Performance of Distributed Algorithms in DTNs
Towards an Analytical Framework for Heterogeneous Mobility

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Outline

1. Motivation & Goals
2. Our New Model
   - State space
   - Local Search
   - Markov Chain Model
3. Correctness Analysis
   - Correctness Conditions
   - In Practice
4. Convergence Analysis
   - Calculating Convergence
   - In Practice
5. Conclusion & Outlook
Two Key Points
Two Key Points

Greedy (combinatorial) optimization
Two Key Points

Greedy (combinatorial) optimization

Heterogeneous mobility
Greed Is Not a Sin in DTNs

Sporadic, partial connectivity
\[\downarrow\]
Highly dynamic graph
\[\downarrow\]
Greedy algorithms for everything

This present contact  Potential future contacts

*Proverb: A bird in hand is worth two in the bush.*
Current DTN Analytical Models

Motive & Goals

Our New Model

Correctness Analysis

Convergence Analysis

Conclusion & Outlook

Current DTN Analytical Models

A. Picu, Dr. T. Spyropoulos

Performance of Distributed Algorithms in DTNs
Current DTN Analytical Models

Outline
- Motive & Goals
- Our New Model
- Correctness Analysis
- Convergence Analysis
- Conclusion & Outlook

A. Picu, Dr. T. Spyropoulos
Performance of Distributed Algorithms in DTNs
Studies of Real Mobility

\[ P^c = \{ p^c_{ij} \} \]
New Algorithms/Protocols (still greedy)

- **E. Daly and M. Haahr**
  *Social network analysis for routing in disconnected delay-tolerant MANETs*
  MobiHoc 2007

- **P. Hui, J. Crowcroft and E. Yoneki**
  *BubbleRap: Social-based Forwarding in Delay Tolerant Networks*
  MobiHoc 2008

Models (still individual)

- **T. Spyropoulos, T. Turletti and K. Obraczka**
  *Routing in Delay-Tolerant Networks Comprising Heterogeneous Node Populations*
  TMC 2009
Decouple

- **mobility** (contact probability matrix $P_{ij}^c$)
- **algorithms** (most of the time greedy)

to obtain

- correctness conditions on mobility scenario
- convergence probability and convergence delay

Illustrative example: Content Placement
Content/service placement
Content/service placement

\[ \mathbf{X} = (X_1, X_2, \ldots, X_N) = (1, 0, 1, 1, 1, 0, \ldots, 0) \]
Content/service placement

Node state space: \{0, 1\}  
Network state space: \left( \begin{array}{c} N \\ L \end{array} \right)
Node state space: \{0, 1\} \quad \text{Network state space: } \binom{N}{L}
Network State Difference

\[ \delta(x, y) = \sum_{1 \leq i \leq N} \{x_i \neq y_i\} \]

\[ x = (1, 0, 1, 1, 1, 0, 0, 0, 0) \]

\[ y = (1, 0, 1, 0, 1, 0, 0, 1, 0) \]

\[ z = (1, 0, 0, 0, 1, 1, 0, 1, 0) \]
Network State Difference

\[ \delta(x, y) = \sum_{1 \leq i \leq N} 1\{x_i \neq y_i\} \]
Network State Difference

\[ \delta(x, y) = \sum_{1 \leq i \leq N} \mathbb{1}\{x_i \neq y_i\} \]
Let $U_x > U_y$:

Transition probability $p_{xy}$:
Local Optimization Algorithms

Let $U_x > U_y$:

$x = (1, 0, 1, 1, 0, 0, 0, 0, 0)$

$y = (1, 0, 1, 0, 1, 0, 1, 0, 0)$

$p_{xy} = p^c_{48}$.

Transition probability $p_{xy}$:

- contact probability $p^c_{ij}$
Let $U_x > U_y$:

Transition probability $p_{xy}$:

- contact probability
- acceptance probability

$p_{xy} = p^c_{48} \cdot A_{xy}$
Local Optimization Algorithms

Let $U_x > U_y$:

Transition probability $p_{xy}$:

1. contact probability
2. acceptance probability

\[ A_{xy} = \begin{cases} 1 \{ U_x < U_y \} \\ f(U_x, U_y) \end{cases} \]

\[ p_{xy} = p^c_{48} \cdot 0 \]
Utilities

- node mobility
  ⇒ degree, contact probability
- node features
  ⇒ buffer space, battery
- content
  ⇒ demand for content

Content Placement

- node utility: 
  \( U \propto \) node degree
- state utility:
  sum of individual utilities

A. Picu and T. Spyropoulos

Minimum Expected \(^\ast\)-cast Time in DTNs
BIONETICS 2009
Putting the pieces together

Pieces of our new model:

1. **Mobility**
   (heterogeneous)

   ⇒ contact probability matrix
   \[ P^c = \{p_{ij}^c\} \]

2. **Solution space**
   (node & network)

   ⇒ e.g., combinations of L nodes

3. **Utilities**

   ⇒ e.g., node degree

4. **Algorithm**

   ⇒ acceptance probability matrix
   \[ A = \{A_{xy}\} \]
The Markov Chain

Complicated problem ⇒ Transition matrix, \( P = \{p_{xy}\} \)

\[
\begin{bmatrix}
0 & p_{12} & \cdots & p_{1N} \\
p_{21} & 0 & \cdots & p_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
p_{N1} & \cdots & \cdots & 0
\end{bmatrix}
\circ
\begin{bmatrix}
1 & A_{x_1x_2} & \cdots & A_{x_1x_M} \\
A_{x_2x_1} & 1 & \cdots & A_{x_2x_M} \\
\vdots & \vdots & \ddots & \vdots \\
A_{x_Mx_1} & \cdots & \cdots & 1
\end{bmatrix}
\]
The Markov Chain

Complicated problem $\Rightarrow$ Transition matrix, $P = \{p_{xy}\}$

\[
P = \begin{bmatrix}
p_{x_1x_1} & p_{x_1x_2} & \cdots & p_{x_1x_M} \\
p_{x_2x_1} & p_{x_2x_2} & \cdots & p_{x_2x_M} \\
\vdots & \vdots & \ddots & \vdots \\
p_{x_Mx_1} & \cdots & \cdots & p_{x_Mx_M}
\end{bmatrix}
\]

\[
P = \{p_{xy}\} = p_{ij}^c \cdot A_{xy}
\]
Why Is This Useful?

- Correctness analysis
  - Feasability of (mobility + algorithm)

- Convergence analysis
  - Achieve trade-off (e.g., delay vs. # copies)
  - Tune parameters
  - Assess performance
Every node has at least one higher utility neighbor?
When Is Greedy Content Placement Correct?

Utility

$L = 3$
(copies)
When Is Greedy Content Placement Correct?

$L = 3$
(copies)
When Is Greedy Content Placement Correct?

Every node has at least one higher utility neighbor?
When Is Greedy Content Placement Correct?

When Is Greedy Content Placement Correct?

$L = 3$
(copies)
Multihop greedy paths in traces
Impact of $L$ and utility

Node-to-relay greedy paths vs TTL (eth)

- $U_{deg}:L=3$
- $U_{deg}:L=10$
- $U_{rnd}:L=3$
- $U_{rnd}:L=10$

Node-to-relay greedy paths vs TTL (info)

- $U_{deg}:L=3$
- $U_{deg}:L=10$
- $U_{rnd}:L=3$
- $U_{rnd}:L=10$
What about convergence?

<table>
<thead>
<tr>
<th>Correct Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>⇒ Convergence probability = 1!</td>
</tr>
<tr>
<td>⇒ Convergence delay = ?</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-correct Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>⇒ Convergence probability = ?</td>
</tr>
<tr>
<td>⇒ Convergence delay = ?</td>
</tr>
</tbody>
</table>
Recall Our *Absorbing* Markov Chain

Complicated problem $\Rightarrow$ Transition matrix, $P = \{p_{xy}\}$

<table>
<thead>
<tr>
<th>$P^c$</th>
<th>mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{11}$</td>
<td>$p_{12}$</td>
</tr>
<tr>
<td>$p_{21}$</td>
<td>$p_{22}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$p_{N1}$</td>
<td>$\ldots$</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$P^c$</th>
<th>algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{11}$</td>
<td>$A_{x_1x_2}$</td>
</tr>
<tr>
<td>$A_{x_2x_1}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$A_{x_Mx_1}$</td>
<td>$\ldots$</td>
</tr>
</tbody>
</table>
Recall Our Absorbing Markov Chain

Complicated problem ⇒ Transition matrix, $P = \{p_{xy}\}$

$$P = \begin{bmatrix}
p_{x_1x_1} & p_{x_1x_2} & \cdots & p_{x_1x_M} \\
p_{x_2x_1} & p_{x_2x_2} & \cdots & p_{x_2x_M} \\
\vdots & \vdots & \ddots & \vdots \\
p_{x_Mx_1} & \cdots & \cdots & p_{x_Mx_M}
\end{bmatrix}$$

$$P = \{p_{xy}\} = p_{ij}^c \cdot A_{xy}$$
Transient Markov Chain Analysis

\[ P = \begin{pmatrix} TR & LM & x^* \\ Q & R_1 & R_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

Fundamental matrix: \( N = (I - Q)^{-1} \)
### Convergence Probability

<table>
<thead>
<tr>
<th></th>
<th>Optimum pred.</th>
<th>Optimum meas.</th>
<th>Local max. pred.</th>
<th>Local max. meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETH</td>
<td>1.0000</td>
<td>1.0000</td>
<td>N/A</td>
<td>N/A</td>
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<td>0.5267</td>
<td>0.5435</td>
<td>0.41321</td>
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<td>TVCM104</td>
<td>0.x</td>
<td>0.x</td>
<td>0.x</td>
<td>0.x</td>
</tr>
</tbody>
</table>

**Table:** Absorption probabilities
Average Convergence Delays

Convergence to optimum (PCA utility)

Convergence to local max (PCA utility)
Conclusion

- New unified model for DTNs
  - Heterogeneous node mobility
  - Greedy algorithms
  - Randomized gradient-ascent algorithms

- Model usage
  - Mobility conditions for algorithm correctness
  - Convergence probability
  - Convergence delay
Outlook

- Model generality
  - routing
  - resource allocation (buffer management)
  - etc.

- Practical issues: state space size
  - State lumping
  - Approximations
  - Petri-nets
The End

Thank you for your attention.

Please, ask questions.