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Resource Allocation Problems in Multifiber WDM Tree Networks

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Abstract

All-optical networks with multiple fibers lead to several interesting optimization problems. In this paper, we consider the problem of minimizing the total number of fibers necessary to establish a given set of requests with a bounded number w of wavelengths, and the problem of maximizing the number of accepted requests for given fibers and bounded number w of wavelengths. We study both problems in undirected tree networks $T = (V, E)$ and present approximation algorithms with ratio $1 + 4|E| \log |V|/OPT$ and 4 for the former and with ratio 2.542 for the latter. Our results can be adapted to directed trees as well.

Keywords: *algorithms, all-optical networks, path coloring, parallel fiber links, trees*

1 Introduction

Optical communication networks are the most promising technology for satisfying the ever-increasing demand for communication bandwidth. A single optical fiber allows transmission of data at rates up to several Terabits per second. *Wavelength-division multiplexing* (WDM) is used to partition the huge optical bandwidth into channels that operate at different wavelengths. Each channel can then carry data at the rate of several Gigabits per second, a speed that can still be handled by electronic network elements. If optical switches (crossconnects) are employed at the nodes of the network, signals can remain in optical form on the whole path from transmitter to receiver, without any need for intermediate

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conversion to or from electrical form. Such networks are called *all-optical* networks. Advantages of all-optical networks include transparent transport of data in arbitrary formats, low delays, low bit-error rates, and high transmission rates [14].

If a connection is to be established in an all-optical network, a path in the network from the transmitter to the receiver must be determined (*routing*) and a certain wavelength must be reserved for the connection on all links of that path (*wavelength assignment*) [25]. If wavelength conversion is not available, the reserved wavelength must be the same on all links of the path. Since wavelength converters are expensive devices that are not yet commonly available, we will consider only all-optical networks without wavelength converters in this paper.

As the number of wavelengths supported in an all-optical network is a scarce resource, graph-theoretical optimization problems modeling routing and wavelength assignment have been studied intensively over the last 5–10 years. Wavelengths are conveniently represented as colors and the routing and wavelength assignment problem is often treated as a path coloring problem: Given a set of terminal pairs in a graph, connect each terminal pair by a path and assign a color to each path such that no two paths with the same color pass through the same edge and such that the number of colors is minimized. We refer the reader to [3, 15, 12] for extensive surveys of known results for this problem.

More recently, researchers have begun to investigate optical networks with multiple parallel fibers between adjacent nodes [23, 18, 17]. In such networks, two paths on the link from u to v can use the same wavelength if they are carried on different fibers. At each node, the signal arriving on any fiber of an incoming link can be forwarded to an arbitrary fiber of an outgoing link, but without changing the wavelength. We model an all-optical network with multiple fibers as a simple graph $G = (V, E)$ with a function $\mu : E \to \mathbb{N}$, where $\mu(e)$ specifies the number of fibers on the link $e \in E$.

We consider all-optical tree networks with multiple fibers. Since the path for a terminal pair is uniquely determined in this case, we will mostly talk about paths instead of terminal pairs in the following. Three optimization problems arise naturally: minimizing the total number of fibers needed (for given paths in G and given number of available colors), minimizing the number of colors (for given paths in G and given μ), and maximizing the number of accepted paths (for given paths in G and given μ and number of available colors). We refer to these problems as MinCollisions-PMC, MinColors-PMC, and Max-PMC, respectively, where PMC is short for "path multicoloring." Formal definitions of the problems appear in Section 1.2. In this paper, we study MinCollisions-PMC and Max-PMC for tree networks, with the goal of devising efficient approximation algorithms. For each of the problems, we consider also the *uniform* variant where all edges have the same number of fibers and indicate this by using "UPMC" instead of "PMC" in the problem name.

1.1 Our Results

In Section 2, we consider the problem MinCollisions-PMC. First, we give an algorithm achieving a total number of fibers that is at most $OPT + 3|E|$ for undirected trees in which all paths touch the same node. Then we extend the algorithm to general sets of paths in trees, using at most $OPT + 4|E| \log |V|$ fibers in total. These algorithms perform well if the optimal solution is large. Furthermore, we show how to derive a 4-approximation algorithm for MinCollisions-PMC from an algorithm for MinColors-PMC given in [4]. In addition, we give an efficient algorithm for MinCollisions-UPMC that uses at most $\lceil 3OPT/2 \rceil$ fibers on every edge. We also obtain similar approximation results for the directed case.

In Section 3, we show that the existence of a ρ-approximation algorithm for Maximum Path Packing (Max-PP) implies a $1/(1 - e^{-1/\rho})$-approximation algorithm for Max-PMC. Using a known 2-approximation for Max-PP in trees, this implies a 2.542-approximation algorithm for Max-PMC in undirected and directed trees.

1.2 Definitions and Preliminaries

A multifiber network is modeled as a graph $G = (V, E)$ with edge weights $\mu : E \rightarrow \mathbb{N}$ that represent the number of fibers on each link of the network. The weight function μ may be given in advance, meaning that the number of fibers available on each link is predetermined, or it may be sought, representing the number of fibers that should be reserved or installed on each link in order to establish a set of connections.

Connections in the network are represented as paths on G. (In general networks, the connections would be represented as terminal pairs, but since we deal with trees, we consider paths instead of terminal pairs.) We will mainly consider the case that G is an undirected graph and that the connections are undirected paths, but we will also discuss how our results extend to the directed case (for which we assume that G is a bidirected graph, i.e., a directed graph in which $(u, v) \in E$ implies $(v, u) \in E$, and the connections are directed paths).

We say that a path *touches* a node v if it contains an edge that has v as an endpoint. For a set of paths $\mathcal{P}$ on G the load $L(e, \mathcal{P})$ of edge e is the number of paths in $\mathcal{P}$ that go over e. The load $L(\mathcal{P}) = \max_{e \in E} L(e, \mathcal{P})$ of G is the maximum number of paths that go over the same edge of G. We will omit $\mathcal{P}$ and simply write $L(e)$ and L whenever it is clear from the context which set $\mathcal{P}$ we mean. A wavelength assignment for a set of connections in a multifiber network corresponds to a *multicoloring* of the corresponding paths; in a multicoloring, color collisions are allowed among paths that share an edge (meaning that the corresponding connections will use the same wavelength on different fibers). A path multicoloring is *valid with respect to* μ, if for each edge e the number of times any color appears on paths through e (we refer to this number also as the *maximum number of color collisions* on e) is at most $\mu(e)$. The following optimization problems arise naturally:

Minimum Collisions Path Multicoloring (MinCollisions-PMC). *Given a graph $G = (V, E)$, a set of paths $\mathcal{P}$ on G and a number w, find a path multicoloring of $\mathcal{P}$ with w colors such that $\sum_{e \in E} \mu(e)$ is minimized, where $\mu(e)$ denotes the maximum number of times that any color appears on edge e.*

Minimum Colors Path Multicoloring (MinColors-PMC). *Given a graph $G = (V, E)$ with edge weights $\mu : E \to \mathbb{N}$ and a set of paths $\mathcal{P}$ on G, find a valid, with respect to μ, path multicoloring of $\mathcal{P}$ such that the number of colors used is minimized.*

Maximum Path Multicoloring (Max-PMC). *Given a graph $G = (V, E)$ with edge weights $\mu : E \to \mathbb{N}$, a set of paths $\mathcal{P}$ on G and a number w, find a valid, with respect to μ, path multicoloring of a subset $\mathcal{P}' \subseteq \mathcal{P}$ with w colors such that $|\mathcal{P}'|$ is maximized.*

For all three problems above we also consider the case in which the weight function μ is (required to be) a constant, corresponding to situations where the number of fibers has to be the same over all links of the network. We thus obtain three new problems, Minimum Collisions Uniform Path Multicoloring (MinCollisions-UPMC), Minimum Colors Uniform Path Multicoloring (MinColors-UPMC), and Maximum Uniform Path Multicoloring (Max-UPMC). Observe that for MinCollisions-UPMC, the objective reduces to minimizing the unique number μ. All these problems are $\mathcal{NP}$-hard for undirected and directed trees, as follows easily from the $\mathcal{NP}$-hardness of path coloring (MinColors-UPMC with $\mu = 1$, abbreviated as PC) [13, 7].

For bidirected networks, the edges (u, v) and (v, u) are considered to be separate edges, i.e., each color can appear at most $\mu(u, v)$ times on directed paths containing (u, v) and at most $\mu(v, u)$ times on directed paths containing (v, u). For MinCollisions-PMC, the objective value is the sum of $\mu(e)$ over all directed edges of the graph.

Notice that in general networks, for all these problems one could look at the routing versions as well, that is, instead of paths only the connection endpoints are given (pairs of nodes) and a solution asks for a routing (a connecting path for each pair of nodes) as well. However, since we are dealing with tree networks we will not consider these variants here, assuming that a connection between two nodes in a tree always uses the (unique) simple path.

An algorithm A for a minimization problem Π is a ρ-approximation if for every instance I of Π, A runs in time polynomial in $|I|$ and delivers a solution with objective value at most $\rho \cdot OPT(I)$, where $OPT(I)$ denotes the objective value of an optimal solution for I. Similarly, if Π is a maximization problem, A is a ρ-approximation if, for every instance I of Π, it delivers a solution with objective value at least $1/\rho \cdot OPT(I)$. If the problem and the instance under consideration are clear from the context, we will simply use OPT to denote the objective value of an optimal solution for the instance.

1.3 Related Work

Wavelength assignment problems in optical networks have been studied extensively in the literature. In this section we review the known results in the area with emphasis on those related to multifiber networks.

To the best of our knowledge, the MinCollisions-PMC problem was first studied in [23]. It was shown that it can be solved optimally in polynomial time in chain networks, while for star and ring networks 2-approximation algorithms were given. A different algorithm for MinCollisions-PMC in chain networks was presented in [28]. Nomikos et al. [21, 22] study a generalization of MinCollisions-PMC with different fiber costs and give constant approximation algorithms for rings and spiders. In [9], we consider MinCollisions-PMC for caterpillars. Caterpillars are trees in which all nodes of degree larger than 2 lie on a single path. We present approximation algorithms with ratio $1 + 5|E|/OPT$ for undirected caterpillars and with ratio $1 + 4|E|/OPT$ for directed caterpillars.

Li and Simha [17] and Margara and Simon [18] study the MinColors-UPMC problem in ring and star networks. In [17] it is shown that MinColors-UPMC in rings remains $\mathcal{NP}$-hard for every fixed number of available fibers. An upper bound of approximately $(\mu + 1)L/\mu^2$ wavelengths for ring networks with μ available fibers per link and load L is given. Several restrictions of MinColors-UPMC in ring and star networks are considered as well. Similar results are shown independently in [18]. The latter paper also studies the (multiplicative) gap between the optimal number of colors and the lower bound $\lceil L/\mu \rceil$. For example, it is proved for every $\mu \geq 1$ that there exists an undirected tree where this gap can be at least $1 + 1/(2\mu^2)$. This line of research is continued in [19] where it is proved that for any network G there exists a μ such that any set $\mathcal{P}$ of paths on G can be colored with $\lceil L(\mathcal{P})/\mu \rceil$ colors with respect to μ. In [10], MinCollisions-UPMC and MinColors-UPMC are studied for general graphs and a connection of these problems with hypergraph coloring is established.

If $\mu = 1$, MinColors-UPMC reduces to the well studied path coloring problem (PC). An algorithm using at most $3L/2$ colors for undirected trees was given in [24]. The currently best known algorithm for PC in undirected trees achieves an asymptotic approximation ratio of 1.1. This follows from the algorithm of Nishizeki and Kashiwagi [20] for edge coloring of multigraphs since approximation algorithms for PC in undirected trees and edge coloring of multigraphs are interchangeable [13, 7]. In directed trees, the best known algorithm achieves approximation ratio 5/3 (using at most $5L/3$ colors) and is due to Erlebach et al. [8].

Li and Simha [16] studied MinColors-UPMC for undirected trees and showed that a valid multicoloring using at most $\lceil 3L/(2\mu) \rceil$ colors can be computed efficiently. For the general problem MinColors-PMC in trees, a 4-approximation algorithm for undirected and directed trees was given recently by Chekuri, Mydlarz and Shepherd [4]. Motivated by an integer multicommodity flow problem on trees they show that a set $\mathcal{P}$ of paths on a (directed or undirected) tree with $\mu(e)$ available fibers on each link e can be colored using at most $4 \max_{e \in E} \lceil L(e, \mathcal{P})/\mu(e) \rceil$ colors.

Nomikos et al. [22] present constant approximation algorithms for MinColors-PMC in rings and spiders, with approximation ratios 2 (for directed and undirected rings as well as for directed spiders) and 5/2 (for undirected spiders).

As far as we know, Max-PMC has not been studied in the literature. For its single-fiber variant, however, known as Max-PC, a 1.58 approximation is known for the undirected case [27] while for directed trees there exists a slightly worse 2.22-approximation [6]. These algorithms follow from a reduction from Max-PC with an arbitrary number of wavelengths to Max-PC with one wavelength [1].

2 Minimizing the Number of Fibers (MinCollisions-PMC)

In the setting of this problem we are given a graph G, a set of paths $\mathcal{P}$ on G, and w colors. Recall that the objective is to minimize the maximum number of color collisions on each edge (in fact, we want to minimize the sum of these maximum numbers over all edges but, as we will see, local optimization implies total optimization). For an edge e we denote by $OPT(e, \mathcal{P})$ the minimum $\mu(e)$ over all possible multicolorings of $\mathcal{P}$ with w colors; when referring to an algorithm, we denote by $SOL(e, \mathcal{P})$ the number $\mu(e)$ that corresponds to a multicoloring returned by the algorithm.

A lower bound for $OPT(e, \mathcal{P})$ is $OPT(e, \mathcal{P}) \geq \lceil L(e, \mathcal{P})/w \rceil$. For an instance $I = (G, \mathcal{P}, w)$ we have $OPT(I) \geq \sum_{e \in E} OPT(e, \mathcal{P})$ and $OPT(I) \geq |E|$ (we assume w.l.o.g. that every edge is used by at least one path).

Before proceeding to the algorithm for trees with arbitrary path sets we will present an algorithm for a special class of instances: trees with centered path sets.

2.1 Trees with Centered Path Set

Here we deal with trees with a set $\mathcal{P}$ of paths such that all paths touch the same node, called the *center* of $\mathcal{P}$. We consider the path center as root of the tree and denote it by r.

MinCollisions-PMC is clearly $\mathcal{NP}$-complete in trees with centered path set, since the path set of a star has always a center and it is known [23] that MinCollisions-PMC in stars is $\mathcal{NP}$-hard. We present an approximation algorithm for MinCollisions-PMC in trees with centered path set.

Algorithm 1 returns a correct path multicoloring of $\mathcal{P}$ with w colors. This is because H is a w-degree bipartite graph, hence it can be edge-colored with w colors. Since each path corresponds to an edge $e \in A$, after the execution of the edge coloring algorithm all paths have been colored.

The most costly part of the algorithm is bipartite edge coloring (step 4), therefore its time complexity is $O(|\mathcal{P}| \log w)$ [5].

We next show that this algorithm is a 4-approximation for MinCollisions-PMC in trees with centered path set.

2.1.1 Approximation Ratio

In step 1 the subset of paths in $\mathcal{P}$ that pass through edge e is partitioned in two sets: one called $\mathcal{P}_{in}(e)$, containing paths that have direction towards the root, and one called $\mathcal{P}_{out}(e)$, containing the remaining paths (with opposite direction). Let $L_i(e, \mathcal{P}) = |\mathcal{P}_i(e)|$, $i \in \{in, out\}$.

Lemma 1 *For each edge e and each direction i, $i \in \{in, out\}$, the algorithm for the problem* MinCollisions-PMC *in trees with centered path set $\mathcal{P}$ colors paths that use e in direction i with at most $\lceil L_i(e, \mathcal{P})/w \rceil + 1$ color collisions.*

Algorithm 1 MinCollisions-PMC in trees with centered path set.
Input: tree $G = (V, E)$, set of paths $\mathcal{P}$ with center r, number of colors w.
Output: path multicoloring of $\mathcal{P}$ with w colors.

1. Assign an arbitrary direction to every path in $\mathcal{P}$.
2. Create two (initially empty) lists of paths Q_s, Q_f. Traverse the nodes of the tree using postorder traversal and update Q_s, Q_f as follows:

   For each node visited do

   > append all paths that start at this node to list Q_s and all paths that finish at this node to list Q_f.

3. Create a set S of starting groups of paths and a set F of finishing groups of paths:

   While Q_s, Q_f not empty do

   > delete w elements from Q_s and group them together to form a new group s and add s to S; delete w elements from Q_f and group them together to form a new group f and add f to F.

   Construct a bipartite graph $H(S, F, A)$, where there is an edge $e \in A$ for each path $p \in \mathcal{P}$ (connecting the groups that contain p).

4. Color the edges of H with w colors using the algorithm in [5]. Assign to each path the color of the corresponding edge e of H.

Proof: Consider an edge $e = \{u, v\}$; w.l.o.g. assume that v is the parent of u (w.r.t. the root r). We will prove the claim for paths in direction *in* ($\mathcal{P}_{in}(e)$); the proof for direction *out* is completely symmetric.

Notice that paths in $\mathcal{P}_{in}(e)$ are exactly the paths that start at u or at some descendant of u. Hence, these paths are consecutive in the list Q_s because postorder visits the subtrees of u and u itself consecutively (of course, any other standard tree traversal would do, as long as it visits a node and its subtrees in a consecutive manner). Therefore the algorithm puts paths in $\mathcal{P}_{in}(e)$ in several full groups (i.e. consisting of w paths each) and at most two semi-full groups. Let the number of paths in the first group (which can be semi-full or full) be k, and let the total number of paths in the remaining groups be k'. The number of groups is clearly $1 + \lceil k'/w \rceil = \lceil k/w \rceil + \lceil k'/w \rceil \leq \lceil (k + k')/w \rceil + 1 = \lceil L_{in}(e, \mathcal{P})/w \rceil + 1$. The number of color repetitions, for any color, is at most the number of groups. Hence, we have $SOL_{in}(e, \mathcal{P}) \leq \lceil L_{in}(e, \mathcal{P})/w \rceil + 1$. □

Proposition 1 *The algorithm for* MinCollisions-PMC *in trees with centered path set* $\mathcal{P}$ *gives a path multicoloring such that on each edge e the maximum number of color collisions is at most* $\lceil L(e, \mathcal{P})/w \rceil + 3 \leq OPT(e, \mathcal{P}) + 3$.

Proof: In step 1 the algorithm partitions the load on each edge e in two parts: $L_{in}(e, \mathcal{P})$ paths that go towards the root and $L_{out}(e, \mathcal{P})$ paths in opposite direction. The number of collisions of any color on edge e is at most $SOL_{in}(e, \mathcal{P})$ for paths in $\mathcal{P}_{in}(e)$ and at most $SOL_{out}(e, \mathcal{P})$ for paths in $\mathcal{P}_{out}(e)$. Thus, the total number of collisions of any color on e is:

$$\begin{aligned} SOL(e, \mathcal{P}) &\leq SOL_{in}(e, \mathcal{P}) + SOL_{out}(e, \mathcal{P}) \leq \left\lceil \frac{L_{in}(e, \mathcal{P})}{w} \right\rceil + 1 + \left\lceil \frac{L_{out}(e, \mathcal{P})}{w} \right\rceil + 1 \\ &\leq \left\lceil \frac{L(e, \mathcal{P})}{w} \right\rceil + 3 \leq OPT(e, \mathcal{P}) + 3. \end{aligned}$$

□

Concluding we have the following:

Theorem 1 *The algorithm for* MinCollisions-PMC *in trees with centered path set is a* $(1 + \frac{3|E|}{OPT})$*-approximation in the undirected case.*

Proof: Adding the solutions for all edges and using Proposition 1, we obtain for any instance $I = (G, \mathcal{P}, w)$: $SOL(I) \leq \sum_{e \in E} SOL(e, \mathcal{P}) \leq \sum_{e \in E} (\lceil L(e, \mathcal{P})/w \rceil + 3) \leq OPT(I) + 3|E|$. □

Remark: The above algorithm is a 4-approximation in the worst case, since $OPT \geq |E|$; however, the approximation ratio improves as the load increases.

A useful observation is that in trees with centered path set where the center is of degree 2 the above algorithm achieves a better ratio:

Corollary 1 *The algorithm for* MinCollisions-PMC *in trees with centered path set where the center is of degree 2, is a* $(1 + \frac{|E|}{OPT})$*-approximation in the undirected case, that is a* 2*-approximation in the worst case.*

Sketch of Proof: This is due to an improvement of step 1 of Algorithm 1. Since there are only two subtrees of the root, assigning to each path a direction from the left subtree to the right subtree we end up with edges that have only paths in one direction, thus for each edge it holds that $SOL(e, \mathcal{P}) \leq OPT(e, \mathcal{P}) + 1$. □

2.1.2 MinCollisions-PMC in Directed Trees with Centered Path Set

For the directed case of the problem we take advantage of the fact that directions of paths are given, therefore there is no loss due to arbitrary selection of direction. This leads to a better approximation.

Proposition 2 *The algorithm for* MinCollisions-PMC *in directed trees with centered path set* $\mathcal{P}$ *gives a path multicoloring such that on each edge* e *the maximum number of color collisions is at most* $\lceil L_{in}(e, \mathcal{P})/w \rceil + \lceil L_{out}(e, \mathcal{P})/w \rceil + 2 \leq OPT(e, \mathcal{P}) + 2$*. Here,* e *is to be understood as an edge in the underlying undirected tree, corresponding to two directed edges with opposite directions in the directed tree.*

Proof: The key property is that, now, $OPT(e,\mathcal{P}) \geq \lceil\frac{L_{in}(e,\mathcal{P})}{w}\rceil + \lceil\frac{L_{out}(e,\mathcal{P})}{w}\rceil$. Therefore, using Algorithm 1, $SOL(e,\mathcal{P}) \leq \lceil\frac{L_{in}(e,\mathcal{P})}{w}\rceil + 1 + \lceil\frac{L_{out}(e,\mathcal{P})}{w}\rceil + 1 \leq OPT(e,\mathcal{P}) + 2$. □

Theorem 2 *The algorithm for* MinCollisions-PMC *in directed trees with centered path set is a 2-approximation.*

Proof: Let E_1 be the set of edges which have paths in only one of the two directions and E_2 be the set of edges with paths in both directions. An obvious lower bound is $OPT(I) \geq |E_1| + 2 \cdot |E_2|$. Note also that for an edge $e \in E_1$, $SOL(e,\mathcal{P}) \leq OPT(e,\mathcal{P}) + 1$. Combining this with Proposition 2, we have $SOL(I) \leq \sum_{e\in E_1}(OPT(e,\mathcal{P})+1) + \sum_{e\in E_2}(OPT(e,\mathcal{P}) + 2) \leq OPT(I) + |E_1| + 2 \cdot |E_2| \leq 2 \cdot OPT(I)$. □

2.2 MinCollisions-PMC in Trees with Arbitrary Path Set

Now we present an algorithm for trees with arbitrary path sets. It is based on a partitioning of the path set $\mathcal{P}$ into $O(\log|V|)$ disjoint sets $\mathcal{P}_i$ using a separator-based approach (as in [2]), such that each $\mathcal{P}_i$ consists of disjoint sets of paths with common center. Then the algorithm MinCollisions-PMC for trees with centered path set can be used for each component of $\mathcal{P}_i$. Combining the solutions, we will obtain a $(1+O(\frac{|E|\log|V|}{OPT}))$-approximation for MinCollisions-PMC in trees.

2.2.1 Separator-Based Partitioning of Paths

First, we explain the details of the separator-based partitioning of the path set. The approach uses tree separators. Given a tree network G there always exists a node r such that its removal leaves a forest of trees, each containing at most $2|V|/3$ nodes. Any tree has a 2/3-separator of size 1 which can be found efficiently [26]. In fact, since we are not interested in separating the tree into two parts only, we use a linear-time procedure that always finds a 1/2-separator.

Let T_v represent the subtree rooted at node v and $c(v)$ = number of nodes of T_v. It can be easily verified that the following procedure finds a vertex v in a tree $T = (V,E)$ such that $T - \{v\}$ is a forest consisting of trees with size at most $|V|/2$:

Procedure 1 Tree separation

set $v :=$ root of the tree (* $c(v) = |V| > |V|/2$ *)
while v has a child x with $c(x) > |V|/2$ do (* $\exists$ at most one *)
    set $v := x$
return v

The partition of the path set is then based on the tree-separation procedure. In each phase the partition procedure selects those paths that touch the separation node(s).

Procedure 2 Partition of the path set

Set $i := 1$, $G_1^1 := G$, $c_1 := 1$;
while $c_i \neq 0$ do (* c_i represents the number of trees of size > 1 in phase i *)

  Phase i:
  set $\mathcal{P}_i := \emptyset$;
  for each tree G_i^j, $j = 1, \ldots, c_i$, of size > 1 in G do

    find a 1/2-separator r_{ij} in G_i^j;
    set $\mathcal{P}_{ij}$:= paths in $\mathcal{P}$ that touch r_{ij};
    set $\mathcal{P}_i := \mathcal{P}_i \cup \mathcal{P}_{ij}$;
    remove r_{ij} from G

  remove $\mathcal{P}_i$ from $\mathcal{P}$;
  set c_{i+1} := number of trees of G of size > 1; (* G is now a forest *)
  set $i := i + 1$

Lemma 2 *Procedure 2 has the following properties:*

1. *The number of phases t is at most* $\log |V|$.
2. $\mathcal{P} = \cup_{1 \leq i \leq t} \mathcal{P}_i$ *and for all* $i \neq j$, $\mathcal{P}_i \cap \mathcal{P}_j = \emptyset$.
3. *Each path $p \in \mathcal{P}_i$ touches nodes of exactly one tree of phase i.*
4. *Each path $p \in \mathcal{P}_i$ touches exactly one separator of phase i.*
5. *Two paths $p_1, p_2 \in \mathcal{P}_i$ that don't touch the same separator are disjoint.*

Proof: 1: due to the use of 1/2-separators.

2: consider any path p. Clearly, the algorithm places this path in some set $\mathcal{P}_i$. This is because p contains at least two connected nodes, say u, v, which are disconnected only if one of them is selected as a separator at some phase i. Then, p is placed in $\mathcal{P}_i$ if not already removed.

3: suppose for the sake of contradiction that p touches two nodes, say $u \in G_i^j$, $v \in G_i^{j'}$, $j \neq j'$. Since G_i^j and $G_i^{j'}$ are not connected at phase i, p touches some separator of a phase $i' < i$. Then, it would have been already removed.

4 and 5: immediate corollaries of the previous. □

2.2.2 The Algorithm

As explained in Section 2.2.1, we can partition $\mathcal{P}$ into $t \leq \log |V|$ subsets $\mathcal{P}_i$, where each subset $\mathcal{P}_i$ is a disjoint union of centered path sets that do not interfere with each other. Therefore, Algorithm 1 can be applied to $\mathcal{P}_i$ by applying it to each centered path set in $\mathcal{P}_i$

separately, and Proposition 1 still holds for this case. Thus we get the following algorithm for MinCollisions-PMC.

Algorithm 2 MinCollisions-PMC in trees
Input: tree G, set of paths $\mathcal{P}$, number of colors w.
Output: path multicoloring of $\mathcal{P}$ with w colors.

1. Partition the path set $\mathcal{P}$ into subsets $\mathcal{P}_i$, $1 \leq i \leq t$, as described in the text.

2. Find a path multicoloring with w colors for each subset $\mathcal{P}_i$ using the algorithm for trees with centered path set (Algorithm 1).

2.2.3 Approximation Ratio

The total load of each edge is $L(e, \mathcal{P}) = \sum_{i=1}^{t} L(e, \mathcal{P}_i)$. For the multicoloring returned by Algorithm 2 let $SOL(e, \mathcal{P}_i)$ denote the maximum number of color collisions on edge e among paths that belong to the subset $\mathcal{P}_i$. Since each $\mathcal{P}_i$ is colored by using Algorithm 1 we have $SOL(e, \mathcal{P}_i) \leq \lceil L(e, \mathcal{P}_i)/w \rceil + 3$ by Proposition 1. Hence, in each edge e, for any color, the total number of color collisions is at most

$$\begin{aligned} SOL(e, \mathcal{P}) &\leq \sum_{i=1}^{t} SOL(e, \mathcal{P}_i) \leq \sum_{i=1}^{t} \left(\left\lceil \frac{L(e, \mathcal{P}_i)}{w} \right\rceil + 3 \right) \leq \left\lceil \frac{\sum_{i=1}^{t} L(e, \mathcal{P}_i)}{w} \right\rceil + 4 \cdot t - 1 \\ &= \left\lceil \frac{L(e, \mathcal{P})}{w} \right\rceil + 4 \cdot t - 1 \leq OPT(e, \mathcal{P}) + 4 \cdot t - 1 \end{aligned}$$

Thus, for any instance $I = (G, \mathcal{P}, w)$, $SOL(I) \leq \sum_{e \in E} SOL(e, \mathcal{P}) \leq \sum_{e \in E} (OPT(e, \mathcal{P}) + 4 \cdot t - 1) \leq OPT(I) + (4 \cdot t - 1) \cdot |E| < (1 + \frac{4|E| \log |V|}{OPT(I)}) \cdot OPT(I)$. Therefore, we have proved the following:

Theorem 3 *The algorithm for the problem* MinCollisions-PMC *in trees is a* $(1 + \frac{4|E| \log |V|}{OPT})$*-approximation in the undirected case.*

Remark: If $OPT(I) = \Omega(|E| \log |V|)$, then Algorithm 2 is a constant approximation; for example, if $OPT(I) \geq 4|E| \log |V|$, Algorithm 2 achieves approximation ratio 2. This renders the algorithm particularly useful for heavily loaded instances. However, in the worst case the approximation guarantee is not better than $O(\log |V|)$; an algorithm that always achieves a constant approximation is presented in Subsection 2.3.

2.2.4 The Directed Case

By Proposition 2, we have $SOL(e, \mathcal{P}_i) \leq \lceil \frac{L_{in}(e, \mathcal{P}_i)}{w} \rceil + \lceil \frac{L_{out}(e, \mathcal{P}_i)}{w} \rceil + 2$ for each centered path set. So the overall solution is: $SOL(e, \mathcal{P}) \leq \sum_{i=1}^{t} (\lceil \frac{L_{in}(e, \mathcal{P}_i)}{w} \rceil + \lceil \frac{L_{out}(e, \mathcal{P}_i)}{w} \rceil + 2) \leq$

$OPT(e, \mathcal{P}) + 4 \cdot t - 2$. Let E_1 and E_2 be defined as in the proof of Theorem 2. For edges $e \in E_1$ it holds that $SOL(e, \mathcal{P}) \leq OPT(e, \mathcal{P}) + 2 \cdot t - 1$. Therefore, $SOL(I) \leq \sum_{e \in E_1}(OPT(e, \mathcal{P}) + 2 \cdot t - 1) + \sum_{e \in E_2}(OPT(e, \mathcal{P}) + 4 \cdot t - 2) \leq OPT(I) + (2 \cdot t - 1)(|E_1| + 2 \cdot |E_2|) < OPT(I) + 2|E'| \log |V|$, where E' is the set of active directed edges (we call a directed edge *active* if there are paths that use it). We have thus proved the following:

Theorem 4 *The algorithm for the problem* MinCollisions-PMC *in trees is a* $(1 + \frac{2|E'| \log |V|}{OPT(I)})$*-approximation in the directed case, where* E' *is the set of active directed edges.*

2.3 A 4-Approximation for MinCollisions-PMC in Trees

The algorithm is based on a recent 4-approximation for the problem MinColors-PMC. For MinColors-PMC, it is clear that $\max_{e \in E} \lceil L(e, \mathcal{P})/\mu(e) \rceil$ is a lower bound on the number of colors used by the optimal solution. Chekuri, Mydlarz, and Shepherd [4] gave a polynomial algorithm that comes within a factor of 4 of this lower bound. We can state their result as follows:

Theorem 5 (Chekuri, Mydlarz, and Shepherd, 2003) *There is a* 4*-approximation algorithm for* MinColors-PMC *in trees. It uses at most* $4 \max_{e \in E} \lceil L(e, \mathcal{P})/\mu(e) \rceil$ *colors.*

Using this algorithm as a subroutine, we can obtain a 4-approximation algorithm for the problem MinCollisions-PMC.

Corollary 2 *There is a* 4*-approximation algorithm for* MinCollisions-PMC *in undirected trees.*

Proof: Let an instance of MinCollisions-PMC be given. Set $\mu'(e) = \lceil L(e, \mathcal{P})/w \rceil$ for every edge $e \in E$. Next, compute a valid path multicoloring with respect to μ' using the algorithm mentioned in Theorem 5. The algorithm will need at most $4 \max \lceil L(e, \mathcal{P})/\mu'(e) \rceil \leq 4w$ colors. Now, we identify four colors at a time (i.e., assuming that the colors are numbered $1, 2, \ldots$, we map each color i to color $\lceil i/4 \rceil$). This reduces the number of colors by a factor of 4 and increases the number of color repetitions on every edge at most by a factor of 4. Thus, we have a path multicoloring with at most w colors that is feasible with respect to μ, where $\mu(e) = 4\mu'(e)$ for every $e \in E$. Since $OPT(e, \mathcal{P}) \geq \mu'(e)$, the claim follows. □

The result extends to the directed case as well, since the algorithm in [4] applies both to undirected and directed trees.

2.4 Uniform Number of Fibers: MinCollisions-UPMC

In this section we consider the problem MinCollisions-UPMC, where the number of fibers has to be the same over all links of the network. The objective of MinCollisions-UPMC is to minimize the uniform μ (equivalently, to compute a path multicoloring that minimizes $\max_{e \in E} \mu(e)$).

Algorithm 3 uses as a subroutine the algorithm for path coloring (PC) in undirected trees by Raghavan and Upfal [24] that colors a path set $\mathcal{P}$ on a tree with no more than $3L(\mathcal{P})/2$ colors.

Algorithm 3 MinCollisions-UPMC in trees.
Input: tree G, set of paths $\mathcal{P}$, number of colors w.
Output: path multicoloring of $\mathcal{P}$ with w colors minimizing $\mu = \max_{e \in E} \mu(e)$.

1. Color all paths in $\mathcal{P}$ using colors in the range $0, \dots, s-1$, $s \leq 3L(\mathcal{P})/2$.

2. Replace each color x with color $x \bmod w$.

Theorem 6 *The algorithm for* MinCollisions-UPMC *in trees computes a multicoloring with at most* $\lceil 3 \cdot OPT/2 \rceil$ *color collisions on any edge in the undirected case.*

Proof: Given an instance $I = (G, \mathcal{P}, w)$, we have that $\max_{e \in E} \mu(e) \geq \lceil L(\mathcal{P})/w \rceil$ holds for any path multicoloring of $\mathcal{P}$, hence $OPT(I) \geq \lceil L(\mathcal{P})/w \rceil$. After using the algorithm for PC in trees it holds that all paths using an edge have different colors in the range $0, \dots, s-1$, $s \leq 3 \cdot L(\mathcal{P})/2$. Hence, the number of color collisions for any color is at most $\lceil \frac{3}{2} L(\mathcal{P})/w \rceil \leq \lceil \frac{3}{2} \cdot OPT(I) \rceil$ in the resulting multicoloring. □

The approximation ratio of the above algorithm is thus $\frac{3}{2} + \frac{1}{2OPT} \leq 2$.

For MinCollisions-UPMC in directed trees, we follow a similar approach using an algorithm for PC in directed trees that colors all paths with at most $5 \cdot L(\mathcal{P})/3$ colors [8]. This leads to at most $\lceil \frac{5}{3} \cdot OPT \rceil$ collisions and approximation ratio $\frac{5}{3} + \frac{2}{3OPT}$ which is at most 2 for $OPT \geq 2$.

3 Maximizing the Number of Connections

In this section we consider the Max-PMC problem and propose a constant-ratio approximation algorithm for tree networks. Our algorithm uses as a subroutine an approximation algorithm for the Maximum Path Packing problem, formally defined as follows.

Maximum Path Packing (Max-PP). *Given a graph* $G = (V, E)$ *with edge capacities* $c : E \to \mathbb{N}$ *and a set* $\mathcal{P}$ *of paths on* G*, find a valid, with respect to* c*, path packing of* $\mathcal{P}$*, i.e., a subset* $\mathcal{P}' \subseteq \mathcal{P}$ *such that no more than* $c(e)$ *paths in* $\mathcal{P}'$ *go over edge* e*. The goal is to maximize* $|\mathcal{P}'|$.

Theorem 7 *Algorithm 4 is a* $1/(1-e^{-1/\rho})$*-approximation algorithm for the problem* Max-PMC *if a* ρ*-approximation algorithm for* Max-PP *is used as a subroutine.*

Algorithm 4 MAX-PMC

Input: graph $G = (V, E)$ with edge weights $\mu : E \rightarrow \mathbb{N}$, set $\mathcal{P}$ of paths, number of colors w.
Output: a valid, with respect to μ, path multicoloring of a subset $\mathcal{P}' \subseteq \mathcal{P}$ with w colors.

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set $\mathcal{P}' := \emptyset$;
for $i = 1$ to w do

  use a MAX-PP algorithm to find a valid, with respect to μ, path packing $\mathcal{P}_i$ of $\mathcal{P}$;
  assign color i to all paths in $\mathcal{P}_i$ and set $\mathcal{P}' := \mathcal{P}' \cup \mathcal{P}_i$;
  remove all paths in $\mathcal{P}_i$ from $\mathcal{P}$;

Proof: The proof is a straightforward adaptation of the technique used by Awerbuch et al. [1] for reducing MAX-PC (the single-fiber version of MAX-PMC) with an arbitrary number of wavelengths to MAX-PC with one wavelength.

Let A' be a ρ-approximation algorithm for MAX-PP that is used as a subroutine in Algorithm 4. Algorithm 4 accepts all paths in $\mathcal{P}_i, 1 \leq i \leq w$, and assigns color i to each path in $\mathcal{P}_i$. Since $\mathcal{P}_i$ is a feasible solution for MAX-PP, no more than $\mu(e)$ paths get color i for all e, i and hence the solution returned by Algorithm 4 is a feasible solution for MAX-PMC. We claim that this solution is away from the optimal by at most a factor of $1/(1 - e^{-1/\rho})$.

Consider an optimal solution to the instance of MAX-PMC and let OPT denote the number of paths accepted in that solution. Furthermore, let $t_i = |\mathcal{P}_i|$ denote the number of paths accepted by algorithm A' during the i-th iteration of Algorithm 4. We will show that

$$\sum_{i=1}^{k} t_i \geq OPT \cdot (1 - (1 - \frac{1}{\rho \cdot w})^k), \quad \text{for } 1 \leq k \leq w. \tag{1}$$

The claim will then follow since

$$1 - (1 - \frac{1}{\rho \cdot w})^w = 1 - \left((1 - \frac{1}{\rho \cdot w})^{\rho \cdot w}\right)^{1/\rho} \geq 1 - (e^{-1})^{1/\rho} = 1 - e^{-1/\rho},$$

and thus we have that

$$\sum_{i=1}^{w} t_i \geq OPT \cdot (1 - e^{-1/\rho}).$$

The proof of (1) is by induction on k. At the k-th iteration there are at least $OPT - (t_1 + \cdots + t_{k-1})$ paths from the optimal solution remaining in $\mathcal{P}$ that can be multicolored with w colors with respect to μ and hence there are at least $(OPT - (t_1 + \cdots + t_{k-1}))/w$ paths that form a valid packing with respect to the edge capacity function μ. Hence, since A' is a ρ-approximation we have that

$$t_k \geq \frac{OPT - (t_1 + \cdots + t_{k-1})}{w \cdot \rho}.$$

For $k = 1$ the base of the induction follows. Let $1 < k \leq w$ and assume that the inequality holds for $1 \leq i \leq k-1$. We have that

$$\begin{aligned} t_1 + \cdots + t_k &\geq t_1 + \cdots + t_{k-1} + \frac{OPT - (t_1 + t_2 + \cdots + t_{k-1})}{\rho \cdot w} \\ &= (t_1 + \cdots + t_{k-1}) \left(1 - \frac{1}{\rho \cdot w}\right) + \frac{OPT}{\rho \cdot w} \\ &\geq OPT \cdot \left(1 - (1 - \frac{1}{\rho \cdot w})^{k-1}\right) \cdot \left(1 - \frac{1}{\rho \cdot w}\right) + \frac{OPT}{\rho \cdot w} \\ &= OPT \cdot \left(1 - (1 - \frac{1}{\rho \cdot w})^{k}\right), \end{aligned}$$

and therefore the inequality holds for all $1 \leq k \leq w$. □

Theorem 7 applies to arbitrary graphs; for trees we obtain a 2.542-approximation algorithm by using the existing 2-approximation algorithm for Max-PP from [11].

Corollary 3 *There exists a 2.542-approximation algorithm for* Max-PMC *in tree networks.*

We note that the same result can be obtained for the directed case by using the Max-PP algorithm given in [6] (note that in [6] Max-PP is defined for uniform edge capacities; however, as the authors already note therein, their algorithm can be adapted to work in the case of arbitrary edge capacities).

4 Conclusion

We have studied several optimization problems motivated by multifiber optical networks. For the problem of minimizing the total number of fibers (MinCollisions-PMC), our algorithm computes a solution of cost at most $OPT + 4|E|\log|V|$ in undirected trees $T = (V, E)$, thus achieving approximation ratio $1 + \frac{4|E|\log|V|}{OPT}$. Using the recent 4-approximation algorithm for MinColors-PMC in [4], we also derived a 4-approximation algorithm for MinCollisions-PMC in undirected or directed trees. For the problem of maximizing the number of accepted requests, we presented a general reduction from Max-PMC to Max-PP and obtained a 2.542-approximation algorithm for undirected and directed trees.

We remark that our results can be generalized to MinCollisions-PMC with different fiber costs on different edges (because our analysis compares the number of fibers on each individual edge with the number of optimal fibers on the edge) and to Max-PMC with different profits for different requests (by using a constant-factor approximation for the weighted version of Max-PP such as the 4-approximation for Max-PP in trees given in [4]). Concerning future work, it would be interesting to see whether the approximation ratios can be improved further and whether good results can also be obtained for other network topologies.

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