Evolutionary Algorithms for Multiobjective Optimization

Eckart Zitzler

Computer Engineering and Networks Lab
Swiss Federal Institute of Technology (ETH) Zurich
Single objective optimization is a special case of multiobjective optimization (and not vice versa)
Overview

Focus:
Basic principles of evolutionary multiobjective optimization in contrast to single-objective optimization

Outline:
• Concepts (optimality, decision making)
• Algorithm design (fitness, diversity, elitism)
• Advanced topics (constraints, preference articulation)
• Applications
• Research topics
Maximize \((y_1, y_2, \ldots, y_k) = f(x_1, x_2, \ldots, x_n)\)
Pareto optimality:
defines set of optimal trade-offs
(all objectives equally important)

Decision making:
choose best compromise
(based on preference information)

Decision making before search (define single objective)

Decision making after search (find Pareto optimal set first)

Decision making during search (guide search interactively)

Combinations of the above
A Brief History of EMO

- **Pioneers** (~ 1990)
  - Objective-wise selection
  - Proof-of principle

- **Classics** (~ 1995)
  - Pareto-based selection
  - Niching
  - Visual comparisons

- **Elitists** (~ 2000)
  - Archiving + elitism
  - Quantitative performance metrics

- **Now?**

**Methods**
- VEGA [Schaffer 85]
  - [Fourman 85]
- ESVO [Kursawe 91]
- MOGA [Fonseca, Fleming 93]
- NPGA [Horn, Nafpliotis 93]
- NSGA [Srinivas, Deb 94]
- SPEA [Zitzler, Thiele 99]
- PAES, PESA [Knowles, Corne 99/00]
- NSGA-II [Deb et al. 00]
Issues in EMO

- How to maintain a diverse nondominated set?
  - density estimation

- How to prevent nondominated solutions from being lost?
  - archiving

- How to guide the population towards the Pareto set?
  - fitness assignment
A General Multiobjective EA

population

archive

evaluate
sample
vary

update
truncate

new population

new archive
Fitness Assignment Strategies

aggregation-based
weighted sum

criterion-based
VEGA

dominance-based
MOGA, NSGA, SPEA

parameter-oriented
scaling-dependent

set-oriented
scaling-independent
Dominance-Based Ranking

Types of information:

- **dominance rank**: by how many individuals is an individual dominated?
- **dominance count**: how many individuals does an individual dominate?
- **dominance depth**: at which front is an individual located?

Examples:

- **MOGA, NPGA**: dominance rank
- **NSGA/NSGA-II**: dominance depth
- **SPEA/SPEA2**: dominance count + rank
Potential Problems

2-objective knapsack problem

- Pareto-based: SPEA2
- Criterion-based: VEGA
- Aggregation-based: extended VEGA

Trade-off between distance and diversity?
Diversity Preservation

**Density estimation techniques:** [Silverman 86]

**Kernel**  
*MOGA, NPGA*  
density estimate  
=  
sum of f values  
where f is a function of the distance

**Nearest neighbor**  
*NSGA-II, SPEA2*  
density estimate  
=  
volume of the sphere defined by the nearest neighbor

**Histogram**  
*PAES, PESA*  
density estimate  
=  
number of solutions in the same box
Archiving Strategies

Which solutions should be kept in the archive?

General approaches:

- **Incremental**: candidates are considered one by one for insertion
- **En bloc**: all candidates are treated as a set

Selection criteria:

- **Dominance**: only nondominated solutions are kept
- **Density**: less crowded regions are preferred to crowded regions
- **Time**: old archive members are preferred to new solutions
- **Chance**: each solution has the same probability to enter the archive
Problem: Deterioration

**Goal:** Maintain “good” front (distance + diversity)

**But:** Most archiving strategies may loose Pareto-optimal solutions…
**Definition 1: **$\varepsilon$-Dominance

A $\varepsilon$-dominates B iff $\varepsilon \cdot f(A) \geq f(B)$

**Definition 2: **$\varepsilon$-Pareto set

A subset of the Pareto-optimal set which $\varepsilon$-dominates all Pareto-optimal solutions
**Goal:** Find an $\varepsilon$-Pareto set

**Idea:** $\varepsilon$-grid, i.e. maintain a set of nondominated boxes (one solution per box)

**Algorithm:** [Laumanns et al. 01]

Accept a new solution if the corresponding box is not dominated by any box represented in the archive

AND

any other archive member in the same box is dominated by the new solution
## Constraint Handling

<table>
<thead>
<tr>
<th>Penalty Functions</th>
<th>Constraints as Objectives</th>
<th>Modified Dominance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add penalty term to fitness</td>
<td>Introduce additional objective(s)</td>
<td>Extend to infeasible solutions</td>
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### Overall Constraint Violation
- [Michalewicz 92]
- [Wright, Loosemore 01]
- [Deb 01]

### Constraints Treated Separately
- ?
- [Coello 00]
- [Fonseca, Fleming 98]
How to incorporate preference information in order to focus the search on interesting regions of the Pareto front?

- Incorporating goals and priorities [Fonseca, Fleming 98]
- Generalizing the dominance concept [Miettinen 99]
- Combining weights and dominance [Parmee et al. 00]
Applications

Numerous applications:

Scheduling, Engineering Design, Bioinformatics, …

“Single-objective” applications:

- Making implicit objectives explicit
  - Genetic Programming and Bloat [Bleuler et al. 01]
  - Model Fitting [Hennig et al. 00]
- Treating constraints as objectives [Coello 00]
- Multiple goal programming [Deb 01]
- Escaping from local optima [Knowles et al. 01]
**Problem:** Trees grow rapidly
- Premature convergence
- Overfitting of training data

**Common approaches:**
- Constraint (tree size limitation)
- Penalty term (parsimony pressure)
- Objective ranking (size post-optimization)
- Structure-based (ADF, etc.)

**Multiobjective approach:**
Optimize both error and size

- Keep and optimize small trees (potential building blocks)
Multiobjective approach finds

- a correct solution with higher probability
- a correct solution slightly faster
- more compact (correct) solutions

than alternative approaches [Bleuler et al. 01].
Focus: particular photoreceptor in plant cells [Hennig et al. 00]

Experiments: grow plant cells in darkness; afterwards expose to
- continuous light
- pulse light (5min)

Goal: find a model which explains photoreceptor dynamics

Task: fit candidate model to both scenarios

Approach: each scenario corresponds to one objective

model cannot explain both scenarios well at the same time
Areas of Research

Algorithms:
- Improved techniques (distance/diversity tradeoff)
- Software toolboxes (building blocks)
- Alternative heuristics

Performance Assessment:
- Test problems
- Performance measures
- Statistical framework for performance assessment

Theory:
- Convergence and diversity proofs
- Investigation of concepts (fitness assignment)

Decision Making:
- EMO and classical MCDM methods
Links and Events

- EMO mailing list:
  http://w3.ualg.pt/lists/emo-list/

- EMO bibliography:
  http://www.lania.mx/~ccoello/EMOO/

- EMO test data:
  http://www.tik.ee.ethz.ch/~zitzler/testdata.html

- Conference on Evolutionary Multi-Criterion Optimization (EMO)
  - Zurich, Switzerland 2001: http://www.tik.ee.ethz.ch/emo/
  - Algarve, Portugal 2003: http://conferences.ptrede.com/emo03/

- Special track on EMO at CEC 2002 in Honolulu:
  http://www.tik.ee.ethz.ch/emotrack/