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Output Feedback Control of Discrete Processes under Time Constraint: Application to Cluster Tools

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Practical control of timed behaviors in industrial applications is difficult. One of the few efficient modeling approaches to deal with time constraints on discrete event systems (DES) is to use Timed Event Graph (TEG), a sub-class of Petri Nets. The dynamic behavior of these graphs is represented by a linear equations system over the Max-Plus algebra. Up to date, these models were assumed to be fully observable. In this paper, it is demonstrated that if a TEG is strongly connected, one can derive a realizable output feedback control. In other words, the control law depends solely on the control itself and the system's outputs, considered to be the only observable events. The satisfaction of a set of constraints is addressed in the case of a single input model (i.e., having only one input transition) and the procedure to derive a realizable output feedback control law is provided. Finally, the present control method is applied to a dual-armed cluster tool, a well-known industrial practical application.

Keywords: Manufacturing control systems; Optimal control; Dynamic production control; Semiconductor manufacture; Petri nets; Discrete event simulation; Output feedback control; Time event graph

1. Introduction

A control problem of discrete event systems commonly involves some temporal constraints to satisfy. They can take diverse forms (e.g., deadline, time intervals, validity duration, synchronization. . .), which are encountered in a wide range of applications (e.g., semiconductor industry (Kim and Lee 2016), automotive industry (Atto, Martinez, and Amari 2011), chemical treatments (Kim and Lee 2003), rail transport (Kersbergen et al. 2013), robotics (Lopes et al. 2014), communication networks (Addad, Amari, and Lesage 2010)).

A Timed Event Graph (TEG) is a subclass of Timed Petri nets useful for modeling timed behavior. It turns non-linearity of timed dynamics into a linear system of equations over the Max-Plus algebra(Baccelli et al. 1992). Combined use of TEG and Max-Plus is a well-known approach in the literature initiated back in the 1960's (Cuninghame-Green 1960) and it is still a very active field of research. This framework has been successfully applied to solve diverse control problems, such as dynamic scheduling (Bonhomme 2013; Kersbergen et al. 2013; Kim and Lee 2008), synchronization of switching models (Lopes, De Schutter, and van den Boom 2012; Lopes et al. 2014), the disturbance decoupling problem (Shang et al. 2014), or just-in-time control(Houssin, Lahaye, and Boimond 2007; Lhommeau, Jaulin, and Hardouin 2012). Both open loop (Shang et al. 2014) and feedback (Lhommeau et al. 2004; Maia et al. 2005; Maia, Andrade, and Hardouin 2011) approaches have been considered to solve these control problems.

In this paper, we focus on the output feedback control of Max-Plus linear systems under temporal constraints defined as a maximal elapsed time, set on the system states (*e.g.*, the stripping time of

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a piece by immersion in an acid bath is defined by a time interval, it requires a minimum soak time but must not exceed a maximum one). This kind of problem has been addressed either using a temporal approach based on daters like in (Amari et al. 2012; Maia, Hardouin, and Cury 2013; Maia et al. 2011; Kim and Lee 2016) or its transformed version based on power series(Houssin, Lahaye, and Boimond 2013; Hardouin et al. 2013), which is similar to Z-transform for conventional linear system theory. An alternative formulation was proposed in (Katz 2007), where the constraints satisfaction is expressed as the $(A, B)-$invariance of a semimodule. The control problem is solved by computing the maximal of such semimodules, included in a domain defined by the constraints.

Although they do address the control of temporal constraints, these approaches suffer from a strong limitation: they consider the system to control as fully observable. By that, we mean that they assume that the occurrence of any event (*i.e.*, the firing of a transition in our case) is directly known to the controller. On the physical system, this means all events are detected by a sensor, and on the system model, that all transitions are outputs. As it is never the case in practice (*i.e.*, some transitions are *non-observable*, or also called *silent*), the natural approach to address this issue is to perform state-estimation using an observer. Hardouin et al. presented an observer design for Max-Plus linear systems (Hardouin et al. 2010), however, in the general case, that approach provides an under-estimation of the states (*i.e.*, $\widehat{x} \leq x$), which is not sufficient to solve our control problem (see Rem.7). The condition for equality cannot be verified *a priori*, as it involves the observer itself. Moreover, it is difficult to interpret how to modify the system such that equality holds, leading to a tedious test-and-retry design.

In (Gonçalves et al. 2014), the authors suggested to observe an expression $W \cdot x$ instead of the full state x, which simplifies the implementation but comes down to the same conditions as in (Hardouin et al. 2010). However, the approach of (Gonçalves et al. 2014) tends to increase dramatically the cycle time of the controlled system (see Sec. 6 for an example). More generally, state estimation of Timed Petri nets has been broadly studied in the prospect of fault detection. Some methods could be adapted and leveraged to solve our control problem, like those of (Ghazel, Toguyéni, and Yim 2009; Declerck and Bonhomme 2014; Lefebvre 2014).

In this body of work, (Kim and Lee 2012, 2016) extends (Amari et al. 2012) by allowing for multiple tokens in the constrained places, in the specific setting of cluster tools. This paper further generalizes (Amari et al. 2012) and (Kim and Lee 2016) by (i) relaxing the observability hypothesis, (ii) considering more general systems' model. Instead of using observers to cope with partially observable models, sufficient conditions for the control to satisfy the constraints are first formulated, then expressed into a realizable output feedback control law (*i.e.*, expressed by a causal function). In comparison with previous work, this approach mainly needs the internal subgraph of our TEG (*i.e.*, the subgraph obtained by deleting input and output transitions) to be strongly connected. However, many of the industrial systems relevant to control actually fit in that category, thus this is not a strong restriction in practice. The presented approach does not rely on observers to perform output feedback and provides good time and robustness performances. For illustration, we apply it to the control of a cluster tool, a well-known industrial case study from the literature (Wu and Zhou 2010; Kim and Lee 2012, 2016) and the results are compared with those of previously known approaches.

Section 2 presents useful background on Max-Plus, TEG, and the modeling of temporal constraints. Section 3 summaries the modeling hypothesis and motivations. In Section 4 useful intermediate results are introduced before addressing the control derivation in Section 5. The practical application of this approach is presented in Section 6, including comparison with previous results and performances discussions. Finally, some conclusions and perspectives are drawn in Section 7.

2. Preliminaries

2.1. *Max-Plus Algebra*

A monoid is a set, say $\mathcal{D}$, endowed with an internal law, noted $\oplus$, which is associative and has a neutral element, denoted ε. A semiring is a commutative monoid endowed with a second internal law, denoted $\otimes$, which is associative, distributive with respect to the first law $\oplus$, has a neutral element, denoted e, and admits ε as absorbing element, (i.e., $\forall a \in \mathcal{D}, a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon$). A dioid is a semiring with an idempotent internal law, (i.e., $\forall a \in \mathcal{D}, a \oplus a = a$). The dioid is said to be commutative if the second law $\otimes$ is commutative. Max-Plus algebra is defined as $(\mathbb{R} \cup \{-\infty\}, \max, +)$. This semiring, denoted $\mathbb{R}_{\max}$, is a commutative dioid, the law $\oplus$ is the operator max with neutral element $\varepsilon = -\infty$, and the second law $\otimes$ is the usual addition, with neutral element $e = 0$. $\otimes$ is abbreviated by $\cdot$ (dot). $(+, \times)$ refer to the usual addition and multiplication.

Given matrices of appropriate dimensions, $\oplus$ and $\otimes$ operations are defined as follow:

$$\begin{aligned}(A \oplus B)(i,j) &= \max(A(i,j), B(i,j)) \\ (A \cdot B)(i,j) &= \bigoplus_{k=1}^{n} (A(i,k) \cdot B(k,j)) \\ &= \max_{k \in [1..n]} (A(i,k) + B(k,j))\end{aligned}$$

The Kleene star of a square matrix $M \in \mathbb{R}_{\max}^{n \times n}$, written M^*, is defined as

$$M^* = \bigoplus_{i \in \mathbb{N}} M^i$$

where M^0 equals the unit matrix I, which has diagonal entries equal to e and ε elsewhere. M^p is the p^{th} power of matrix M in $\mathbb{R}_{\max}$ (e.g., $M^2 = M \cdot M$). u^T denotes the transposed of vector u.

Definition 1 (Similar vectors)**.** *Two vectors u and v are similar in $\mathbb{R}_{\max}$ if they share the same neutral elements regarding to the law $\oplus$. If they are of size n then,*

$$\forall r \in [1..n], \; (\, u(r) = \varepsilon \;\Leftrightarrow\; v(r) = \varepsilon \,)$$

Remark 1. *In some references, similar vectors (or matrices) are said to have* same support *(Baccelli et al. 1992).*

Proposition 1. *Similarity is distributive over the law $\otimes$. Given three vectors $(u, v, w) \in (\mathbb{R}_{\max}^n)^3$,*

$$(u, v) \text{ similar} \;\Rightarrow\; (u^T \cdot w, v^T \cdot w) \text{ similar}$$

Proof.

$$\begin{aligned} u^T \cdot w = \varepsilon &\Leftrightarrow \bigoplus_{r=1}^{n} u(r) \cdot w(r) = \varepsilon \\ &\Leftrightarrow [\forall r \in [1..n], u(r) \neq \varepsilon \Rightarrow w(r) = \varepsilon\,] \end{aligned}$$

Since (u, v) are similar,

$$\begin{aligned} &\Leftrightarrow [\forall r \in [1..n], v(r) \neq \varepsilon \Rightarrow w(r) = \varepsilon\,] \\ &\Leftrightarrow v^T \cdot w = \varepsilon \end{aligned}$$

□

2.2. *TEG and Linear Max-Plus Models*

An **event graph** is an ordinary Petri net where each place has exactly one upstream and one downstream transition. It is also referred to as a *decision free Petri net*, as one token never enable more than one transition at a time. A **timed event graph (TEG)** is an event graph with extra delays associated to places (*holding times*) or transitions (*firing times*). Transitions without downstream places are **outputs**, those without upstream places are **inputs**. Others are simply called internal or **standard transitions**. Disturbances are not modeled, thus, all inputs are controllable. They are referred to as the **controls**. They can be fired following any arbitrary non-decreasing sequence of epochs.

The following notations and definitions are further introduced

- $\boldsymbol{p_{ij}}$ denotes the place linking t_j to t_i when it exists,
- A **path** is an oriented alternating sequence of transitions and places successively connected by an arc,
- The **token number** of a path is the sum of tokens in all places along the path,
- The **delay** of a path is the sum of holding and firing time of all places and transitions along the path,
- Given two transitions t_i and t_j and a token number m_{ij}, several m_{ij}-token paths connecting t_j to t_i exist in general.
  $\boldsymbol{t_j} \overset{m_{ij},\tau_{ij}}{\longrightarrow} \boldsymbol{t_i}$ denotes the *maximal* of such paths (i.e., the m_{ij}-token path with maximum delay τ_{ij}),
- A **primal path** contains exactly one token in the first place along the path,
- An **empty path** contains no token,
- A **circuit** around t_i is a path connecting t_i to itself,
- An **elementary path** does not contain any transition more than once,
- An **event** is the firing of a transition,
- A transition t_j is **controllable** if it is linked directly from a control (i.e., a place p_{ju_s} exists),
- A transition t_i is **observable** if it is linked directly to an output, (i.e., a place $p_{y_r i}$ exists).

A Petri net is said to be **live** for an initial marking if all transitions can always be enabled by a future marking (Baccelli et al. 1992). An autonomous event graph contains only internal transitions. A nonautonomous event graph is said to be live if its internal subgraph (or autonomous subgraph, in other words the complete graph pruned of input/output transitions) is live.

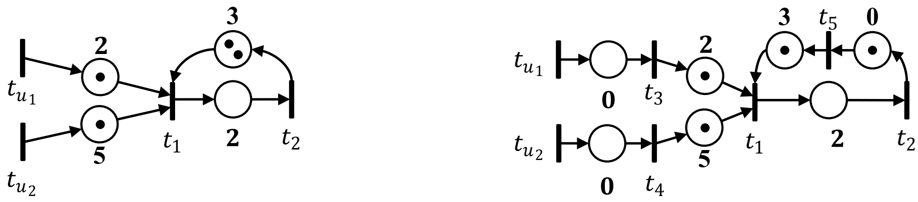
Theorem 1 (from (Baccelli et al. 1992))**.** *An autonomous event graph is live if and only if every circuit contains at least one token with respect to the initial marking.*

Definition 2 (Realizable output feedback)**.** *An output feedback expression is said to be realizable if it depends linearly on controls and outputs events associated to non-negative delays. In other words, $\mathcal{C}$ is realizable if there exist $\{\delta_{m,r}, \mu_{m,s}\}_{m,r,s} \subset \mathbb{R}^+ \cup \{\varepsilon\}$ such that*

$$\mathcal{C} = \bigoplus_{m,r,s} (\delta_{m,r} \cdot y_r(k-m) \oplus \mu_{m,s} \cdot u_s(k-m))$$

The state variable $x_i(k)$ of a TEG is the epoch when standard transition t_i fires for the k^{th} time. We call $x(k) = (x_i(k))$ the system's **state vector**. The **input vector** $u(k)$ is defined likewise. The dynamic behavior of a TEG in Max-Plus is described by the evolution equation

$$x(k) = \bigoplus_{m \geq 0} (A_m \cdot x(k-m) \oplus B_m \cdot u(k-m)) \tag{1}$$



(a) Original model (b) Extended model

Figure 1. Example of TEG; (a) a general model and (b) its extension

where $A_m(i,j) = \begin{cases} \tau & \text{if } p_{ij} \text{ exists, contains } m \text{ tokens, and has holding time } \tau \\ \varepsilon & \text{otherwise.} \end{cases}$

B_m is defined likewise for places between input and standard transitions.
It yields $(A_m, B_m) \in \mathbb{R}_{\max}^{n\times n} \times \mathbb{R}_{\max}^{n\times q}$.

Example 1. *Consider the TEG in Figure 1(a). The firing epochs of t_1 are defined by the following equation*

$$x_1(k) = \max(2 + u_1(k-1),\, 5 + u_2(k-1)\,,\, 3 + x_2(k-2))$$

which rewrites into linear form in Max-Plus

$$x_1(k) = 2 \cdot u_1(k-1) \oplus 5 \cdot u_2(k-1) \oplus 3 \cdot x_2(k-2)$$

Setting $x(k) = [x_1(k), x_2(k)]'$ and $u(k) = [u_1(k), u_2(k)]'$, the full TEG behavior is expressed by

$$x(k) = \underbrace{\begin{pmatrix} \cdot & \cdot \\ 2 & \cdot \end{pmatrix}}_{A_0} x(k) \oplus \underbrace{\begin{pmatrix} \cdot & 3 \\ \cdot & \cdot \end{pmatrix}}_{A_2} x(k-2) \oplus \underbrace{\begin{pmatrix} 2 & 5 \\ \cdot & \cdot \end{pmatrix}}_{B_1} u(k-1)$$

It is subsequently assumed that places connecting input to standard transitions contain no token. It yields $B_k = \varepsilon$ for all $k > 0$. This is no restriction as one can add an extra place between such transitions, such that the first place is empty and the second contains tokens. Moreover, as for regular linear systems, the initial recurrence (1) can be transformed into an equivalent recurrence of order 1 by extending the state vector. This consists in expanding all places with marking $m > 1$ into m places with marking equal to 1. Hence, for each of such places, $(m-1)$ intermediate transitions are added and the resulting extended state vector $\hat{x}(k)$ belongs to $\mathbb{R}_{\max}^{N}$ with $N = n + (m-1)$. Figure 1(b) is a simple example of both model extensions from Figure 1(a). It follows,

$$\hat{x}(k) = \hat{A}_0 \cdot \hat{x}(k) \oplus \hat{A}_1 \cdot \hat{x}(k-1) \oplus \hat{B}_0 \cdot u(k)$$

The $\widehat{(.)}$ notation will be further omitted for the sake of readability.

Furthermore, it is shown that, if the event graph is live, the Kleene star of A_0, A_0^*, is computed by $\bigoplus_{i=0}^{n} A_0^i$ (Baccelli et al. 1992). Hence one can derive the **standard state-space equation**,

$$x(k) = A \cdot x(k-1) \oplus B \cdot u(k) \tag{2}$$

where $A = A_0^* \cdot A_1$ and $B = A_0^* \cdot B_0$. See (Baccelli et al. 1992) for details and Remark 2 for graph interpretation of A_0^*. Finally, for any integer ϕ such that $1 \leq \phi \leq k$, by doing ϕ substitutions in

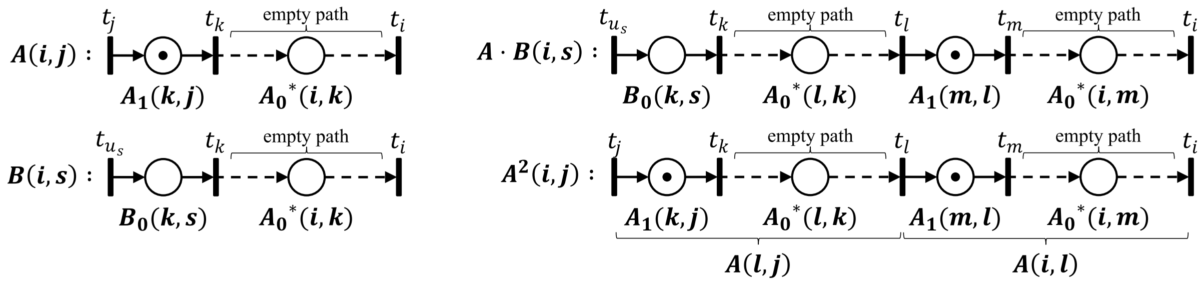


Figure 2. Graph interpretation of A and B matrices

(2), one obtains

$$x(k) = A^{\phi} \cdot x(k-\phi) \oplus \left[\bigoplus_{k'=0}^{\phi-1} A^{k'} \cdot B \cdot u(k-k') \right] \tag{3}$$

Remark 2. *Graph interpretation of A and B is illustrated in Figure 2. Matrices' coefficients represent the maximal delay along some paths, depending on the matrix under consideration. For instance, $A(i,j) = (A_0^* \cdot A_1)(i,j)$, thus $A(i,j)$ is the maximal delay of paths connecting t_j to t_i, primal (because of A_1) and then going through an arbitrary number of empty places (from A_0^*). It equals ε if no such path exists.*

A^k contains delays of "longer" paths (i.e., with more tokens) as k increases. Hence, A^k will tend to "grow", to have bigger values, with k.

Note that these paths do not need to be elementary (i.e., they can pass through the same transition several times).

Example 2. *Back to Example 1, consider the extended TEG represented in Figure 1(b). Its dynamics are expressed by the following standard form*

$$x(k) = \underbrace{\begin{pmatrix} . & . & 2 & 5 & 3 \\ . & . & 4 & 7 & 5 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & 0 & . & . & . \end{pmatrix}}_{A} x(k-1) \oplus \underbrace{\begin{pmatrix} . & . \\ . & . \\ 0 & . \\ . & 0 \\ . & . \end{pmatrix}}_{B} u(k)$$

Remark 3. *One should remember that the state variables (x_i) represent firings of transitions. Hence, when we refer to state-feedback, the* ***state is the set of last firing epochs of the system's transitions****. This differs from the notion of state in the Petri net formalism, which is usually the marking of the net.*

Definition 3 (Cycle time (Baccelli et al. 1992))**.** *The cycle time λ of a live TEG is the maximum of its cycle means over all circuits in the graph. If $\mathcal{S}$ is the number of transitions of the graph, it computes*

$$\lambda = \bigoplus_{j=1}^{\mathcal{S}} \left(trace(A^j)\right)^{1/j} = \bigoplus_{j=1}^{\mathcal{S}} \left(\bigoplus_{i=1}^{\mathcal{S}} A^j(i,i) \right)^{1/j} \tag{4}$$

where $(.)^{1/j}$ represents in Max-Plus the division by j in the conventional sense and the trace is the maximum of the diagonal elements.

(a) Example of a temporal constraint

(b) The same temporal constraint, extended following the same logic as for Figure 1

Figure 3. Modeling of a temporal constraints and graphical representation

One usual control objective for industrial systems is to maximize the throughput (the inverse of the cycle time). Hence, a controller should delay the original system as few as possible while ensuring that all the constraints are meet.

Definition 4 (Optimal control). *A control policy is said to be optimal if the controlled system respects all constraints and have the same cycle time as the autonomous system.*

2.3. *Model of cyclic processes*

Many real life applications follow cyclic sequences of actions (e.g., a manufacturing assembly line). When they are modeled by means of a graph, the resulting graph is said to be strongly connected.

Definition 5 (Strongly connected graph). *A graph is said to be strongly connected if for any pair of nodes i and j, there exists an oriented path from i to j.*

Hence, a timed event graph is strongly connected if for any pair of internal transitions (t_i, t_j), there exists $m_{ij} \in \mathbb{N}$ such that a path $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$ exists, which yields $x_i(k) \geq \tau_{ij} \cdot x_i(k - m_{ij})$.

Remark 4. *If the event graph is live, the token number of every circuit is at least 1 (see Theorem 1). Thus, adding strong connectivity, one can note that for any $m \in \mathbb{N}$ and any pair of transitions, there always exists a (possibly non-elementary) $m'-$token path, $m' \geq m$, connecting these two transitions.*

2.4. *Temporal constraints*

Strict time constraints are frequent in industry. It is crucial to model them efficiently to be able to ensure they are satisfied during the process. In a TEG, a holding time represents the minimal time a token has to sojourn in the place. If one wants to account for a maximal duration, another constraint has to be added.

An approach to solve this modeling problem was suggested in (Amari et al. 2012). The sojourn time of tokens in place p_{ij} is minimized by the holding time τ_{ij}. In addition, it must not exceed another time delay, denoted τ_{ij}^{max}. Hence, a time interval $[\tau_{ij}, \tau_{ij}^{max}]$ can be associated with the place p_{ij} subject to a strict time constraint. This additional temporal constraint is expressed by the following Max-Plus inequality

$$x_i(k) \leq \tau_{ij}^{max} \cdot x_j(k - m_{ij}) \tag{5}$$

This is illustrated in Figure 3.

3. Model description and motivations

In the remaining of this paper, TEGs are considered being nonautonomous and live, having a single input transition t_u, a single output transition t_y, and N standard transitions, having no circuit of null delay, and such that their autonomous subgraph are strongly connected. The output y is connected to x_j for a given $j \in [1..N]$, such that $y = x_j$. The dynamics of such TEG are described by the standard form equation

$$(2)\ :\ x(k) = A \cdot x(k-1) \oplus B \cdot u(k)$$

It yields A and B belong to $\mathbb{R}_{\max}^{N\times N}$ and $\mathbb{R}_{\max}^{N\times 1}$ respectively.

The interest of this work is in the modeling manufacturing systems, in which holding times represent length of processes and, thus, are non-negative. Firing times of transitions are set to 0 and the following evolution rules are considered

- Transitions fire as soon as they are enabled.
- A token starts enabling the downstream transition as soon as it has completed its place's holding time.
- The following compatible (refer to (Baccelli et al. 1992)) initial conditions are assumed $x(k) = \begin{cases} e \text{ if } k = 0 \\ \varepsilon \text{ if } k < 0 \end{cases}$

It is assumed that the firing sequence of the control $u(k)$ can be arbitrary defined. Solving the control problem consists in deriving a realizable output feedback expression for u enforcing time constraints the type of (5), also aiming to be optimal in the sense of Definition 4.

4. Intermediate results

This Section introduces Lemma 1, which demonstrates that, under Section 3's assumptions, the firing epochs of any standard transition can be upper-bounded by a realizable expression. In Section 5, this result is applied to strict sufficient conditions for the controller, which leads to Theorem 2, the main result of this paper. Three intermediate propositions are first presented, which are prerequisites for the demonstration of the Lemma. All proofs are provided in Appendix A.

Proposition 2. *For any $(i,j) \in [1..N]^2$, if there exists a path $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$,*

$$\forall p \in \mathbb{N}^*, \quad A^{p+m_{ij}}(i,:) \geq \tau_{ij} \cdot A^p(j,:)$$

Proposition 3. *For any $(i,j) \in [1..N]^2$ such as there exist a path $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$ and a circuit $t_i \overset{m_{ii},\tau_{ii}}{\longrightarrow} t_i$, and for any $\nu \in \mathbb{R}$,*

$$\exists q_0 \in \mathbb{N}, \forall q \geq q_0, \forall p \in \mathbb{N}^*, \quad \nu \cdot A^{p+m_{ij}+q\times m_{ii}}(i,:) \geq A^p(j,:)$$

Proposition 4. *For any $(i,j) \in [1..N]^2$ such as there exist paths $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$ and $t_i \overset{m_{ji},\tau_{ji}}{\longrightarrow} t_j$, there exists $p_0 \in \mathbb{N}^*$ such that,*

$4.1:\ \forall p \geq p_0, \forall q \in \mathbb{N},\quad (A^{p+q\times m_{ii}}(i,:),\ A^p(i,:))$ *similar,*
$4.2:\ \forall p \geq p_0 + m_{ji},\quad (A^{p+m_{ij}}(i,:),\ A^p(j,:))$ *similar,*
$4.3:\ \forall p \geq p_0 + m_{ji}, \forall q \in \mathbb{N},\quad (A^{p+q\times m_{ii}+m_{ij}}(i,:),\ A^p(j,:))$ *similar,*
where $m_{ii} = m_{ij} + m_{ji}$.

Lemma 1. *For any* $(i,j) \in [1..N]^2$ *such as there exist paths* $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$ *and* $t_i \overset{m_{ji},\tau_{ji}}{\longrightarrow} t_j$*, and for any* $\nu \in \mathbb{R}$*, one can define* $\mu_{k'} = \nu \cdot (A^{k'} \cdot B)(i)$ *and*

$$\exists\, p \in \mathbb{N}^*,\ q \in \mathbb{N},\ \delta \in \mathbb{R}^+,\ \forall k \geq (p+q),$$
$$\nu \cdot x_i(k) \leq \delta \cdot x_j(k-q) \oplus \left[\bigoplus_{k'=0}^{p+q-1} \left(\mu_{k'} \cdot u(k-k')\right)\right]$$

Lemma 1 is a key result of this paper. Given any (even negative) delay ν and state x_i, it guarantees that one can find a **realizable upper bound**, in other words, an expression depending solely on past control events $u(k)$ and any other transition's events $x_j(k)$, associated with a non-negative delay (δ) on x_j.

5. Output feedback control

In this section, the feasibility of a realizable output feedback control is demonstrated. Theorem 2 (see Section 5.1) expresses strict sufficient conditions for the satisfaction of constraints the type of (5) and then, leverages Lemma 1 to derive a realizable upper-bound of these. Ultimately, a realizable output feedback control is derived. The algorithmic procedure to compute that control law is presented in Section 5.2.

5.1. *Demonstration of realizable controllability*

Theorem 2. *Consider a TEG satisfying Section 3's hypothesis, and having a single place* p_{ij} *(containing* m_{ij} *tokens) subjects to a temporal constraint of form (5). Given that,*
(i) There exists an empty path $t_u \overset{0,B(j)}{\longrightarrow} t_j$*,*
(ii) $\forall k',\ 0 \leq k' < m_{ij},\ (A^{k'} \cdot B)(i) = \varepsilon$*,*
(iii) $(A^{m_{ij}} \cdot B)(i) \leq \tau_{ij}^{max} \cdot B(j)$*,*
it is sufficient to set

$$u(k) \geq \mathcal{C} = (-B(j)) \cdot (-\tau_{ij}^{max}) \cdot x_i(k + m_{ij}) \tag{6}$$

for u *to be a control which guarantees to satisfy constraint* (5)*, for all* $k \geq m_{ij}$.

Furthermore, one can always derive an upper-bound of $\mathcal{C}$ *of the following form, which defines a realizable output feedback control,*

$$\mathcal{C} \leq \delta \cdot y(k + m_{ij} - q) \oplus \left[\bigoplus_{k'=1}^{\substack{p+q\\-1-m_{ij}}} \left(\mu^{+}_{k'+m_{ij}} \cdot u(k-k')\right)\right] \tag{7}$$

where $(p,q) \in \mathbb{N}^2$ *with* $q \geq 0 \oplus (1 + m_{ij} - m_{yu})$*,*
m_{yu} *is the smallest token number of paths* $t_u \longrightarrow t_y$*,*
coefficients p*,* q*,* δ *and* $(\mu^{+}_{k'})$ *are returned by Algorithm 1.*

Proof. The time constraint to satisfy is expressed by (5) : $x_i(k) \leq \tau_{ij}^{max} \cdot x_j(k - m_{ij})$.

Existence of an empty path $t_u \overset{0,B(j)}{\longrightarrow} t_j$ is equivalent to $B(j) \neq \varepsilon$ (refer to Remark 2). Therefore, $\mathcal{C}$ is well-defined in $\mathbb{R}_{\max}$.

$(2) : x_j(k) = A(j,:) \cdot x(k-1) \oplus B(j) \cdot u(k)$
$\quad\Rightarrow \quad x_j(k) \geq B(j) \cdot u(k)$
$u(k) \geq \mathcal{C}$
$\quad\Rightarrow \quad x_j(k) \geq B(j) \cdot (-B(j)) \cdot (-\tau_{ij}^{max}) \cdot x_i(k+m_{ij})$
$\quad\Rightarrow \quad x_j(k) \geq (-\tau_{ij}^{max}) \cdot x_i(k+m_{ij})$
$\quad\Rightarrow \quad \forall k \geq m_{ij},\ \boldsymbol{x_i(k) \leq \tau_{ij}^{max} \cdot x_j(k-m_{ij})}$

Moreover, $u(k)$ must respect the state equations system, especially, for any $\phi \in \mathbb{N}$:

$$(3) \qquad \Rightarrow \quad x_i(k) = A^{\phi}(i,:) \cdot x(k-\phi) \oplus \left[\bigoplus_{k'=0}^{\phi-1} (A^{k'} \cdot B)(i) \cdot u(k-k')\right]$$

$$\Rightarrow \quad \forall k' \in \mathbb{N},\ x_i(k) \geq \nu_{k'} \cdot x_i(k+m_{ij}-k') \qquad (*)$$

where $\nu_{k'} = (A^{k'} \cdot B)(i) \cdot (-B(j)) \cdot (-\tau_{ij}^{max})$

Note that $x_i(k)$ is a non decreasing sequence. Hence $x_i(k) \leq x_i(k+p)$ for any $p \geq 0$. Therefore, $(*)$ will be satisfied if

$\forall k' < m_{ij},\ \nu_{k'} = \varepsilon$
$\quad\Leftrightarrow \quad \boldsymbol{(ii) : \forall k' < m_{ij},\ (A^{k'} \cdot B)(i) = \varepsilon}$

and $0 \geq (A^{m_{ij}} \cdot B)(i) \cdot (-B(j)) \cdot (-\tau_{ij}^{max})$
$\quad\Leftrightarrow \quad \boldsymbol{(iii) : (A^{m_{ij}} \cdot B)(i) \leq \tau_{ij}^{max} \cdot B(j)}$

Note that the two conditions are sufficient but the second is also necessary ($(*)$ cannot be true if (iii) does not hold).

Moreover, under the Theorem's assumptions, we can apply Lemma 1,
$\exists \delta \in \mathbb{R}^+,\ \exists (p_0, q_0) \in \mathbb{N}^2,\ \forall p \geq p_0,\ \forall q \geq q_0,$

$$(-B(j)) \cdot (-\tau_{ij}^{max}) \cdot x_i(k) \quad \leq \quad \delta \cdot y(k-q) \oplus \left[\bigoplus_{k'=0}^{p+q-1} (\mu_{k'} \cdot u(k-k'))\right]$$

where $\mu_{k'} = (-B(j)) \cdot (-\tau_{ij}^{max}) \cdot (A^{k'} \cdot B)(i)$

$$\Rightarrow \ \mathcal{C} \leq \delta \cdot y(k+m_{ij}-q) \oplus \left[\bigoplus_{k'=0}^{p+q-1} (\mu_{k'} \cdot u(k+m_{ij}-k'))\right] \ = \mathcal{C}'$$

Furthermore, if (i) and (ii) are satisfied, $\mu_{k'} = \varepsilon$ for $0 \leq k' \leq m_{ij}$, therefore,

$$\begin{aligned}\mathcal{C}' &= \delta \cdot y(k+m_{ij}-q) \oplus \left[\bigoplus_{k'=m_{ij}+1}^{p+q-1} (\mu_{k'} \cdot u(k+m_{ij}-k'))\right] \\ &= \delta \cdot y(k+m_{ij}-q) \oplus \left[\bigoplus_{k'=1}^{p+q-1-m_{ij}} \left(\mu_{k'+m_{ij}} \cdot u(k-k')\right)\right]\end{aligned}$$

Lets set $\mu_{k'}^{+} = \begin{cases} \mu_{k'} & \text{if } \mu_{k'} > 0, \\ \varepsilon & \text{otherwise.} \end{cases}$

$\forall k' \geq 0, \mu \leq 0 \Rightarrow u(k) \geq \mu \cdot u(k-k')$ by definition. Thus, if one defines $\mathcal{C}'^{+}$ as $\mathcal{C}'$ with $\mu_{k'}$ replaced by $\mu_{k'}^{+}$, it follows

$u(k) \geq \mathcal{C}'^{+} \ \Leftrightarrow \ u(k) \geq \mathcal{C}'$
$\qquad\qquad\quad \Rightarrow \ u(k) \geq \mathcal{C}$
$\qquad\qquad\quad \Rightarrow \ u(k)$ satisfies the constraint.

Finally, $u(k) \geq \mathcal{C}'^{+}$ implies $u(k) \geq \delta \cdot y(k+m_{ij}-q)$. Therefore, implementing this control results in adding a path $t_y \overset{(q-m_{ij}),\,\delta}{\longrightarrow} t_u$. According to Theorem 1, for the controlled system to remain live, it ultimately needs to satisfy

$m_{yu} + m_{uy} \geq 1 \Rightarrow \ m_{yu} + (q - m_{ij}) \geq 1$
$\qquad\qquad\qquad \Rightarrow \ \boldsymbol{q \geq 1 + m_{ij} - m_{yu}}$ □

Remark 5. *This method easily extends to multiple constraints, in the exact same fashion as in (Amari et al. 2012) and (Kim and Lee 2016). For each constraint, one derives a suitable control $u_s(k)$. All constraints will be satisfied by any control u such that $u(k) \geq \bigoplus u_s(k)$.*

Remark 6. *Note that, even though demonstrated starting from the standard form* (2)*, Theorem 2* *is not restricted to constraint places with zero or one token.*

Consider the example of Figure 3(b). After expansion, the temporal constraint reads $x_i(k) \leq \tau_{ij}^{max} \cdot x_{j'}(k-1)$*. But since* $x_{j'}(k) = x_j(k-1)$*, it is equivalent to* $x_i(k) \leq \tau_{ij}^{max} \cdot x_j(k-2)$*, from which this method can be applied.*

Remark 7. *In feedback control of Max-Plus models, conditions for the control are expressed as* $u(k) \geq \mathcal{C}$ *where* $\mathcal{C}$ *is a given expression (*e.g., $F \cdot x(k-1)$ *for usual state feedback; or given by Eq.*(6) *in our case). If one uses the observer from (Hardouin et al. 2010) on* $\mathcal{C}$*, it obtains* $\widehat{\mathcal{C}} \leq \mathcal{C}$*. Hence, setting* $u(k) \geq \widehat{\mathcal{C}}$ *does not guarantee* $u(k) \geq \mathcal{C}$ *and thus is not a proper solution for the control problem.*

5.2. *Algorithmic procedure for the control derivation*

Coefficients from control's definition of Theorem 2 are computed by Algorithm 1. It takes as inputs the state equations system (matrices A and B), the input and output transitions of the constrained place (i and j) and its token number (m_{ij}), the constraint value (τ_{ij}^{max}), the output transition of the system (y), and the smallest token number of paths $t_u \longrightarrow t_y$ (m_{yu}). It returns all coefficients defining a realizable output feedback control lower-bound which satisfies the constraint (p, q, δ and ($\mu_{k'}^{+}$)) when sufficient conditions are satisfied, and a failure otherwise. Theorem 2 guarantees that the procedure terminates.

6. Application to the control of cluster tools

Semiconductor manufacturing industry is an good example of practical need for precise temporal control of processing. A commonly used technology is low pressure chemical vapor deposition (LPCVD), which has a strict time limit on the maximum acceptable wafer delays. Pushed by optimization concerns over the past 20 years, there has been a lot of work done oriented toward this type of industry, from process design and performance evaluation (Srinivasan 1998), to optimized scheduling (Chryssolouris, Dicke, and Lee 1991; Lalas et al. 2005; Kim and Lee 2008; Wu and Zhou 2010; Jung and Lee 2012; Jung, Kim, and Lee 2015; Wu et al. 2013), multirobot cluster tools (Zuberek 2001), and control (Kim and Lee 2012, 2016).

In this section, a model of cluster-tool is presented and used as a study case to show how this method applies in practice, plus an evaluation of the control performance. A final discussion includes a comparison with previous results from the literature.

6.1. *Presentation of the system*

Thereafter is introduced the TEG model of a radial dual-armed cluster tool, which description is mostly taken from (Kim and Lee 2016). It is then presented how to apply the method from this paper to derive a realizable feedback control ensuring the satisfaction of wafer residency time constraints.

A cluster tool, as illustrated in Figure 4(a), consists of several single wafer processing chambers, (also called process modules – PMs), a wafer handling robot, and loadlocks (LLs) for loading and unloading of wafer cassettes in a closed environment. Cluster tools are widely used for various semiconductor manufacturing processes including etching, sputtering, chemical vapor deposition and so on. PMs and LLs are mostly radially arranged around a robot. A robot performs loading and unloading a wafer at a PM or a LL through radial moves. The swap sequence (illustrated in Figure 4(b)) is a well-known simple robot task sequence for dual-armed cluster tools. It is known to be optimal for most practical cases. It repeats a swap operation at each PM in order of wafer

Algorithm 1 deriveControlCoef

Input: A, B, i, j, m_{ij}, τ_{ij}^{max}, y, m_{yu}
Output: $(p, q, \delta, (\mu_{k'}^{+}))$

%Tests for sufficient conditions
if $B(j) = \varepsilon$ **or** $(A^{m_{ij}} \cdot B)(i) > \tau_{ij}^{max} \cdot B(j)$ **then**
  print `Failure` ;
  return
end if
for $k' \in [0..m_{ij} - 1]$ **do**
  if $(A^{k'} \cdot B)(i) \neq \varepsilon$ **then**
    print `Failure` ;
    return
  end if
end for

%Initialization and tests definition
$p := 1$; $q := \max(0,\, 1 + m_{ij} - m_{yu})$;
$\nu := -B(j) - \tau_{ij}^{max}$;
`majorationTest` = $(\nu \cdot A^{p+q}(i,:) \geq A^{p}(y,:))$;
`similarityTest` = $(A^{p+q}(i,:),\, A^{p}(y,:))$ similar ;

while `majorationTest` is false **do**
  $q := q + 1$;
end while%$q = q_0$ from Prop. 3 has been reached.

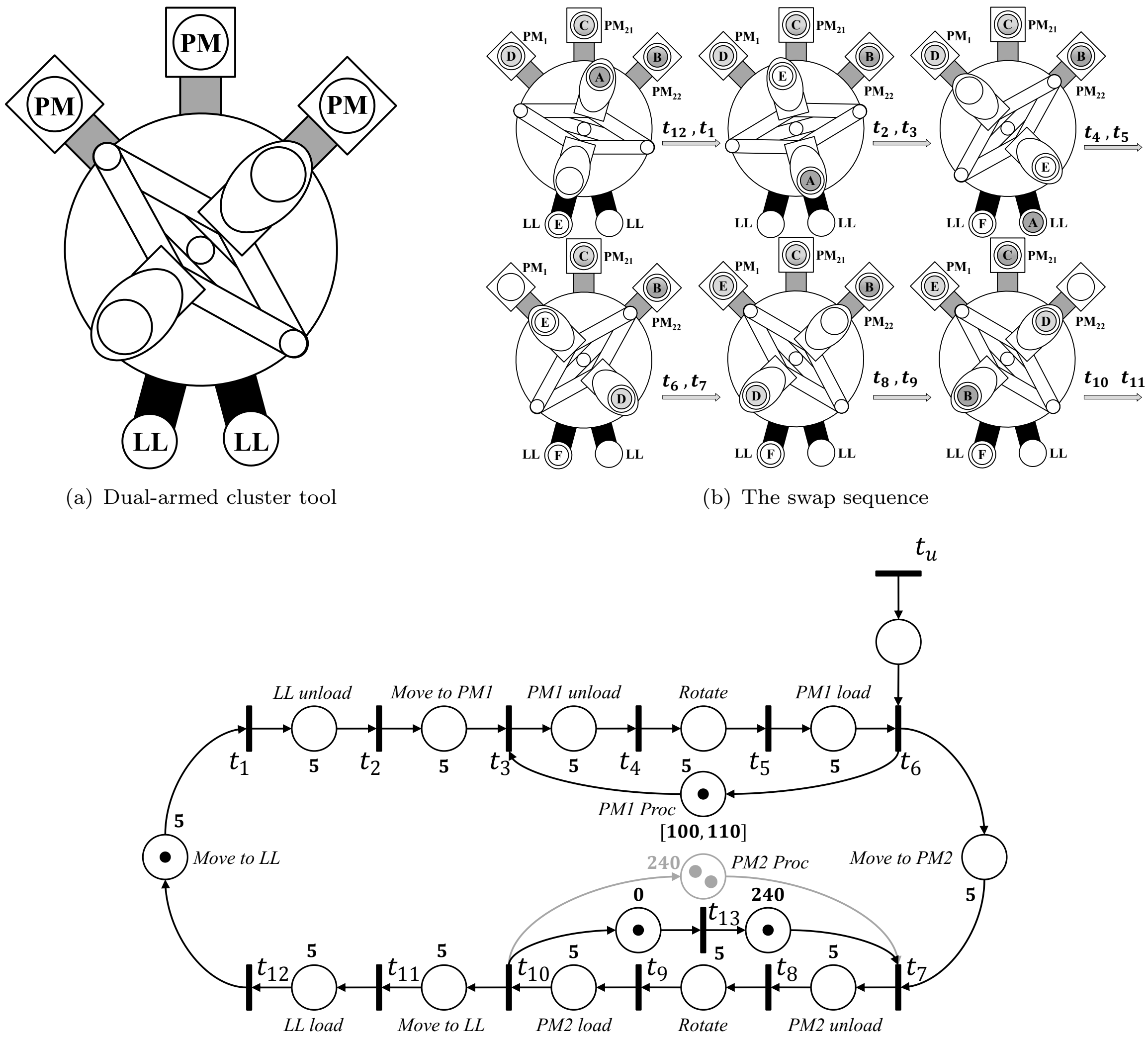
while `similarityTest` is false **do**
  $p := p + 1$;
end while%$p = p_0$ from Prop. 4.3 has been reached.

%Compute the remaining coefficients
$\delta := \max\limits_{\substack{r \in [1..N] \\ A^{p}(y,r) \neq \varepsilon}} (\nu \cdot A^{p+q}(i,r) - A^{p}(y,r))$;
for $k' \in [m_{ij} + 1..p + q - 1]$ **do**
  $\mu_{k'} := (A^{k'} \cdot B)(i) - B(j) - \tau_{ij}^{max}$;
  $\mu_{k'}^{+} := \max(\mu_{k'},\, 0)$
end for

flow. The swap operation unloads a wafer from a PM into the empty arm, rotate the robot arms, and unloads the wafer on the other arm into the PM. This sequence can be modeled by the TEG of Fig. 4(c).

Note that in this model, there are two process tasks to perform. Nominal duration of PM_1 is 100 time units while PM_2 takes 240 times units, but can be processed on two modules, hence the two tokens in the lower-side loop. In order to derive the standard state equation (2), this place (in grey on Figure 4(c)) has been decomposed into two places containing 1 token each, in the fashion of Figure 1.

Processing task PM_1 is subject to a time constraint (i.e., a wafer must not stay longer than 110 time units in PM_1). It is assumed that the firing of transition t_6 (starting PM_1) is a controllable event, which is represented by the input transition t_u connected to t_6 on Figure 4(c). It follows



(a) Dual-armed cluster tool

(b) The swap sequence

(c) TEG model of the uncontrolled system

Figure 4. Modeling of a dual-armed cluster tool performing a swap sequence

that the dynamics of this TEG are expressed by the following state-space equation

$$x(k) = A.x(k-1) \oplus B.u(k)$$

where $A = \begin{pmatrix} . & . & . & . & . & . & . & . & . & . & . & 5 & . \\ . & . & . & . & . & . & . & . & . & . & . & 10 & . \\ . & . & . & . & . & 100 & . & . & . & . & . & 15 & . \\ . & . & . & . & . & 105 & . & . & . & . & . & 20 & . \\ . & . & . & . & . & 110 & . & . & . & . & . & 25 & . \\ . & . & . & . & . & 115 & . & . & . & . & . & 30 & . \\ . & . & . & . & . & 120 & . & . & . & . & . & 35 & 240 \\ . & . & . & . & . & 125 & . & . & . & . & . & 40 & 245 \\ . & . & . & . & . & 130 & . & . & . & . & . & 45 & 250 \\ . & . & . & . & . & 135 & . & . & . & . & . & 50 & 255 \\ . & . & . & . & . & 140 & . & . & . & . & . & 55 & 260 \\ . & . & . & . & . & 145 & . & . & . & . & . & 60 & 265 \\ . & . & . & . & . & . & . & . & . & 0 & . & . & . \end{pmatrix}$, $B = \begin{pmatrix} . \\ . \\ . \\ . \\ . \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ . \end{pmatrix}$ and ε is replaced by a . for the sake of readability.

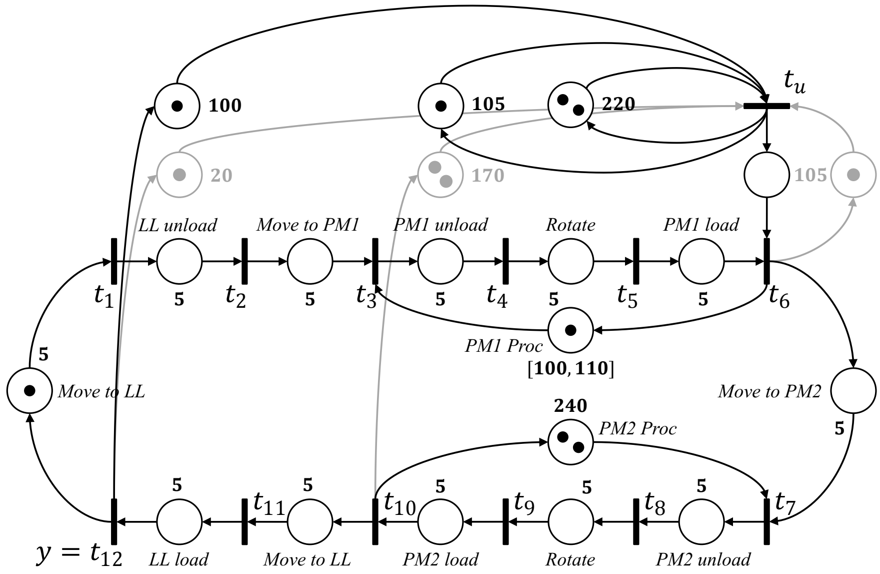


Figure 5. Realizable output feedback control of the cluster-tool– In gray is the controller derived in (Kim and Lee 2016)

6.2. *Realizable feedback control of the system*

The unloading of a processed part in one of the loadlocks LL is defined as the system's output. Hence, it yields $y = x_{12}$. The firing of all t_i, $i \neq 12$, are considered to be non-observable events.

Remark 8. *Note that the choice of x_{12} is arbitrary. The present approach can be applied whatever which transition's firings are observable, as long as Theorem 2's conditions are satisfied.*

PM_1's time constraint is expressed by the following inequality

$$x_3(k) \leq \tau_{36}^{\max} \cdot x_6(k-1) = 110 \cdot x_6(k-1) \tag{8}$$

The system's model satisfies hypothesis of Theorem 2. It is live with no 0-delay circuit and its autonomous subgraph is strongly connected. Furthermore,

(i) There exists an empty path $t_u \overset{0,B(j)}{\longrightarrow} t_j$,
(ii) $(B)(3) = \varepsilon$,
(iii) $(A \cdot B)(3) = 100 \leq 110 + 0 = \tau_{36}^{max} \cdot B(6)$.

It follows that constraint (8) is satisfied by

$$u(k) \geq \mathcal{C} = -(B(6) + \tau_{36}^{\max}) \cdot x_3(k+1) = -110 \cdot x_3(k+1)$$

Applying Algorithm 1 to $\mathcal{C}$ returns $p = 2$, $q = 2$, $\delta = 100$, and $\mu_{k'}^{+} = [105, 220]'$, hence, according to Theorem 2,

$$u(k) \geq 100 \cdot y(k-1) \oplus 105 \cdot u(k-1) \oplus 205 \cdot u(k-2)$$

satisfies constraint (8). The implementation of this control law is represented on Figure 5. One can observe that this control law is indeed realizable, as it only uses the output ($y = t_{12}$) and the past input events, associated with non-negative delays.

6.3. *Discussions on control performance and optimality*

To evaluate control performance in this setting, a natural parameter to consider is the system's cycle time (i.e., the inverse of the throughput). An optimal control (according to Definition 4) is a control which does not increase the cycle time of the uncontrolled system.

Let λ and $\widetilde{\lambda}$ be the cycle times of the uncontrolled and controlled systems respectively. Applying (4) returns

$$\begin{aligned} \lambda &= 127.5 \ \text{ time units,} \\ \widetilde{\lambda} &= 130 \quad \text{ time units.} \end{aligned}$$

Thus, the present control might not be optimal, as it does affect the system's cycle time ($\lambda < \widetilde{\lambda}$).

In this work, as in previous approaches from the literature, sufficient conditions for controllability of temporal constraints the kind of (5) are derived. There are no proof that any of such control strategies are minimally restrictive. This could be true if sufficient conditions from Theorem 2 are also necessary. This remains an open question.

Although it was only partially presented in this paper, the present control method has been implemented in simulation and tested on several manufacturing scenarios from the literature (Atto, Martinez, and Amari 2011; Kim and Lee 2016). As one can expect, compared to the full-observability case, the present control approach tends to increase more the cycle time of the controlled system. This is natural as it copes with some extra restrictions on the information available. It is the price one has to pay to control under partial observability.

Remark 9. *Note that it is not always the case. The cycle time increases depending on the system's architecture, the constraints values, and the choice of the state variable set as output. For instance, for the cluster-tool example presented earlier, one can derive an realizable control law which is also optimal (i.e., which does not increase the system's cycle time) by considering x_5 as the system's output. Applying Algorithm 1 returns the following control law*

$$u(k) = 105 \cdot u(k-1) \oplus 220 \cdot u(k-2) \oplus 335 \cdot u(k-3) \oplus 135 \cdot y(k-1)$$

which preserves $\widetilde{\lambda} = 127.5$ time units. However, up to this point in the authors's research, there is no proof that this can always be achieved. In other word, there is no guarantee that an optimal and realizable controller can be derived.

6.4. *Comparison with previous approaches*

On Figure 5, the control law derived in (Kim and Lee 2016) is also represented, showed in grey. Compared to the present approach, this control uses as inputs the firings of transitions t_{12} but also t_6 and t_{10}. Until the present contribution, the control dependence with these events could not be handled. If only one of such events is non-observable, the controllability of the system could not be guaranteed. In introduction, it was claimed that the present approach is an extension of that of (Kim and Lee 2016). Indeed, while stated otherwise, (Kim and Lee 2016) reduces to reformulate expression (6) (refer to Theorem 2) into feedback form. This is done easily by using relation (3) with $\phi = m_{ij} + 1$,

$$x_i(k + m_{ij}) = (A^{m_{ij}+1})(i,:) \cdot x(k-1) \oplus \left[\bigoplus_{k'=0}^{m_{ij}} A^{k'} \cdot B \cdot u(k + m_{ij} - k') \right]$$

Under hypothesis (ii) and (iii) of Theorem 2, terms in u simplify and one obtains

$$u(k) \geq ((A^{m_{ij}+1})(i,:) \cdot (-B(j)) \cdot (-\tau_{ij}^{max})) \cdot x(k-1)$$

The problem here is that such expression uses possibly all state variables in its definition, while some of them might not be observable. Therefore, the present approach pushes further the order of ϕ. It was shown that, if one looks "far enough" in the past, firing epochs of any state variable of interest (x_i in this setting) can be upper-bounded by previous firings of any other state variable (e.g., the output y) and some positive delays. This is the result from Lemma 1. Theorem 2 shows how to use this result to derive a realizable output feedback control, which can be further implemented on a physical system.

7. Conclusions

Max-Plus models are useful to capture time dependencies of discrete event systems. In the literature, several approaches address the control of such models, especially in cases of strict time constraints (e.g., (Amari et al. 2012; Atto, Martinez, and Amari 2011; Maia, Hardouin, and Cury 2013; Kim and Lee 2016)). It was demonstrated that one can derive a control sequence $u(k)$ which ensures such constraints are met. However, in these works, all events are assumed to be observable, which means all transitions firing epochs can be used to feed the control. This is a strong limitation in practice.

In this paper, that hypothesis is relaxed. In other word, the synthesis of a controller under partial observability was addressed. It was demonstrated that for any strongly connected TEG, one can always derive a realizable output feedback control (i.e., depending on output and the control events only), ensuring a given set of temporal constraints will be satisfied. The procedure to derive such a control law is provided. Furthermore, the practical example of a dual-armed cluster tool was used to demonstrate the use of the present method and discuss its performance.

As a final contribution related to this work, the authors are currently developing a plug-in for the TINA (TIme petri Net Analyzer) software tool (Berthomieu, Ribet, and Vernadat 2004). Taking the TEG to control as input (under textual or graphic form), it would return the control policy, add it to the model and evaluate the cycle time of the controlled system. This tool is also interesting because of another (already existing) plug-in performing LTL model-checking on Time and Timed Petri nets. For example, it allows one to verify the controller we have derived does indeed verify our constraints.

To push further this synthesis approach, it would be interesting to investigate conditions for an optimal control, that is, whether or not one can derive a realizable output feedback control which will not increase the system's cycle time, or at least no more that a fully-observable control does. Other perspectives include the more general study of necessary conditions for controllability, the satisfaction of time constraints on paths instead of places, relaxing the empty-path hypothesis (Hyp. (i) of Theorem 2), or the efficient control of systems using multiple inputs (i.e., multiple control transitions). Finally, the fundamental limitation of the present approach is the underlying inability (or at least inefficiency) of TEGs for handling conflicts. It would be interesting to consider extensions of such control approaches to sets of TEGs with conflicts, in the fashion of (Addad, Amari, and Lesage 2010).

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Appendix A. Proofs of Section 4 properties and lemma

Proof of Proposition 2.

For any $r \in [1..N]$, $A^p(j,r) = \tau_{jr} \neq \varepsilon$

$$\Rightarrow \exists \text{ a primal path } t_r \overset{p,\tau_{jr}}{\longrightarrow} t_j \qquad \{\text{Rem. 2}\}$$

$\exists\, t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$

$$\Rightarrow \exists\, t_r \overset{p,\tau_{jr}}{\longrightarrow} t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$$

$$\Rightarrow \exists\, t_r \overset{p+m_{ij},\tau_{ij}\cdot\tau_{jr}}{\longrightarrow} t_i, \text{ which is primal (but not necessarily unique)},$$

$$\Rightarrow A^{p+m_{ij}}(i,r) \geq \tau_{ij} \cdot \tau_{jr} = \tau_{ij} \cdot A^p(j,r) \qquad \{\text{Rem. 2}\}$$

Else, if $A^p(j,r) = \varepsilon$, the result trivially holds. In any case, property holds true for any r, thus it holds for all. □

Proof of Proposition 3.

There exists a circuit $t_i \overset{m_{ii},\tau_{ii}}{\longrightarrow} t_i$, therefore,

$\forall p \in \mathbb{N}^*,\ A^{p+m_{ii}}(i,:) \geq \tau_{ii} \cdot A^p(i,:)$ {Prop. 2}

One can recursively set $p := p + m_{ii}$ and easily show that

$\Rightarrow\ \forall p \in \mathbb{N}^*, \forall q \in \mathbb{N},\ A^{p+q\times m_{ii}}(i,:) \geq (q \times \tau_{ii}) \cdot A^p(i,:)$

One can further set $p := p + m_{ij}$,

$\Rightarrow\ \forall p \in \mathbb{N}^*, \forall q \in \mathbb{N},\ A^{p+m_{ij}+q\times m_{ii}}(i,:) \geq (q \times \tau_{ii}) \cdot A^{p+m_{ij}}(i,:)$

Plus:

$\exists\, t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$

$\Rightarrow\ \forall p \in \mathbb{N}^*, A^{p+m_{ij}}(i,:) \geq \tau_{ij} \cdot A^p(j,:)$ {Prop. 2}

$\Rightarrow\ \forall p \in \mathbb{N}^*, \forall q \in \mathbb{N},\ A^{p+m_{ij}+q\times m_{ii}}(i,:) \geq (q \times \tau_{ii} + m_{ij}) \cdot A^p(j,:)$

As we assumed no $0-$delay circuits, $\tau_{ii} > 0$, therefore,

$\forall \nu \in \mathbb{R}, \exists q_0 \in \mathbb{N},\ \nu + (q_0 \times \tau_{ii} + m_{ij}) \geq 0$

$\Rightarrow\ \forall q \geq q_0, \nu \cdot A^{p+m_{ij}+q\times m_{ii}}(i,:) \geq (\nu + q \times \tau_{ii} + m_{ij}) \cdot A^p(j,:)$

$\Rightarrow\ \forall \nu \in \mathbb{R}, \forall q \geq q_0, \forall p \in \mathbb{N}^*, \nu \cdot A^{p+m_{ij}+q\times m_{ii}}(i,:) \geq A^p(j,:)$ □

Proof of Proposition 4.

Existence of both $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$ and $t_i \overset{m_{ji},\tau_{ji}}{\longrightarrow} t_j$ implies there exists at least one circuit $t_i \overset{m_{ii},\tau_{ii}}{\longrightarrow} t_i$ where $m_{ii} = m_{ij} + m_{ji}$. For any of such circuit, let $(\eta^i_q)_{q\in\mathbb{N}}$ be the sequel counting the number of $\varepsilon-$element in $A^{q\times m_{ii}}(i,:)$.

$\forall p \in \mathbb{N}^*,\ A^{p+m_{ii}}(i,:) \geq \tau_{ii} \cdot A^p(i,:)$ {Prop. 2}

$\Rightarrow A^{p+m_{ii}}(i,:) \geq A^p(i,:)$ {non-negative delays}

$\Rightarrow A^{(q+1)\times m_{ii}}(i,:) \geq A^{q\times m_{ii}}(i,:)$ $\{p := q \times m_{ii}\}$

$\Rightarrow \forall r \in [1..N], \big[\, (A^{(q+1)\times m_{ii}}(i,r) = \varepsilon\ \Rightarrow\ A^{q\times m_{ii}}(i,r) = \varepsilon) \,\big]$

$\Rightarrow \eta^i_{q+1} \leq \eta^i_q$

$\Rightarrow (\eta^i_q)_{q\in\mathbb{N}}$ is decreasing, plus it is minimized by 0,

$\Rightarrow (\eta^i_q)_{q\in\mathbb{N}}$ converges. Since it takes only discrete values, the sequel reaches its limit at a given step $l \geq 0$.

$\Rightarrow \exists\, l \in \mathbb{N}, \forall q \in \mathbb{N},\ \eta^i_{l+q} = \eta^i_l$

$\Rightarrow [\, A^{(l+q)\times m_{ii}}(i,r) = \varepsilon \Leftrightarrow A^{l\times m_{ii}}(i,r) = \varepsilon \,]$

One can set $p_0 := l \times m_{ii}$,

$$\Rightarrow \exists\, p_0 \in \mathbb{N}, \forall\, q \in \mathbb{N}, \quad [\, A^{q\times m_{ii}+p_0}(i,:) = \varepsilon \Leftrightarrow A^{p_0}(i,:) = \varepsilon \,] \tag{A1}$$

which concludes proof of 4.1 by use of Prop. 1.

Furthermore, as there exist both $t_i \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_j$ and $t_j \overset{m_{ji},\tau_{ji}}{\longrightarrow} t_i$ paths,

$\forall p \in \mathbb{N}^*,\ \begin{cases} A^{p+m_{ij}}(i,:) \geq A^p(j,:) \\ A^{p+m_{ji}}(j,:) \geq A^p(i,:) \end{cases}$ {Prop. 2}

$\Rightarrow\ \forall p \in \mathbb{N}^*,\ A^{p+m_{ij}+m_{ji}}(i,:) \geq A^{p+m_{ji}}(j,:) \geq A^p(i,:)$

$\Rightarrow\ \forall p \in \mathbb{N}^*,\ A^{p+m_{ii}}(i,:) \geq A^{p+m_{ji}}(j,:) \geq A^p(i,:)$

One can set $p := p_0 = l \times m_{ii}$,

$\Rightarrow\ A^{(l+1)\times m_{ii}}(i,:) \overset{①}{\geq} A^{l\times m_{ii}+m_{ji}}(j,:) \overset{②}{\geq} A^{l\times m_{ii}}(i,:)$

Thus, for any $r \in [1..N]$:

$$\begin{array}{ll} \overset{①}{\Rightarrow} & \left[\ A^{(l+1)\times m_{ii}}(i,r) = \varepsilon \quad \Rightarrow \quad A^{l\times m_{ii}+m_{ji}}(j,r) = \varepsilon \ \right] \\ \overset{②}{\Rightarrow} & \left[\begin{array}{lll} A^{l\times m_{ii}+m_{ji}}(j,r) = \varepsilon & \Rightarrow & A^{l\times m_{ii}}(i,r) = \varepsilon \\ & \Leftrightarrow & A^{p_0}(i,r) = \varepsilon \\ & \overset{(A1):q=1}{\Leftrightarrow} & A^{m_{ii}+p_0}(i,r) = \varepsilon \\ & \Leftrightarrow & A^{(l+1)\times m_{ii}}(i,r) = \varepsilon \end{array} \right] \end{array}$$

$\Rightarrow$ [$A^{(l+1)\times m_{ii}}(i,:) = \varepsilon \;\Leftrightarrow\; A^{l\times m_{ii}+m_{ji}}(j,:) = \varepsilon$]
$\Rightarrow$ [$A^{p_0+m_{ij}+m_{ji}}(i,:) = \varepsilon \;\Leftrightarrow\; A^{p_0+m_{ji}}(j,:) = \varepsilon$]
One can set $\widetilde{p_0} := p_0 + m_{ji}$,
$\Rightarrow$ [$A^{\widetilde{p_0}+m_{ij}}(i,:) = \varepsilon \;\Leftrightarrow\; A^{\widetilde{p_0}}(j,:) = \varepsilon$]
Using Prop. 1, we deduce that,
$\Rightarrow \forall p \geq \widetilde{p_0}$, $(A^{p+m_{ij}}(i,:),\, A^p(j,:))$ similar, which concludes 4.2's proof.

Finally, 4.1 holds for all $p \geq p_0$, so it holds in particular for all $p \geq \widetilde{p_0}$. Therefore, for all $q \in \mathbb{N}$ and p $\geq \widetilde{p_0}$,
$(A^{p+q\times m_{ii}}(i,:),\, A^p(i,:))$ similar,
$\overset{\text{Prop. 1}}{\Rightarrow}$ $(A^{p+q\times m_{ii}+m_{ij}}(i,:),\, A^{p+m_{ij}}(i,:))$ similar,
$\overset{4.2}{\Rightarrow}$ $(A^{p+q\times m_{ii}+m_{ij}}(i,:),\, A^p(j,:))$ similar, which proves 4.3. □

Proof of Lemma 1.
If there exists $t_j \overset{m_{ij},\tau_{ij}}{\longrightarrow} t_i$ and $t_i \overset{m_{ji},\tau_{ji}}{\longrightarrow} t_j$ then there exists at least one circuit $t_i \overset{m_{ii},\tau_{ii}}{\longrightarrow} t_i$ where $m_{ii} = m_{ij} + m_{ji}$. Hence, we can apply both Prop.3 and Prop.4.3 and the following relations hold: $\forall \nu \in \mathbb{R}$, $\exists (p_0, q_0) \in \mathbb{N}^* \times \mathbb{N}$,

$$\nu \cdot A^{p_0+m_{ij}+q_0\times m_{ii}}(i,:) \geq A^{p_0}(j,:) \tag{A2}$$
$$(A^{p_0+m_{ij}+q_0\times m_{ii}}(i,:),\, A^{p_0}(j,:)) \text{ similar} \tag{A3}$$

From (A2), the right-hand side is contained in the left-hand side and there are both similar according to (A3). These two terms are then comparable. The remaining of the proof consists in deriving an upper-bound for the left-hand side using the right-hand side.

Let set $p \leftarrow p_0$ and $q \leftarrow m_{ij} + q_0 \times m_{ii}$ for convenience and define,
$\delta \leftarrow \max\limits_{\substack{r\in[1..N]\\ A^p(j,r)\neq\varepsilon}} (\nu \cdot A^{p+q}(i,r) - A^p(j,r))$
Note that we need to be careful here in the definition of δ because $(-\varepsilon) = +\infty$ does not belong to $\mathbb{R}_{\max}$.
From (A2), we have $\delta \geq 0$ and it follows that
$\forall r \in [1..N], A^p(j,r) \neq \varepsilon,$ $\qquad \nu \cdot A^{p+q}(i,r) \leq \delta \cdot A^p(j,r)$
Taking (A3) into account, this holds for any $r \in [1..N]$, hence we deduce,

$$\nu \cdot A^{p+q}(i,:) \;\leq \delta \cdot A^p(j,:) \tag{A4}$$

Furthermore, the state equation of form (3) with $\phi = p$ gives, for any $k \geq p$,
$$x_j(k) = A^p(j,:) \cdot x(k-p) \;\oplus\; \left[\bigoplus_{k'=0}^{p-1} (A^{k'} \cdot B)(j) \cdot u(k-k')\right]$$
By neglecting the terms from the sum and shifting the index k to $k-q$, one deduces the following inequalities
$$\begin{aligned} &\forall\, k \geq (p+q),\; x_j(k-q) \;\geq\; A^p(j,:) \cdot x(k-(p+q)) \\ \Leftrightarrow\quad & A^p(j,:) \cdot x(k-(p+q)) \;\leq\; x_j(k-q) \\ \Leftrightarrow\quad & \delta \cdot A^p(j,:) \cdot x(k-(p+q)) \;\leq\; \delta \cdot x_j(k-q) \end{aligned}$$
Taking into account (A4), one deduces

$$\nu \cdot A^{p+q}(i,:) \cdot x(k-(p+q)) \;\leq\; \delta \cdot x_j(k-q) \tag{A5}$$

Finally, using again the state equation of form (3) but with $\phi = p+q$, we obtain that for any $\nu \in \mathbb{R}$ and any $k \geq (p+q)$,

$$\nu \cdot x_i(k) = \nu \cdot A^{p+q}(i,:) \cdot x(k-(p+q)) \oplus \left[\bigoplus_{k'=0}^{p+q-1} \nu \cdot (A^{k'} \cdot B)(i) \cdot u(k-k') \right]$$

Using relation (A5), one obtains

$$\nu \cdot x_i(k) \leq \delta \cdot x_j(k-q) \oplus \left[\bigoplus_{k'=0}^{p+q-1} \nu \cdot (A^{k'} \cdot B)(i) \cdot u(k-k') \right]$$

One can then set $\mu_{k'} \leftarrow \nu \cdot (A^{k'} \cdot B)(i)$ for all $k \in [0..p+q-1]$ and conclude the proof of the lemma. $\square$