Anonymous Networks: Randomization = 2-Hop Coloring

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Anonymous Networks
Anonymous Networks
Maximal Independent Set
Coloring
Coloring $\rightarrow$ Maximal Independent Set
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Computability

Deterministic | Randomized | Impossible
Computability

D. Angluin.
Local and global properties in networks of processors (extended abstract).
*STOC*, 1980.
Computability

- Deterministic
- Randomized
- Impossible

- Maximal Independent Set
- Coloring
- Leader Election
Computability

Degree

Maximal Independent Set

Coloring

2-Hop Coloring

Leader Election

Deterministic

Randomized

Impossible
2-Hop Coloring?!
Theorem

Randomized Algorithm

Deterministic Algorithm

2-Hop Coloring

\[ \text{Randomized Algorithm} + \text{Deterministic Algorithm} = \text{2-Hop Coloring} \]
1. Obtain Local View
Recipe

1. Obtain
   Local View

2. Canonical Representation
Recipe

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   Local View

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Recipe

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1. Obtain Local View

2. Canonical Representation


4. “Lift” Output
Obstacles

1. Obtain Local View

Universal covers of graphs: Isomorphism to depth $\infty - 1$ implies isomorphism to all depths.

Obstacles

1. Obtain Local View

Obstacles

1. Obtain Local View

N. Norris.
Universal covers of graphs: Isomorphism to depth $n - 1$ implies isomorphism to all depths.
Obstacles

1. Obtain Local View

2. Canonical Representation

\[ n \text{ unknown!} \]

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N. Norris.
Universal covers of graphs: Isomorphism to depth \( n - 1 \) implies isomorphism to all depths.
Obstacles

1. Obtain *Local View*
2. Canonical Representation

\( n \) unknown! keep guessing (converges)

Obstacles

1. Obtain *Local View*

2. Canonical Representation

\[ n \text{ unknown!} \]

keep guessing (converges)

---

Obstacles

1. Obtain *Local View*
2. Canonical Representation

\[ n \text{ unknown!} \]

keep guessing (converges)

---

Obstacles

1. Obtain *Local View*

2. Canonical Representation

\( n \) unknown!

keep guessing (converges)

---

Obstacles

2. Canonical Representation
   Guess

3. Simulate Randomized Algorithm

4. "Lift" Output
Obstacles

2. Canonical Representation

Guess

Educated

3. Simulate Randomized Algorithm

4. “Lift” Output

Li/f_t Output

Educated
Obstacles

2. Canonical Representation
   Guess

3. Simulate Randomized Algorithm

4. “Lift” Output
   Stick to previously used random bits

Educated
Theorem

Randomized Algorithm

Deterministic Algorithm

2-Hop Coloring
Computability

Degree

Maximal Independent Set

Coloring

2-Hop Coloring

Leader Election

Deterministic

Randomized

Impossible
Leader Election

"Promise" must be decidable with anonymous algorithm.
Leader Election

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Leader Election

"Promise" must be decidable with anonymous algorithm.
Leader Election

▶ “Promise” must be *decidable* with anonymous algorithm
Recipe

1. Obtain Local View
2. Canonical Representation
4. “Lift” Output

...
Summary

Randomized Algorithm = Deterministic Algorithm

Degree

Maximal Independent Set

Coloring

2-Hop Coloring

Leader Election

Deterministic

Randomized

Impossible