A Path Availability Model for Wireless Ad-Hoc Networks

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***Abstract*—Ad-hoc networks are expected to play an important role in future commercial and military communications systems. As such, scalable routing strategies capable of supporting greater user mobility and a wide range of applications are needed. This paper proposes a novel routing metric, which defines a probabilistic measure of the availability of network paths that are subject to link failures caused by node mobility in ad-hoc networks. It is shown how this measure can be used to select more stable paths and reduce the routing overhead caused by node mobility. A mobility model is first proposed and used to characterize the movement of ad-hoc network nodes. This model is then used to derive expressions for link and path availability. Finally, simulation results are reported which validate the proposed analytical model.**

I. Introduction

Wireless ad-hoc networks are self-organizing, rapidly deployable, and require no fixed infrastructure. They are comprised of wireless nodes, which can be deployed anywhere, and must cooperate in order to dynamically establish communications using limited network management and administration. Nodes in an ad-hoc network may be highly mobile, or stationary, and may vary widely in terms of their capabilities and uses. The objective of this *new* network architecture is to achieve increased flexibility, mobility and ease of management relative to *infrastructured* wireless networks.

Ad-hoc networks have no fixed network infrastructure for routing traffic. Furthermore, transmission range is limited by power constraints, frequency reuse and channel effects. Consequently, store-and-forward packet routing is required over multiple-hop wireless paths. Because communication end-points can move freely and independently of one another, routing is a difficult challenge.

Although routing must be responsive to dynamics in the network topology, efficiency is of paramount importance due to the scarcity and variability of network resources in ad-hoc networks. However, traditional routing algorithms designed to operate in infrastructured networks exhibit their poorest behavior and incur their greatest overhead when the topology is highly dynamic. Consequently, the need for greater efficiency limits the responsiveness of these algorithms to dynamic changes.

To address this problem, routing strategies which attempt to minimize the cost of reaction to topological changes [1], have been designed for operation in ad-hoc networks. However, it is not certain that any of the currently proposed strategies are sufficiently scalable, or capable of adapting effectively to high rates of node mobility [2]. One method that has been advocated to improve routing performance in ad-hoc networks is based on choosing the most stable paths [3, 4]. Specifically, a routing algorithm capable of choosing paths based, in part, on a probabilistic model of the future availability of the path could decrease the frequency of path failure. This, in turn, limits the need of the routing algorithm to respond to topological changes. Therefore, mobility models that can be used to predict the future state of ad-hoc network links are needed.

Previously, mobility models have focused on a single node moving relative to a fixed base station or network of base stations [5, 6]. To our knowledge, however, no work which addresses the *two-body* mobility problem encountered in ad-hoc networks has been published. To address this need, this paper proposes a well-defined mobility-based routing metric for ad-hoc networks. Analytical expressions are presented for the probability that a wireless link exists between two mobile nodes at time $t_0 + t$, given that a link exists between them at time t_0. This probability, defined as *link availability*, provides the basis for path selection based on the probability the path will remain available over a specified interval of time.

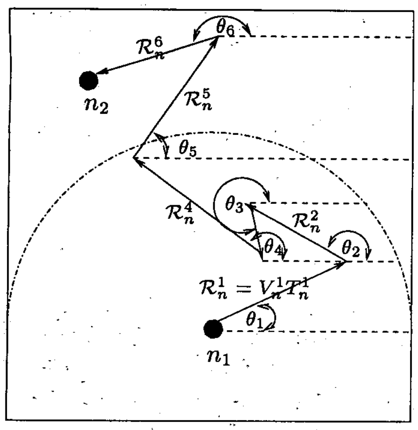
II. Ad-Hoc Mobility Model

In this section, a random walk-based mobility model for node movement in ad-hoc networks is proposed, and expressions characterizing the distribution of aggregate distance and direction covered by a node over an interval of time, t, are presented. These expressions provide the basis for deriving link and path availability.

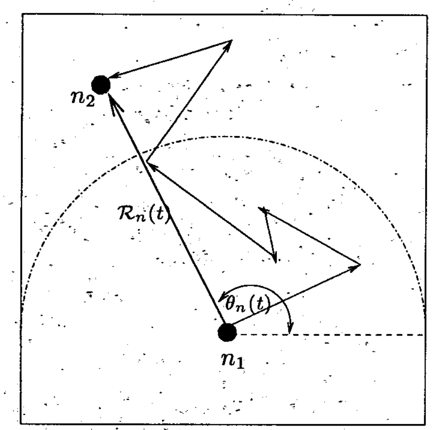
The mobility model developed in this section characterizes the aggregate behavior of nodes in a large network—an analogy can be drawn between this model and common communications models for packet and connection arrivals in which long-term average behavior is of interest.

A. Random Ad-Hoc Mobility

The random ad-hoc mobility model proposed in this section is a continuous-time stochastic process that characterizes the movement of nodes in a two-dimensional space. Based on this mobility model, each node's movement consists of a sequence of random length intervals called *mobility epochs* during which a node moves in a constant direction at a constant speed. The speed, V_n^i, and direction, θ_n^i, of each node varies randomly from epoch to epoch. Consequently, during epoch i of duration T_n^i, node n moves a distance of $V_n^i T_n^i$ in a straight line at an angle of θ_n^i. The number of epochs during an interval of length t is the discrete random process $\mathcal{N}(t)$. Fig. 1-a illustrates the movement of node n over six mobility epochs, each of which is characterized by its direction, θ_n^i, and distance, $V_n^i T_n^i$.



(a) Epoch mobility vectors.



(b) Random mobility vector $(0, T)$.

Figure 1: Ad-hoc mobility model node movement.

The *mobility profile* of a given node n moving according to a random ad-hoc mobility model is specified based on three parameters: λ_n, μ_n and σ_n^2. The following list defines these parameters for node n, and describes the assumptions made in developing this model:

- The epoch lengths, T_n^i, are IID exponentially distributed with mean $1/\lambda_n$.
- The speed, V_n^i, during each epoch is an IID distributed random variable with mean μ_n and variance σ_n^2, and remains constant only for the duration of the epoch.
- The direction, θ_n^i, of the mobile during each epoch is IID uniformly distributed over $(0, 2\pi)$, and remains constant only for the duration of the epoch.
- Speed, direction and epoch length are uncorrelated.
- Node mobility is uncorrelated and links fail independently.

The assumptions reflect a network environment in which there are a large number of autonomous nodes operating in a truly ad-hoc fashion, such that aggregate node movement can be modeled reasonably well as random mobility. Furthermore, correlation in movement among the nodes is assumed to be small enough to model each node's movement as an independent process. We are currently developing a heuristic methodology for enhancing the model to capture the effects of group mobility.

In order to characterize the availability of a link between two nodes over a period of time $(t_0, t_0 + t)$, the distribution of the mobility of one node with respect to the other must be determined. To characterize this distribution, it is first necessary to derive the mobility distribution of a single node in isolation. The single node distribution is extended to derive the joint mobility distribution which accounts for the mobility of one node with respect to the other. Using this joint mobility distribution, the link availability distribution is derived. If the link availability metric is known for each link along a path between two mobile nodes, assuming that links fail independently, the path availability is easily determined as the product of the individual link availability metrics.

B. Single Node Mobility

Two random vectors are central to the development of the single node mobility model. These vectors characterize the direction and distance moved by a mobile node during a single epoch and over an interval of length t respectively.

Definition 1 $\vec{\mathcal{R}}_n^i$ *is the* epoch random mobility vector *for node n. This vector represents the direction and distance moved by node n during mobility epoch i. Its magnitude,* $\mathcal{R}_n^i = |\vec{\mathcal{R}}_n^i| = V_n^i T_n^i$, *is the distance covered by node n during epoch i, and its phase,* θ_n^i, *is the direction of node n during epoch i.*

Definition 2 $\vec{\mathcal{R}}_n(t)$ *is the* random mobility vector *for node n. Its magnitude,* $\mathcal{R}_n(t)$, *is equal to the distance from* $(X(t_0), Y(t_0))$ *to* $(X(t_0 + t), Y(t_0 + t))$, *where* $(X(\tau), Y(\tau))$ *is the position of the node at time* τ. *Its phase angle,* $\theta_n(t)$, *is the angle of the line joining the node's initial position,* $(X(t_0), Y(t_0))$, *to its position at time* t_0+t, $(X(t_0+t), Y(t_0+t))$. *The random mobility vector can be expressed as a random sum of the epoch random mobility vectors:* $\vec{\mathcal{R}}_n(t) = \sum_1^{\mathcal{N}(t)} \vec{\mathcal{R}}_n^i$.

Fig. 1-a shows the movement of node n over an interval of length t, as it moves from position n_1 to position n_2. For each epoch, the figure shows the epoch vector $\vec{\mathcal{R}}_n^i$ with magnitude $V_n^i T_n^i$ and direction θ_n^i of the node during the epoch. The resulting random mobility vector $\vec{\mathcal{R}}_n(t)$ is shown in Fig. 1-b; it can be seen that it is the vector sum of the individual epoch vectors.

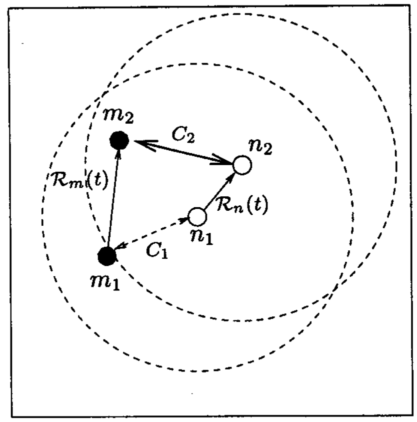
It is shown in [7] that the magnitude of $\vec{\mathcal{R}}_n(t)$ is approximately Raleigh distributed, and that its direction is uniformly distributed over $(0, 2\pi)$. Under reasonable conditions, these results are shown to hold for distributions of speed that are not identical during each epoch, and for more arbitrary distributions for the epoch length.

C. Joint Node Mobility

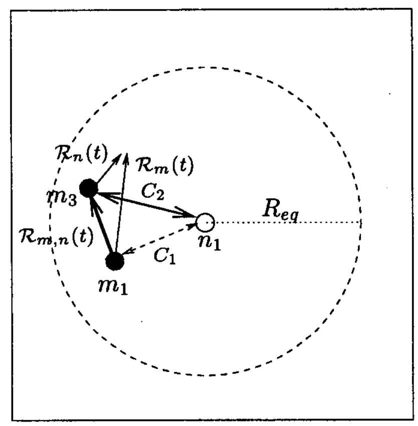
The characterization of mobility metrics for cellular networks relies on the analysis of the movement of a single node with respect to a fixed point of reference. The ad-hoc problem can be transformed into the cellular problem by considering the mobility of two nodes at a time, and fixing the frame of reference of one node with respect to the other. This transformation is accomplished by logically treating one of the nodes as if it were the *base station* of a cell, keeping it at a fixed position. For each movement of this node, the other node is translated an equal distance in the opposite direction. The result is the *equivalent random mobility vector*, which is characterized by the following definition:

Definition 3 *$\vec{\mathcal{R}}_{m,n}(t)$ is the* equivalent random mobility vector *of node m with respect to node n. It is defined by fixing m's frame of reference to n's position, and moving m relative to that point.*

Fig. 2 demonstrates the mobility of two nodes initially separated by a distance C_1. In Fig. 2-a, the two nodes move over an interval of length t: Node n moves according to its random mobility vector $\vec{\mathcal{R}}_n(t)$, from position n_1 to n_2, and node m moves according to $\vec{\mathcal{R}}_m(t)$ from position m_1 to m_2. The final distance between the two nodes is C_2. In Fig. 2-b, the position of node n is fixed at n_1 while node m is moved according to the equivalent random mobility vector, $\vec{\mathcal{R}}_{m,n}(t)$, from position m_1 to m_3. The transformation from single node random mobility vectors to the equivalent random mobility vector can be seen by noting that $\vec{\mathcal{R}}_{m,n}(t)$ is the vector difference $\vec{\mathcal{R}}_m(t) - \vec{\mathcal{R}}_n(t)$, and by observing how the progression of the distance between the nodes proceeds in an identical manner in Fig. 2-a and Fig. 2-b. It is shown in [7] that the magnitude of $\vec{\mathcal{R}}_{m,n}(t)$ is approximately Raleigh distributed, and the direction is uniformly distributed over $(0, 2\pi)$.



(a) Joint node case.



(b) Single node transformation.

Figure 2: Joint mobility transformation.

III. Link Availability Model

Link availability is a probabilistic model which predicts the future status of a wireless link. This section begins with a statement of our assumptions and definitions related to link status, and continues with the development of the distribution of link availability.

The status of a wireless link depends on numerous system and environmental factors that affect transmitter and receiver range. In general, a node's transmission range is neither fixed, nor symmetric—it demonstrates temporal and spatial variability. In this paper, a widely applied [5], albeit optimistic model has been adopted. Specifically, it is assumed that transmissions from a node, n, originate at the center of hexagonal region, which can be approximated by a circle of radius R_{eq}. If node m is located within this circle, it is assumed to correctly receive node n's transmissions.

Due to unpredictable propagation effects and power variability, unidirectional links are expected to be common in ad-hoc wireless environments. It is shown in [8] that this can have severe effects on ad-hoc routing algorithms. Consequently, R_{eq} applies to transmissions in one direction only—out-bound from the center of the circular transmission region. The value of R_{eq} could differ for each pair of nodes, and for the transmission direction. Therefore, a link in one direction does not imply a link in the other direction. The following definitions are needed to complete the development of the link availability model:

Definition 4 *An* active link, *from node n to node m, is defined as a directed communications link over which node n can transmit data directly to node m without any intermediate nodes. Bidirectional communications requires an active link in* both *directions.*

Definition 5 *$\mathcal{L}_{m,n}(t)$ is an indicator variable which reflects the state of the link directed from node n to node m at time t. $\mathcal{L}_{m,n}(t) = 1$ if the link is active and $\mathcal{L}_{m,n}(t) = 0$ if the link is inactive.*

Definition 6 Link availability *is defined as the probability that there is an active link between two mobile nodes at time $t_0 + t$, given that there is an active link between them at time t_0. Note that a link is considered available at time $t_0 + t$ even if it experienced failures during one or more intervals (t_i, t_j); $t_0 < t_i < t_j < t_0 + t$. More specifically, for nodes n and m,* link availability *is defined as:*

$$\mathcal{A}_{m,n}(t) \equiv Pr(\mathcal{L}_{m,n}(t_0 + t) = 1 | \mathcal{L}_{m,n}(t_0) = 1)$$

Depending on the initial status and location of nodes n and m, two cases of link availability are identified. Assuming node n is active at time t_0, Theorems 1 and 2 characterize the link availability between two mobile nodes, n and m, as reflected by the following initial conditions:

1. Node Activation: Node m activates at time t_0 at a random location within range of node n.
2. Link Activation: Node m moves within range of node n at time t_0 by reaching the boundary defined by R_{eq}, and is located at a random point around the boundary.

The analysis in [5] presents the derivation of the the distance, Z, a mobile node must travel before reaching the boundary of a cell with an effective radius of R_{eq}, when the mobile moves in a random uniform direction. Distributions of Z are derived for cases which are equivalent to the two cases described above. Link availability as given in Definition-6 can, therefore, be expressed as follows:

$$\mathcal{A}_{m,n}(t) \equiv Pr(\mathcal{R}_{m,n}(t) < Z) \quad (1)$$

This probability can be evaluated by substituting appropriate distributions for Z and $\mathcal{R}_{m,n}(t)$, and conditioning on $Z = z$. Complete derivations appear in [7].

The following two theorems characterizes the the link availability between two mobile nodes m and n at time $t \geq t_0$ for the two cases described above:

Theorem 1 Node Activation: *If node n moves according to a random ad-hoc mobility profile, $<\lambda_n, \mu_n, \sigma_n^2>$, and node m activates at time t_0 within a uniform random distance from node n and moves according to a random ad-hoc mobility profile, $<\lambda_m, \mu_m, \sigma_m^2>$, then the distribution of the link availability over time is given approximately by the following expression, where $\Phi(a, b, z)$ is the Kummer-Confluent Hypergeometric function:*

$$\mathcal{A}_{m,n}(t) \approx 1 - \Phi(\frac{1}{2}, 2, \frac{-4R_{eq}^2}{\alpha_{m,n}}) \quad (2)$$

$$\alpha_{m,n} = 2t(\frac{\sigma_m^2 + \mu_m^2}{\lambda_m} + \frac{\sigma_n^2 + \mu_n^2}{\lambda_n}) \quad (3)$$

Theorem 2 Link Activation: *Let $< \lambda_n, \mu_n, \sigma_n^2 >$ be the random ad-hoc mobility profile of node n, and let $< \lambda_m, \mu_m, \sigma_m^2 >$ be the random ad-hoc mobility profile of node m. Assume that a link activates between n and m at time t_0 such that m is located at a uniform random point exactly R_{eq} from n, then the link availability is distributed according to the following expression, where I_0 is a modified Bessel function of the first kind, and $\alpha_{m,n}$ is expressed in equation 3:*

$$\mathcal{A}_{m,n}(t) = \frac{1}{2}(1 - I_0(\frac{-2R_{eq}^2}{\alpha_{m,n}}) \exp(\frac{-2R_{eq}^2}{\alpha_{m,n}})) \quad (4)$$

A. Random Ad-Hoc Path Availability

The following definition extends the link availability concept to an entire path and Corollary-1 completes the model developed in this section by relating path availability to the individual link availabilities.

Definition 7 *$\mathcal{P}_{m,n}^k(t)$ is an indicator variable which reflects the status of directed path k from node n to node m at time t. $\mathcal{P}_{m,n}^k(t) = 1$ if all the links in the path are active at time t, and $\mathcal{P}_{m,n}^k(t) = 0$ if one or more links in the path are inactive at time t. The directed* path availability *$\Pi_{m,n}^k(t)$ between two nodes, n and m, at time $t \geq t_0$ is given by the following probability expression:*

$$\Pi_{m,n}^k(t) \equiv Pr(\mathcal{P}_{m,n}^k(t_0 + t) = 1 \mid \mathcal{P}_{m,n}^k(t_0) = 1)$$

Corollary 1 *Let $\mathcal{A}_{i,j}(t)$ be the availability for link $(i, j) \in$ path k between nodes m and n as defined in Definition-6. The directed path availability at time $t_0 + t$ is denoted $\Pi_{m,n}^k(t)$. According to the assumption of independent link failures, path availability is given by:*

$$\Pi_{m,n}^k(t) = \prod_{(i,j)\in k} \mathcal{A}_{i,j}(t) \quad (5)$$

Corollary-1 provides a metric which represents a probabilistic measure of path availability. This metric can be used by the routing algorithm to favor more stable paths, or to select paths which support a lower bound, α, on availability over an interval of length t as specified in the following expression:

$$\Pi_{m,n}^k(t) \geq \alpha \quad (6)$$

IV. Model Validation

Discrete-event simulation was used to validate the analytical models presented in this paper. Selected results are shown in this section. In general, the simulation results demonstrate excellent agreement with the analytical model and converge, as expected, as λt increases. Furthermore, for large values of λt, convergence of the link availability model is very rapid. In the worst case, for small values of λt, the results from analytical model are generally within 10% of the simulation. The path availability model faired slightly worse, as the path length was increased, largely because it is the product of the link availabilities; however, it also converges with increasing λt and provides a lower bound on the simulated path availability. Although there are some profiles which result in larger error for small values of λt, they demonstrate rapid convergence as λt increases.

A. Simulation Model

A simulation model was developed consisting of two processes representing the nodes at each end of a wireless link. To model the initial conditions for Theorem-1 the two nodes were initially located randomly within range of one another. Whereas, for Theorem-2, one node was initially located at a random point, a distance of R_{eq} from the other. Each simulated node moved according to its own random ad-hoc mobility profile as defined in Subsection-II.A.. During each simulation run, the nodes moved over a range of observation intervals. Specifically, each experiment consisted of 40 evenly spaced observation intervals ranging in length from $t = 0$ to $t = 120$ minutes. 100000 independent replications were run for each observation interval.

B. Simulation Output Analysis

Simulation results are compared to the analytical model for several of the more important experiments in Fig. 3. A more complete presentation and analysis, which is consistent with results reported here, appears in [9]. Results are shown for Theorem-1 and Theorem-2 for nodes moving at 10 and 20 kilometers-per-hour (kph). For each case, the effective transmission range was varied from 100 to 1000 meters (m)—a range considered reasonable for low to moderate powered radio-frequency devices.

Fig. 3-a and Fig. 3-b reflect the node activation model. As the mean speed increases, the link availability decreases for each value of R_{eq}. This result is consistent with intuitive expectations. It is interesting to observe, however, that as the speed increases the analytical results for small values of t become less precise. This is due to the fact the actual number of epochs that complete prior to a link failure is inversely proportional to the speed. In contrast to this effect, the convergence between the two models as a function of time is far more rapid as the speed is increased.

Similar effects can be observed, in Fig. 3-c and Fig. 3-d, for the link activation model as the speed is increased. However, in these cases the overall availabilities are lower than in the node activation case. This is due to the high probability of a node effectively turning away and a link failing sooner since the nodes always start at the threshold distance. Furthermore, for small values of t the accuracy of link availability model suffers for the same reason. Since the nodes are near their threshold separation, the probability of a link failing quickly is very high. This means that the conditions for satisfying the Central Limit Theorem, which is the basis for the derivation of this model, are more difficult to meet since the number of mobility epochs will be small in comparison to the node activation case. As t increases the initial positions become less of a factor and the models converge rapidly as expected.

Fig. 3-e and Fig. 3-f show results for the simulation of multi-hop path availability for path lengths of 4 hops and 8 hops respectively. The path availability model demonstrates convergence characteristics similar to those of the link availability cases. Although the relative error increases with increasing path length for small values of λt, this effect is expected and the analytical model does converge to the simulated values.

V. Conclusions

In this paper a random-walk based mobility model was proposed to characterize node movement in ad-hoc networks. To our knowledge, no other work has been published specifically treating the problem of mobility modeling for ad-hoc networks. Based on this model, expressions for the probability of link and path availability were derived for different initial conditions. The analytical results were validated using discrete event simulation. The results generally showed excellent agreement between the simulation and the analytical model.

The expressions for link availability provide the basis for a novel routing metric. This routing metric places a bound on the probability of path failure. The ability to probabilistically determine the path availability could be used to support increased node mobility, and improve the scalability of routing algorithms for ad-hoc networks. As an enhancement to this model, techniques are being developed to capture the effects of group and non-uniform mobility patterns. Group dynamics are expected to be a crucial property of many ad-hoc network applications.

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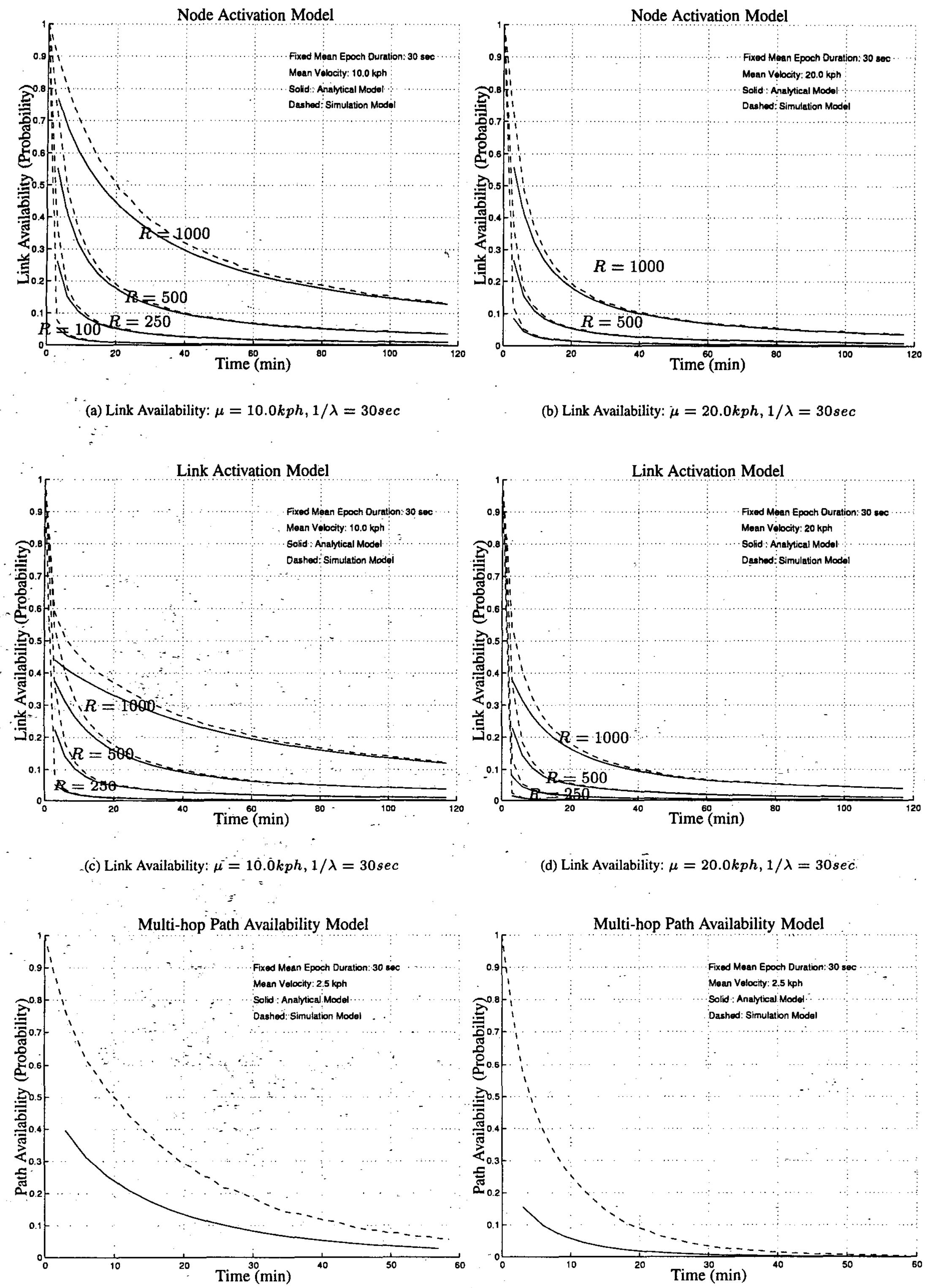

(a) Link Availability: $\mu = 10.0kph, 1/\lambda = 30sec$

(b) Link Availability: $\mu = 20.0kph, 1/\lambda = 30sec$

(c) Link Availability: $\mu = 10.0kph, 1/\lambda = 30sec$

(d) Link Availability: $\mu = 20.0kph, 1/\lambda = 30sec$

(e) Path Availability: Path length = 4 hops

(f) Path Availability: Path length = 8 hops

Figure 3: Comparison of analytical and simulation results.