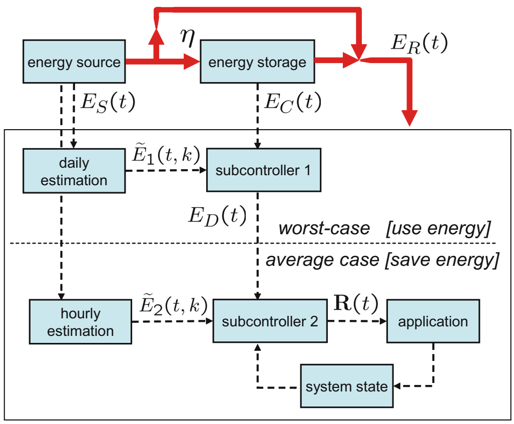

Fig. 1. Illustration of the hierarchical control model

3 Approximate Multiparametric Linear Programming

At first, we define a state vector $\mathbf{X}$ consisting of the actual system state, the level of the energy storage as well as the estimation of the incoming energy over the finite prediction horizon (cp. Fig.1). For subcontroller 1, e.g., the state vector $\mathbf{X}$ can be written as

$$\mathbf{X}(t) = \Big(E_C(t)\,,\, \widetilde{E}_1(t,0)\,,\, \ldots, \widetilde{E}_1(t,N-1)\Big)^T \tag{1}$$

where $\widetilde{E}_1(t,0)\,,\, \ldots, \widetilde{E}_1(t,N-1)$ denote the energy predictions for N time intervals. Furthermore, let us define the vector of optimal control inputs $\mathbf{U}^*(\mathbf{X})$, i.e., the vector of optimal rates $\mathbf{R}$ for the N prediction intervals.

In the following, we present a new algorithm for approximative multiparametric linear programming. The basic idea is

– to take a large number of samples $\mathbf{X}_i$ of the state space of $\mathbf{X}$,
– to solve a linear program for each sample $\mathbf{X}_i$ to get the optimal control $\mathbf{U}_i^*$,

– to find a (preferably simple) fitting function $\hat{\mathbf{U}}^*(\mathbf{X})$ for the multidimensional data $(\mathbf{X}_i, \mathbf{U}_i^*)$,
– and finally to use $\hat{\mathbf{U}}^*(\mathbf{X})$ (which has been calculated off-line) as approximation for $\mathbf{U}^*(\mathbf{X})$ in the on-line case.

At first, a random number generator is used to generate the samples $\mathbf{X}_i$, $1 \leq i \leq N_S$, where N_S denotes the total number of samples. We used independent, uniformly distributed random values as samples for the single elements of $\mathbf{X}$. As fitting algorithm, we opted for the algorithm proposed in [5]. This algorithm attempts to fit data samples to a set of convex, piece-wise linear candidate functions. The optimal control rates $\mathbf{U}^*(\mathbf{X})$ are not necessary convex over the state space $\mathbf{X}$. However, it has been shown that the optimal objective value $J^*(\mathbf{X})$ exhibits the wished convexity property. For each sample $\mathbf{X}_i$, we now solve a linear program and determine the optimal control vector $\mathbf{U}^*(\mathbf{X}_i)$ as well as the optimal objective value $J^*(\mathbf{X}_i)$. This can be done using common simplex-based or interior-point solvers. Next, we implement the heuristic algorithm in [5] to fit the objective $J^*(\mathbf{X_i})$, i.e. to solve the least square fitting problem

$$\text{minimize} \quad \sum_{i=1}^{N_S} \left(\max_{j=1,\ldots,\hat{N}_{CR}} (\hat{\mathbf{T}}_j^T \cdot \mathbf{X}_i + \hat{\mathbf{V}}_j) - J^*(\mathbf{X}_i) \right)^2 \tag{2}$$

We obtain the approximated objective function $\hat{J}^*(\mathbf{X}) = \max_{j=1,\ldots,\hat{N}_{CR}} \{\hat{\mathbf{T}}_j^T \cdot \mathbf{X} + \hat{\mathbf{V}}_j\}$ Next, we group the samples $\mathbf{X}_i$ according to the region j they belong to. For each region j, we perform a simple least square fitting of the respective samples to compute the coefficients $\hat{\mathbf{A}}_j$ and $\hat{\mathbf{B}}_j$ of the approximated control rates $\hat{\mathbf{U}}^*$. As a result, we have derived an explicit form for the control rates $\hat{\mathbf{U}}^*(\mathbf{X})$ as a function of the current state $\mathbf{X}$:

$$\hat{\mathbf{U}}^*(\mathbf{X}) = \hat{\mathbf{A}}_j \mathbf{X} + \hat{\mathbf{B}}_j \quad \text{if } \hat{\mathbf{H}}_j \mathbf{X} \leq \hat{\mathbf{K}}_j, j = 1, \ldots, \hat{N}_{CR} \tag{3}$$

Everything done so far has to be done off-line. The approximated control law in (3) can now be used in an on-line controller instead of the exact solution.

4 Simulation Results

Table 1 displays the evaluation of an approximate control law with $\hat{N}_{CR} = 4$ regions for both subcontroller 1 and subcontroller 2. In comparison, the optimal solution exhibits 30 and 161 critical regions, respectively. The primary optimization objective of regulating the sensing rate $\hat{r}_1$ is met almost as well as for the exact solution. We define the average efficiency η_{avg} of the energy utilisation as a metric to quantify the performance of the sensor node. Obviously, the approximated algorithm manages to save even slightly more energy then its exact counterpart. The stored energy $\hat{E}_C$ is varying up to 11.57% from E_C. However, the peak of $\hat{E}_C$ is just 4.03% above the one of E_C. That is, the capacity of the energy storage is required to be approximately 5% higher if the system is controlled by an approximated algorithm. Showing a comparable performance during runtime, the

Table 1. Comparison of multiparametric and approximate-mp control design, sc = subcontroller, storage in real numbers, ops in the worst case

control design	$\max_t \left\| \frac{\hat{r}_1(t)}{r_1(t)} - 1 \right\|$	$\max_t \left\| \frac{\hat{E}_C(t)}{E_C(t)} - 1 \right\|$	η_{avg}	N_{CR} (or $\hat{N}_{CR}$)	storage	ops
optimal, sc1	0%	0%	93.00%	30	1920	3689
sc2				161	2898	4829
approximate, sc1	1.52%	11.57%	93.75%	4	256	308
sc2				4	108	69
approximate, sc1	0.82%	5.47%	92.97%	4	256	308
sc2				9	243	173

main advantage of the approximation becomes obvious considering the complexity of the control laws. According to Table 1, the storage demand is significantly reduced by 92.44% compared to the optimal solution. In terms of worst case computation demand, the reduction even amounts 95.57%. Here, worst case refers to the situation where the currently active region is the last region to be tested. Table 1 also outlines the results for a second low-complexity approximation.

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