

Time-Optimal Information Exchange on Multiple Channels



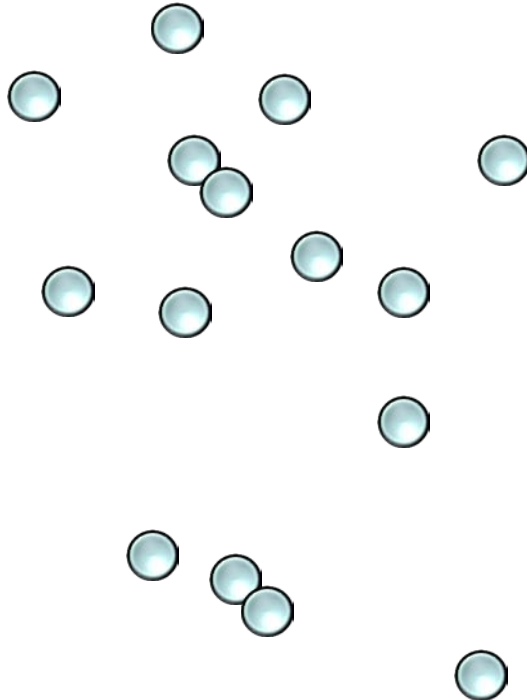
*Stephan Holzer
Yvonne Anne Pignolet
Jasmin Smula
Roger Wattenhofer*

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

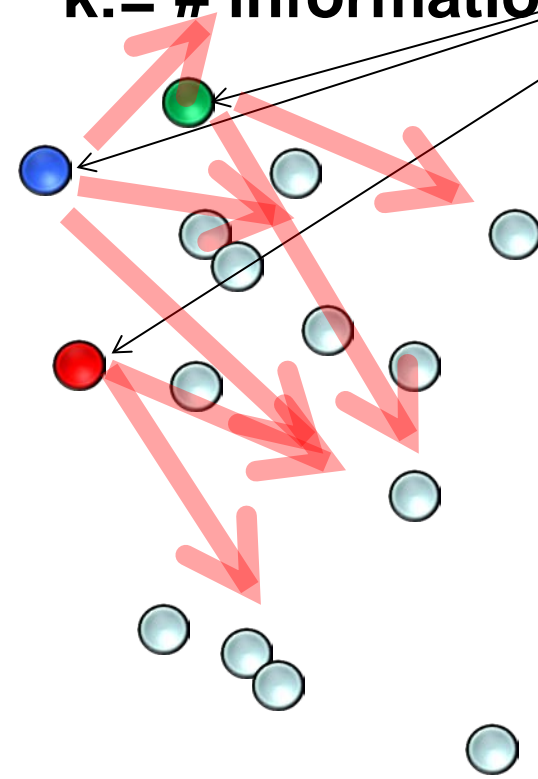
Problem:



Time-Optimal Information Exchange on Multiple Channels



Problem: $n := \# \text{ nodes}$
 $k := \# \text{ information}$ Have information

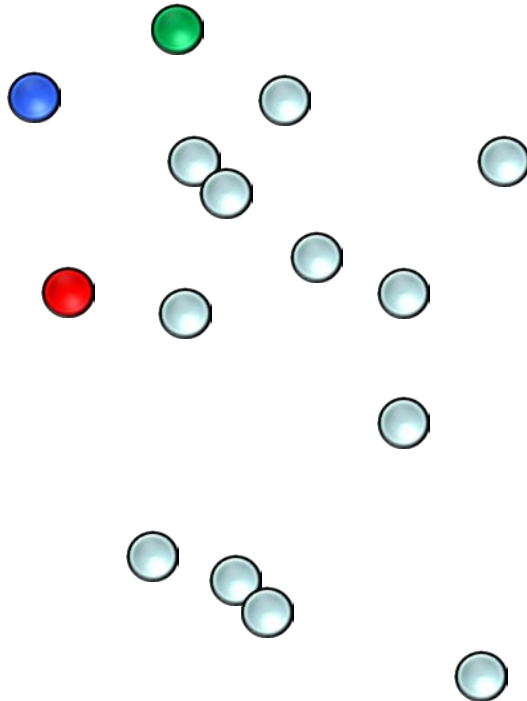



Disseminate to all! ?

Time-Optimal Information Exchange on Multiple Channels



Problem:

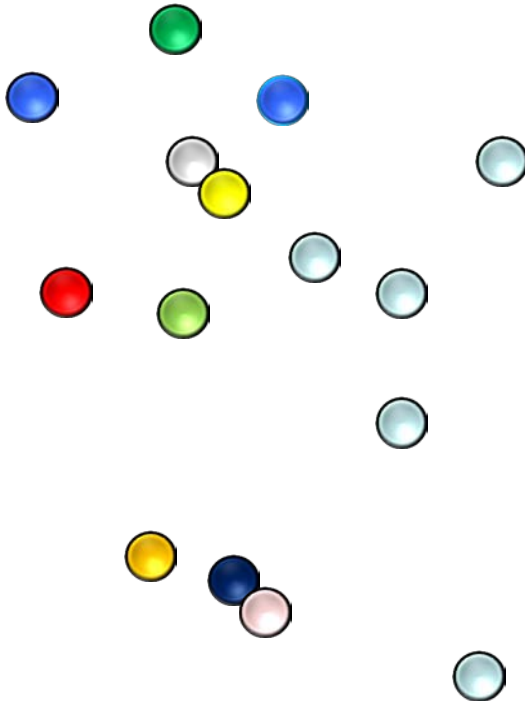


Disseminate to all! 

Time-Optimal Information Exchange on Multiple Channels



Problem:



Disseminate to all! ?

Easy: $O(n)$

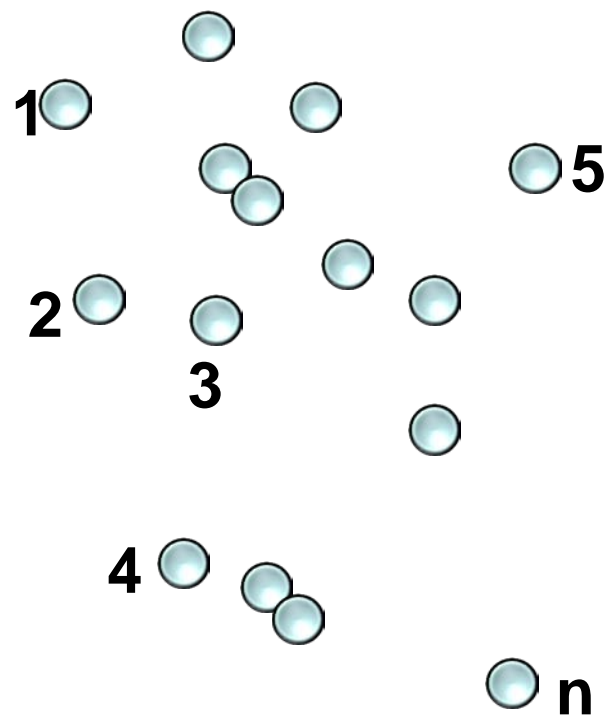
Faster?

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

Problem:



Unique IDs 1...n

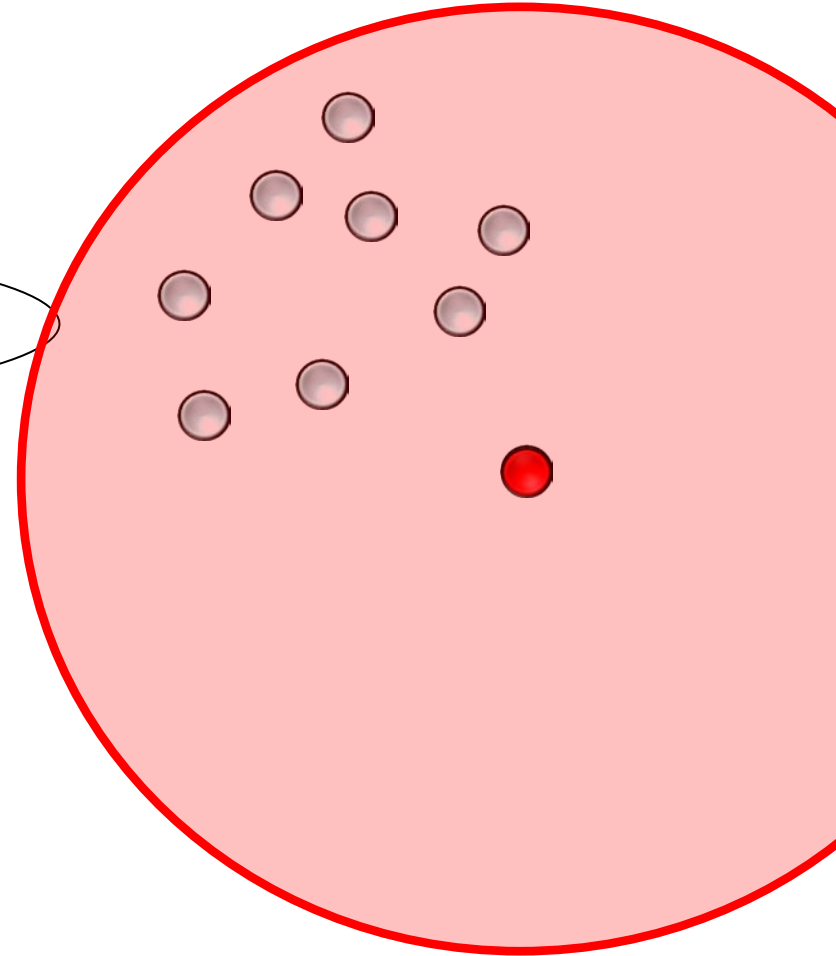
Time-Optimal Information Exchange Channels



I can:

send / receive

reach each node



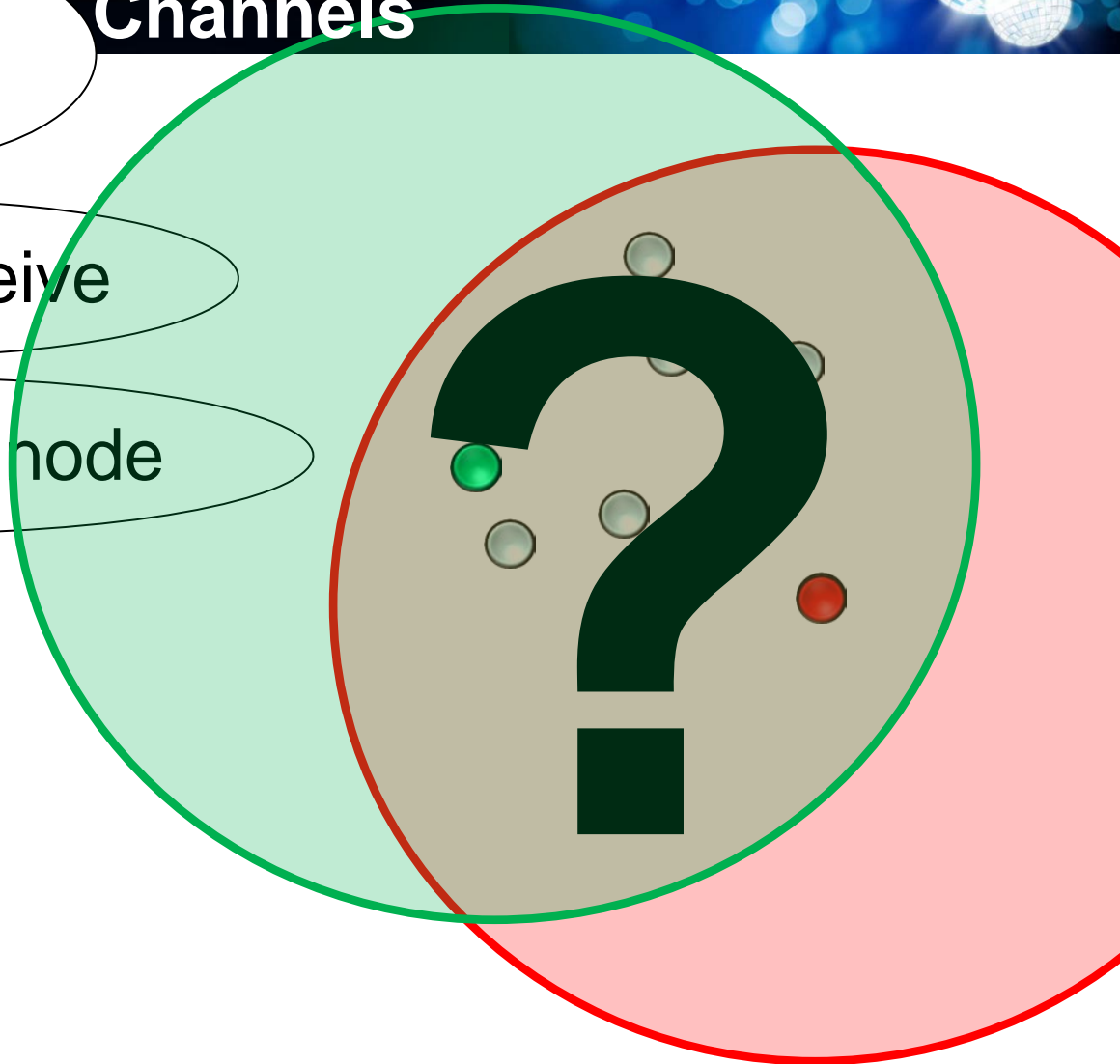
Time-Optimal Information Exchange Channels



I can:

send / receive

reach each node



Time-Optimal Information Exchange Channels

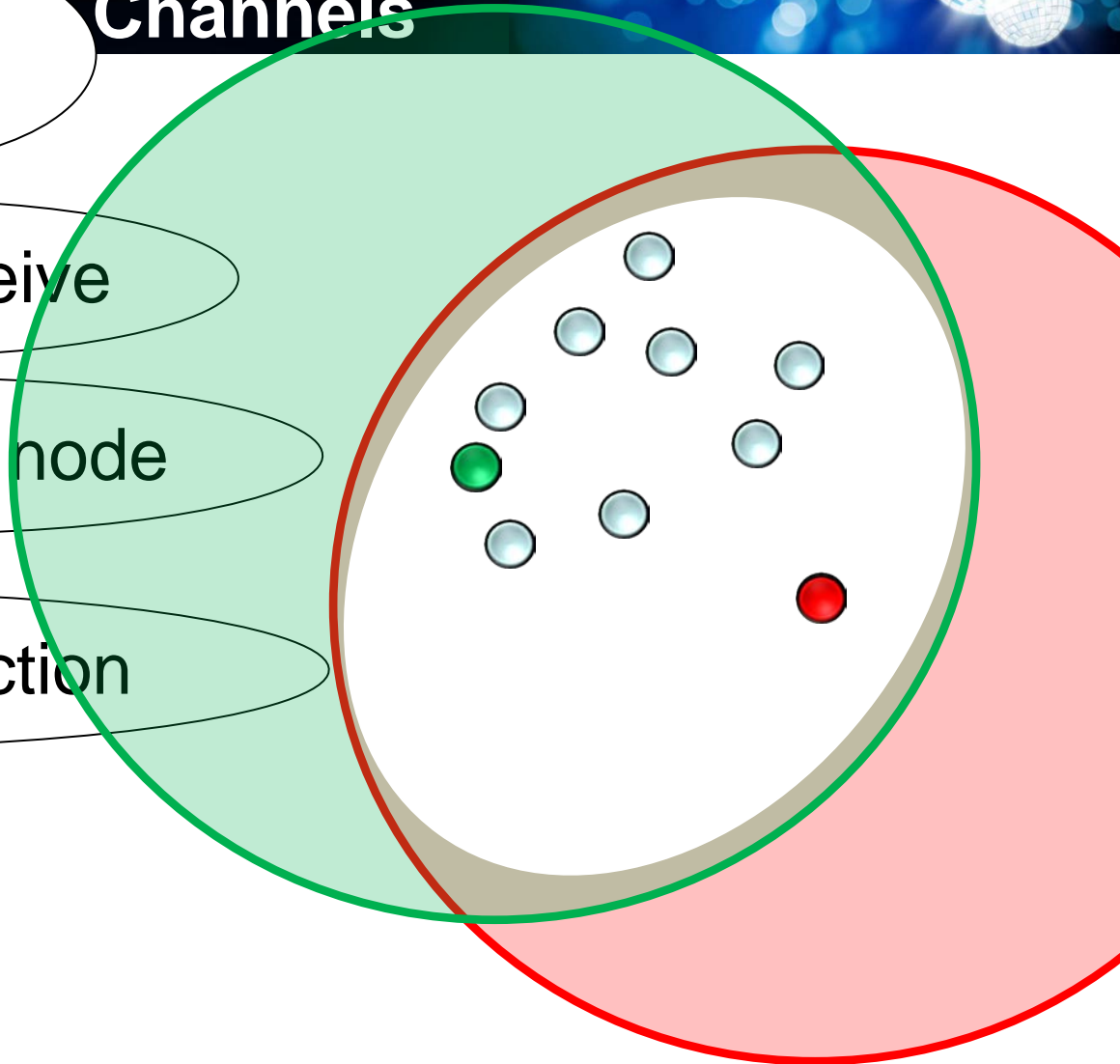


I can:

send / receive

reach each node

no collision detection



Time-Optimal Information Exchange Channels



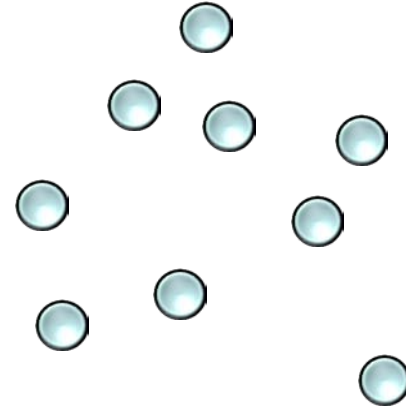
I can:

send / receive

reach each node

no collision detection

switch channels



101 Mhz
117 Mhz
132 Mhz

synchronous ...

Time-Optimal Information Exchange Channels



I can:

send / receive

reach each node

no collision detection

switch channels

complexity

computation: free

radio: time 1

synchronous

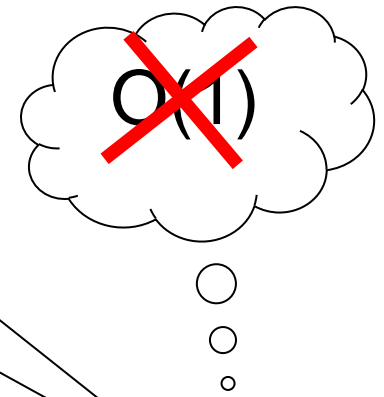
Time-Optimal Information Exchange on Multiple Channels



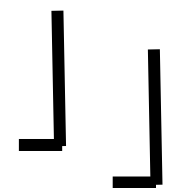
$n := \# \text{ nodes}$

$k := \# \text{ information}$

One Information / $\log n$ bits per message



$$\Rightarrow \Omega(k)$$



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

$k := \# \text{ information}$

Multi channel

$$\Theta(k)$$

one channel

$$\Rightarrow \Omega(k + \log n)$$

[Kushilevitz, Mansour SIAM JComp 1998]

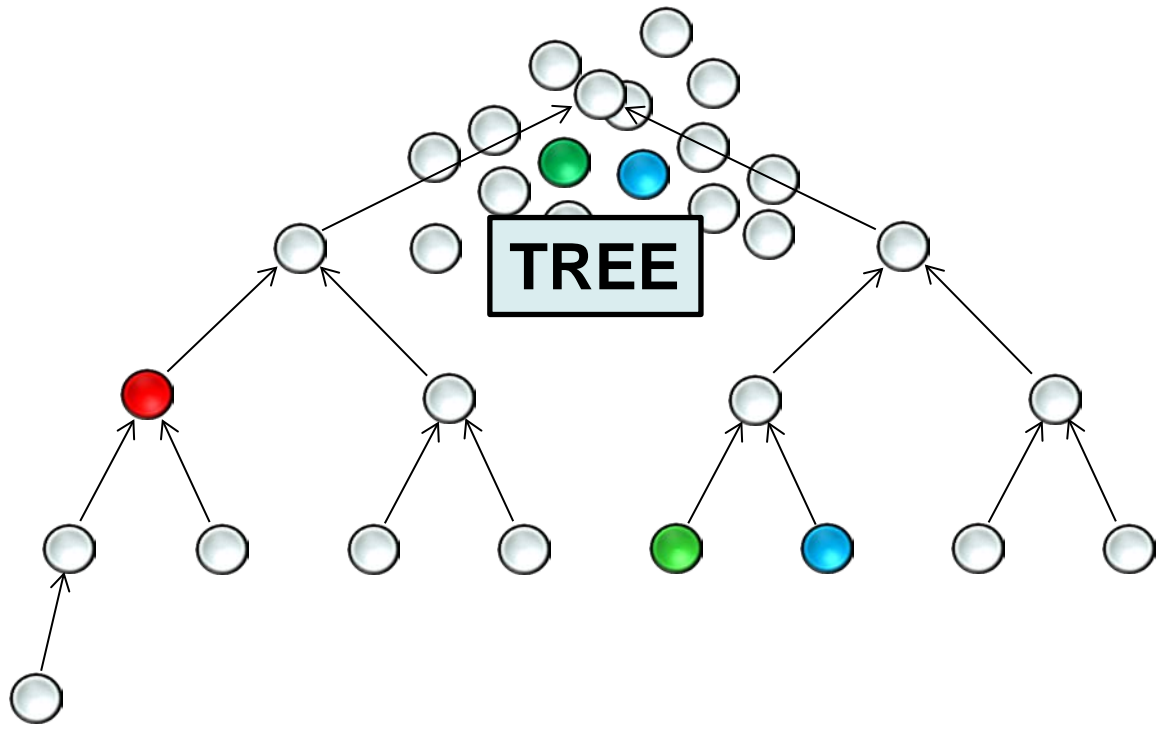
?

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$

What can I do?

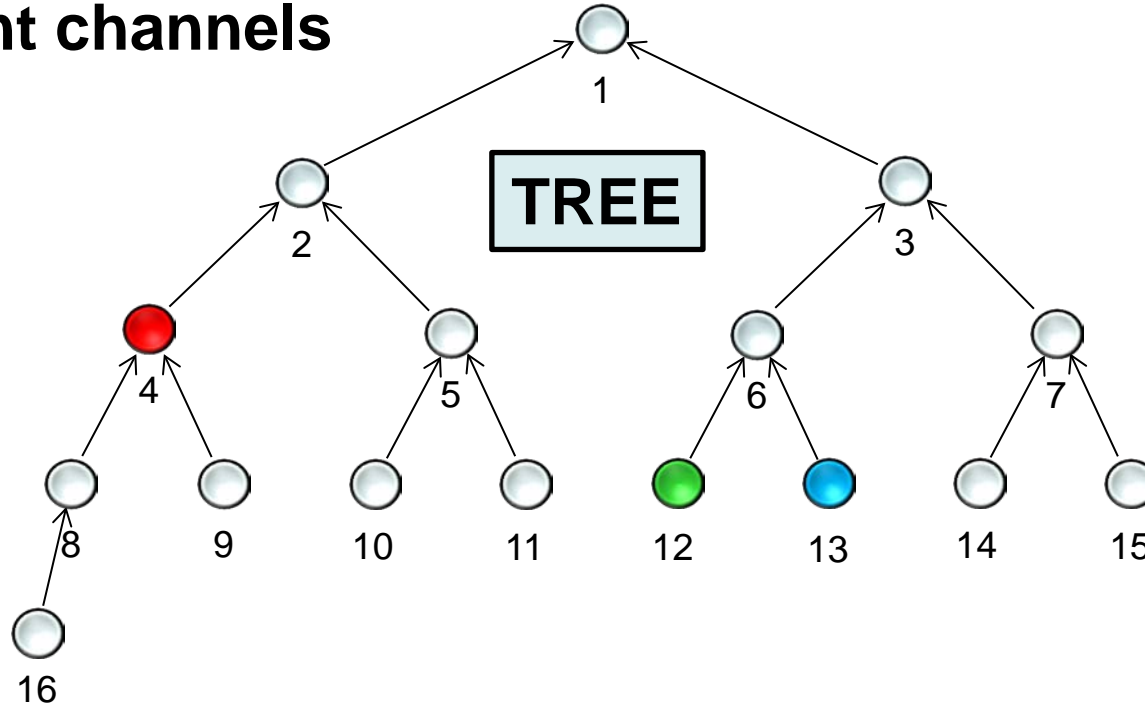


Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$

Communicate in parallel
on different channels

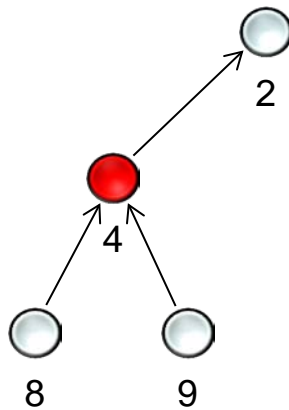


Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$

Communicate in parallel
on different channels

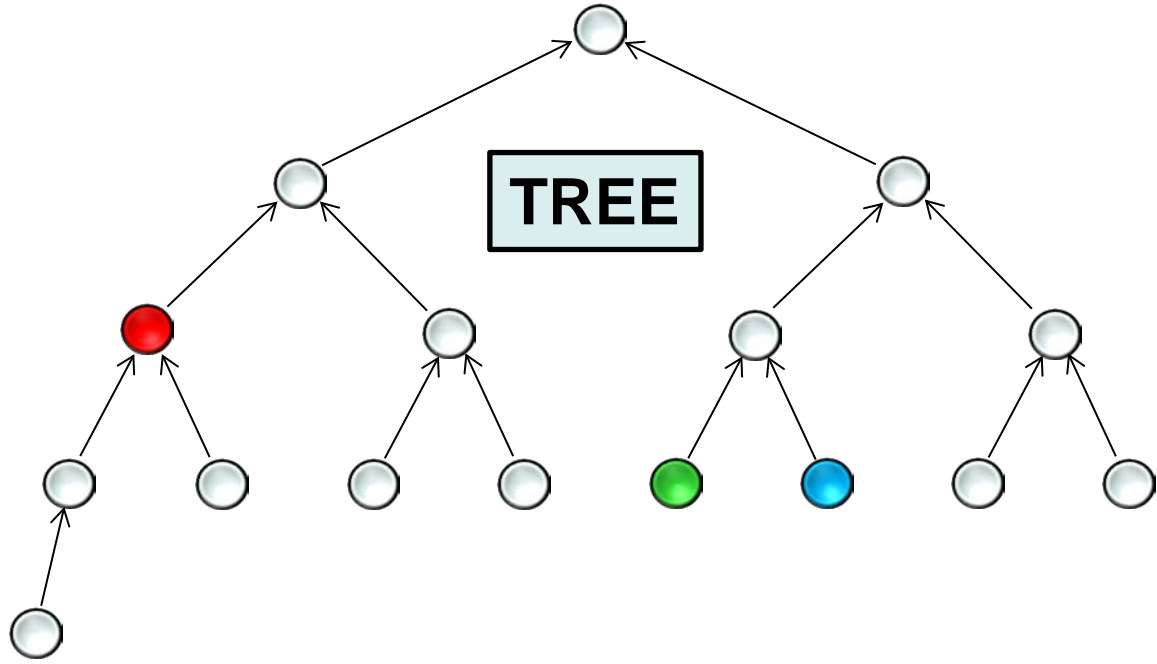


TREE

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$

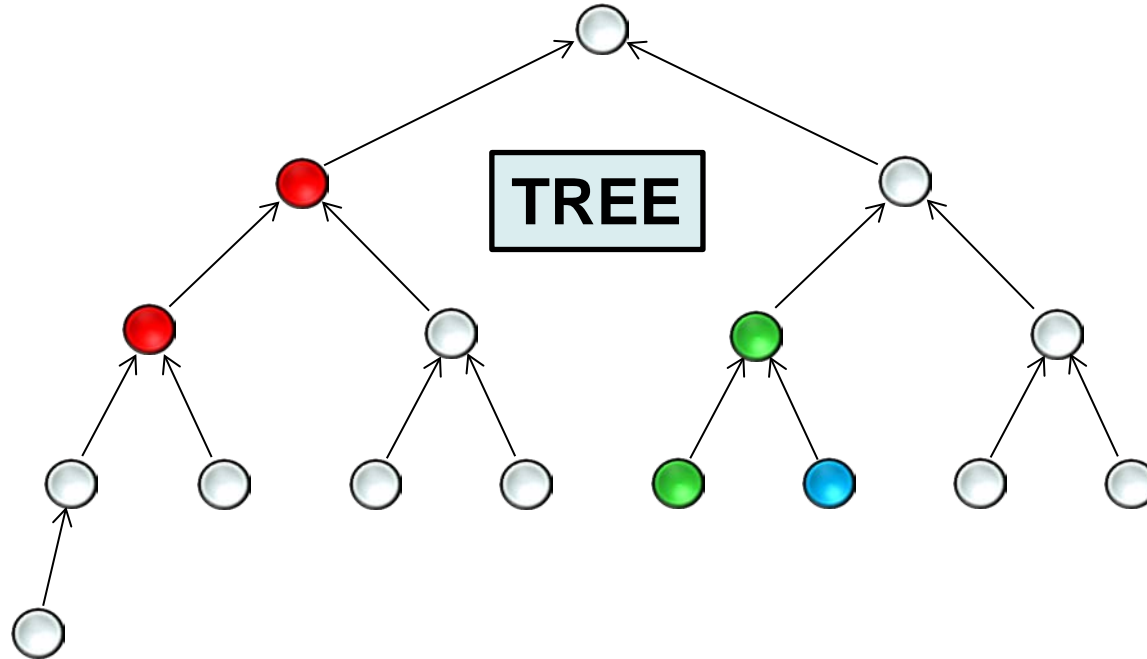


Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

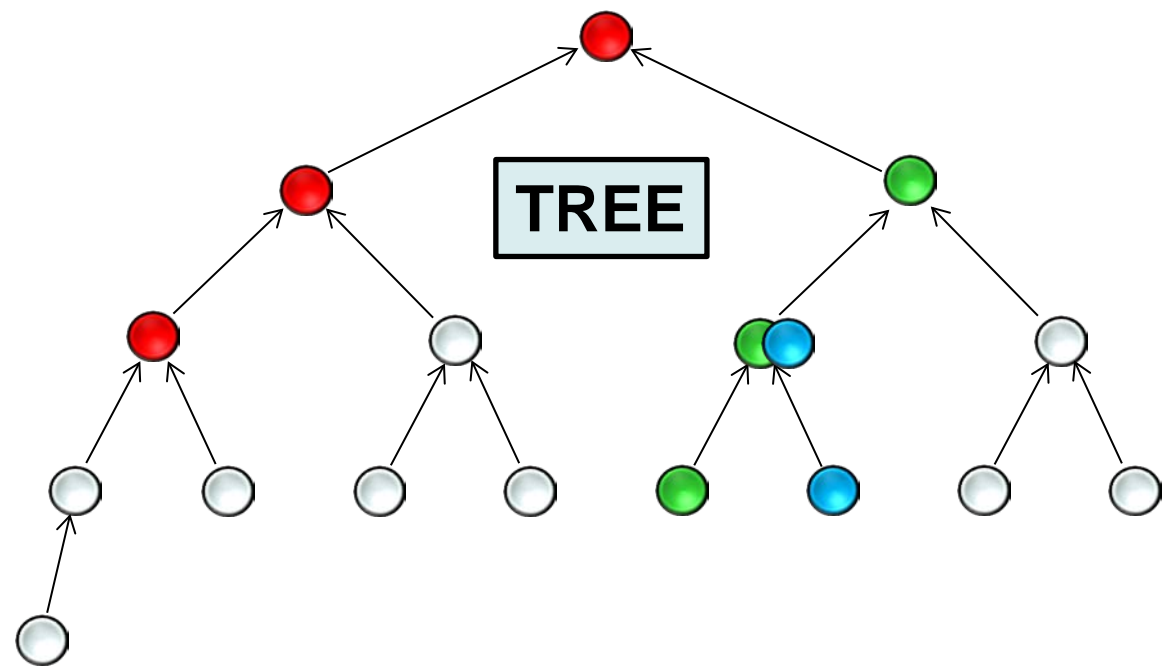
$k := \# \text{ information}$



Time-Optimal Information Exchange on Multiple Channels



n := # nodes
k := # information

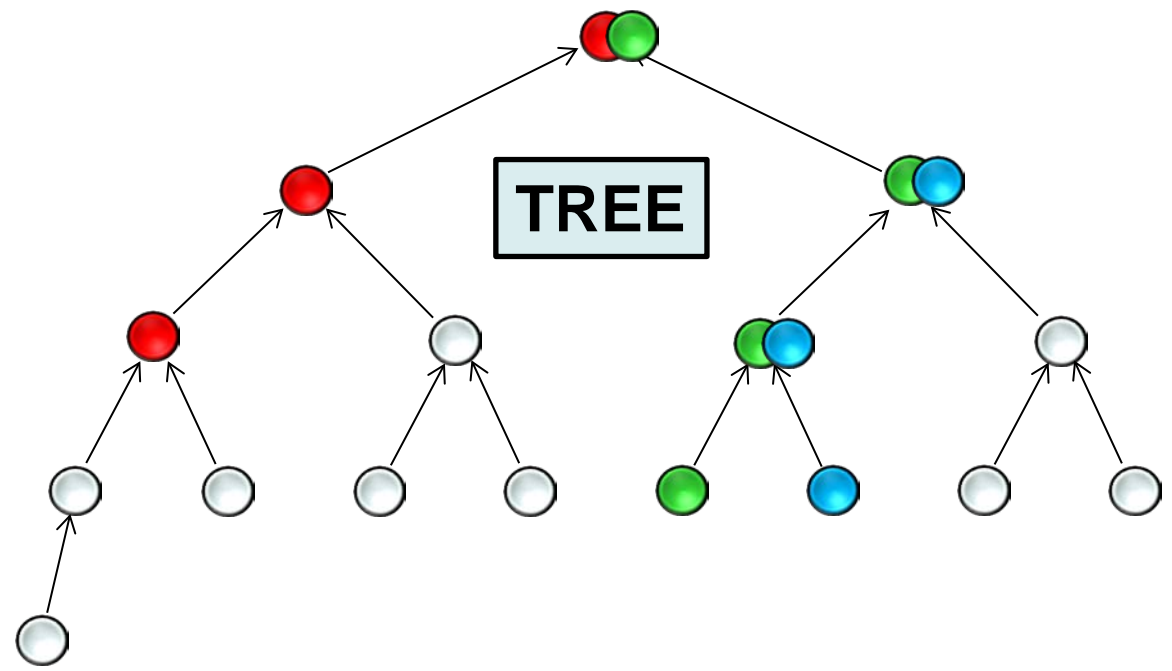


Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

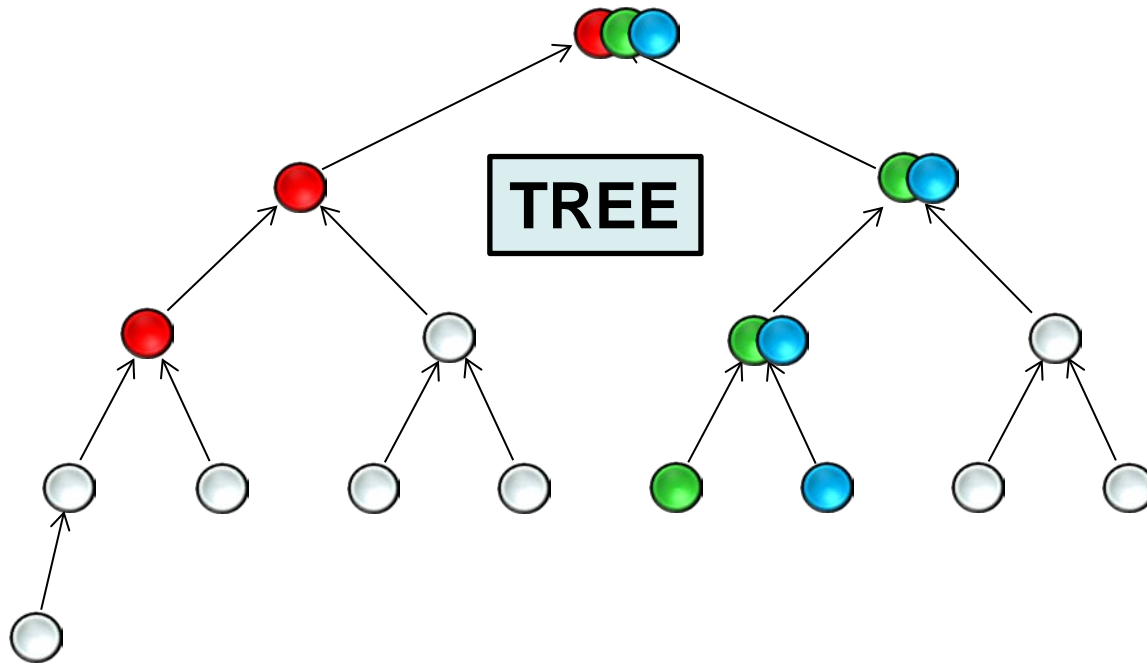
$k := \# \text{ information}$



Time-Optimal Information Exchange on Multiple Channels



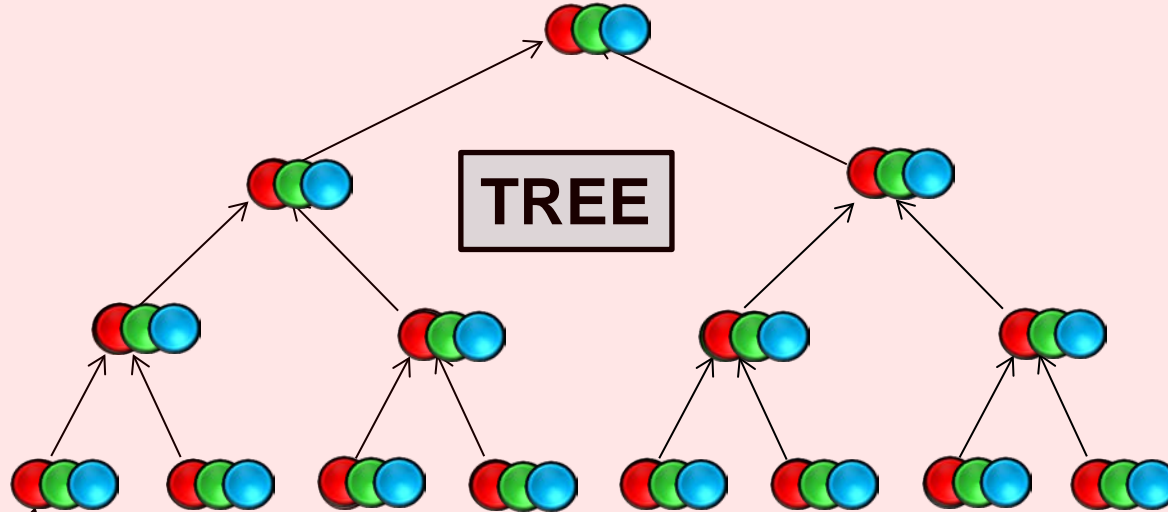
$n := \# \text{ nodes}$
 $k := \# \text{ information}$



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$



$$O(k + \log n)$$

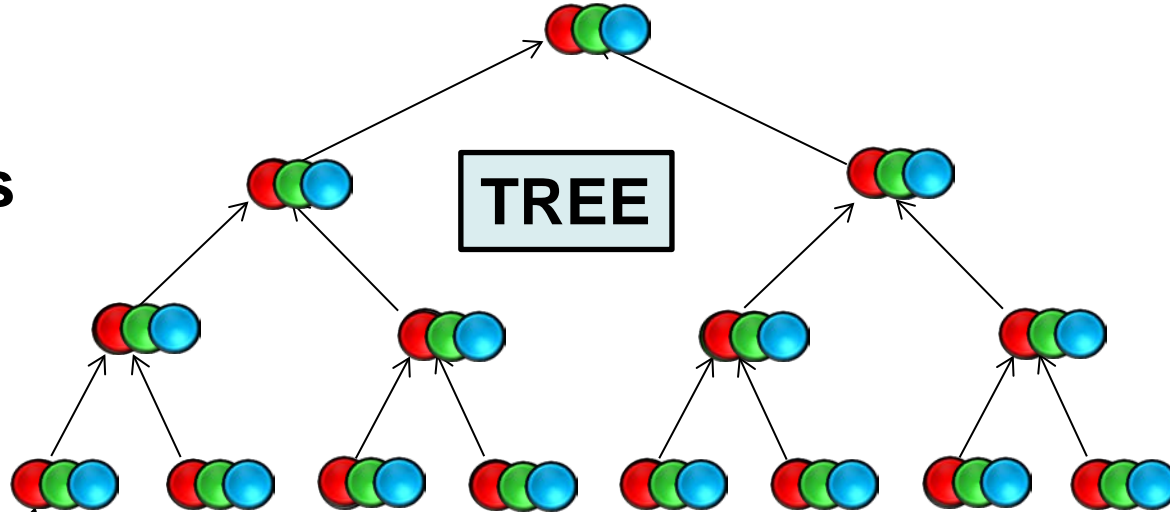
Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$

What if $k < \log n$?

n channels



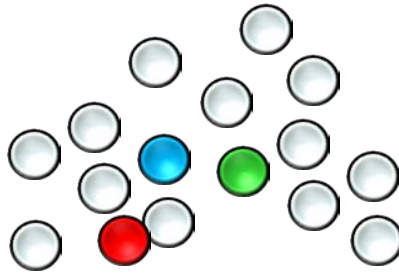
$O(k)$ if $k > \log n$

Time-Optimal Information Exchange on Multiple Channels



$n :=$ # nodes
 $k :=$ # information

What if $k < \log n$?



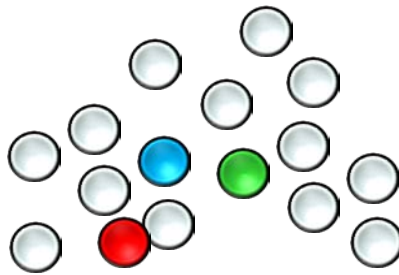
Time-Optimal Information Exchange on Multiple Channels



n:= # nodes

k:= # information

Assume: k known



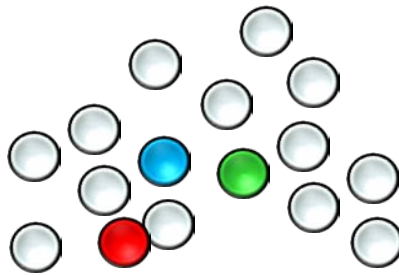
Time-Optimal Information Exchange on Multiple Channels



$n := \#$ nodes

$k := \#$ information

Assume: k known



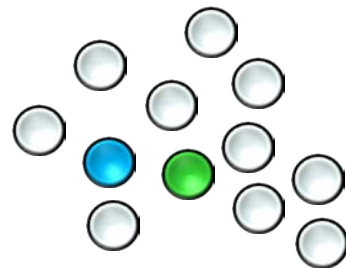
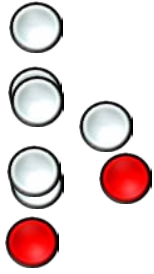
Balls into Bins

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$
Assume: k known

ID=1... 2^k



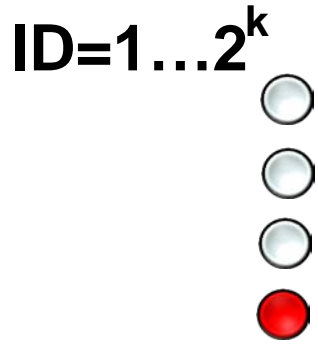
Balls into Bins



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$
Assume: k known



Balls into Bins



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

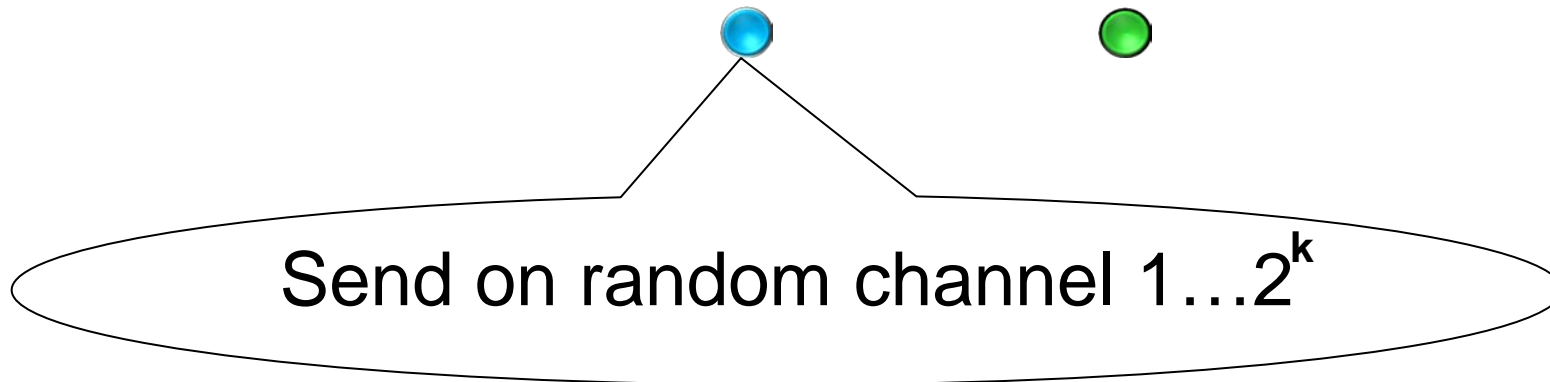
$k := \# \text{ information}$

Assume: k known

$ID = 1 \dots 2^k$

- Listens on channel 1
- Listens on channel 2
- Listens on channel 3
- Listens on channel 4

Balls into Bins



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

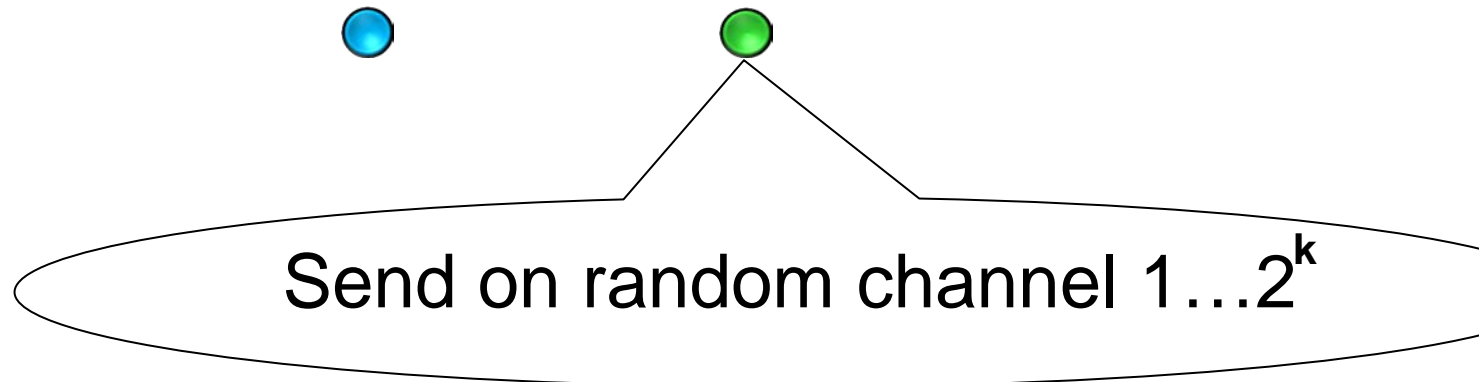
$k := \# \text{ information}$

Assume: k known

$ID = 1 \dots 2^k$

- Listens on channel 1
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Balls into Bins



Time-Optimal Information Exchange on Multiple Channels

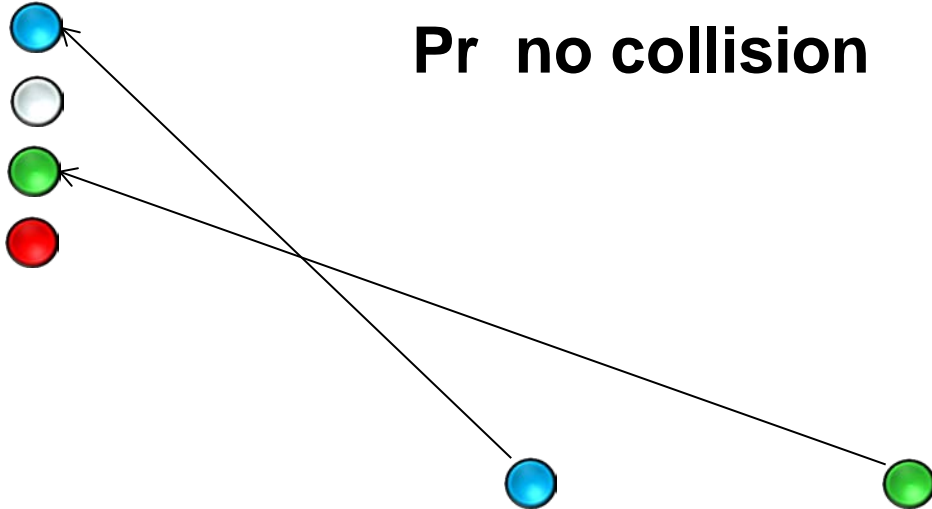


$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

ID=1... 2^k



Balls into Bins

Time-Optimal Information Exchange on Multiple Channels



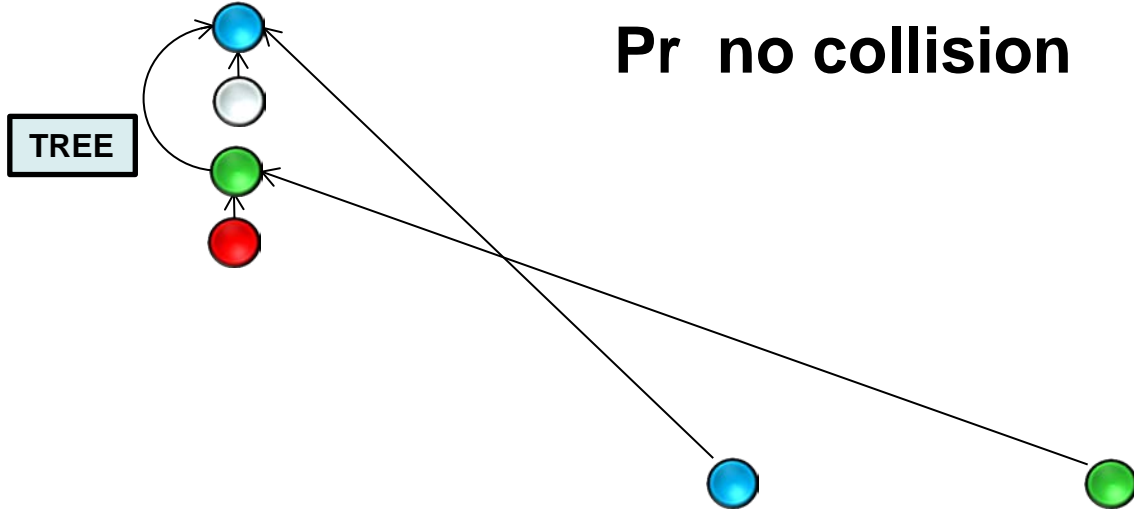
$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

$ID = 1 \dots 2^k$

Pr no collision



Balls into Bins

Time-Optimal Information Exchange on Multiple Channels

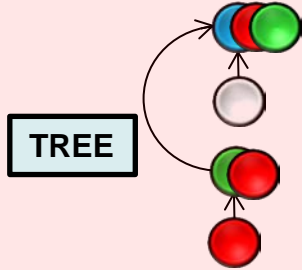


$n := \# \text{ nodes}$

$k := \# \text{ information}$

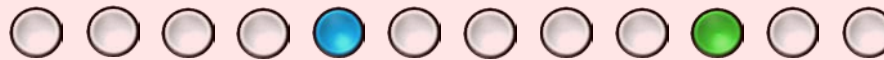
Assume: k known

$ID = 1 \dots 2^k$



Pr no collision

Balls into Bins



Time-Optimal Information Exchange on Multiple Channels

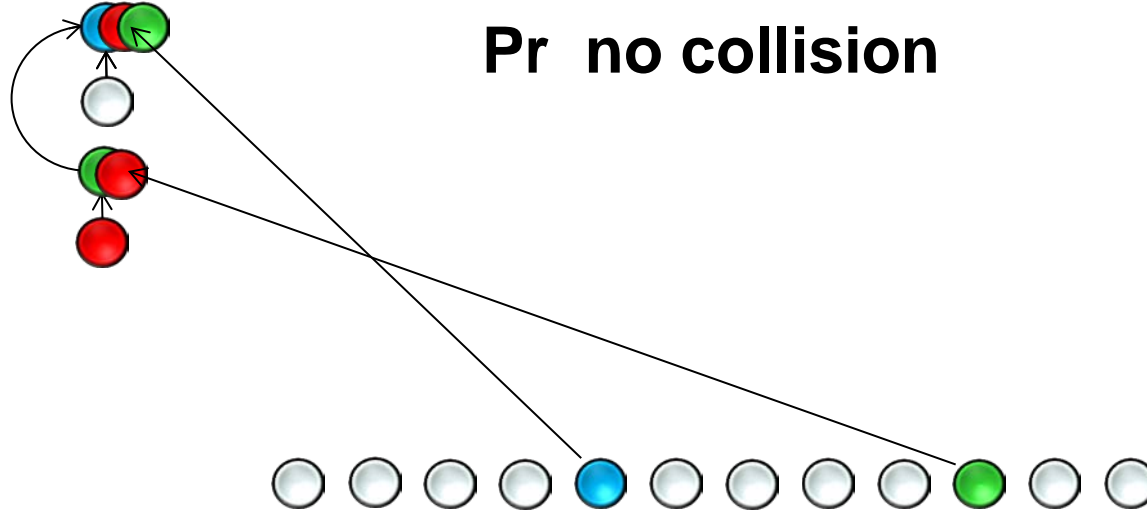


$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

ID=1... 2^k



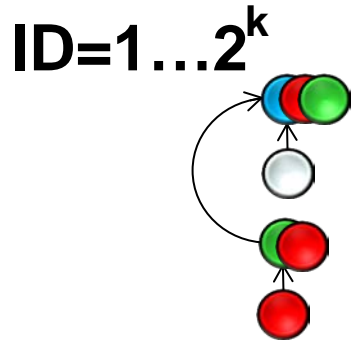
Balls into Bins

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$
Assume: k known

What if $k < \sqrt{\log n}$?



Repeat k times

$$\Pr [\text{no collision}] > 1 - \frac{1}{n^c}$$

If $k > \sqrt{\log n}$

2^k channels

Balls into Bins

TREE



$$O(k) \text{ if } k > \sqrt{\log n}$$

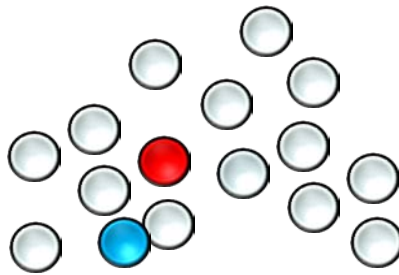
Time-Optimal Information Exchange on Multiple Channels



n := # nodes
k := # information
Assume: k known

What if $k < \sqrt{\log n}$?

Size: $\sqrt{n \log n}$



Time: $O(k)$

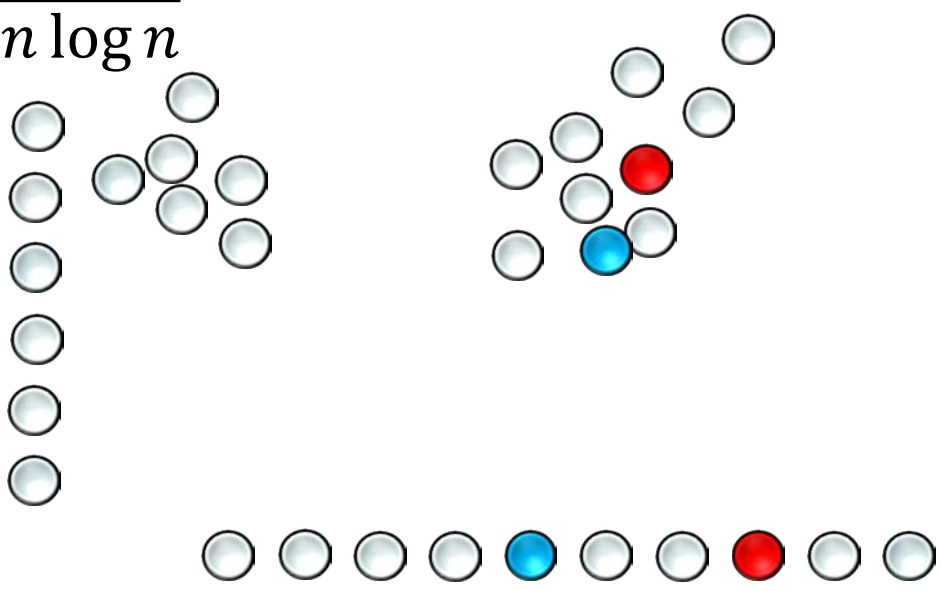
Unique Subset

Time-Optimal Information Exchange on Multiple Channels



n := # nodes
k := # information
Assume: k known

Size: $\sqrt{n \log n}$



Unique Subset

Time-Optimal Information Exchange on Multiple Channels



n := # nodes

k := # information

Assume: k known

Size: $\sqrt{n \log n}$



Unique Subset

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

Size: $\sqrt{n \log n}$



Unique Subset



Send on random channel $\in \{1, \dots, 2^{\frac{\log n}{2k}}\}$.

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

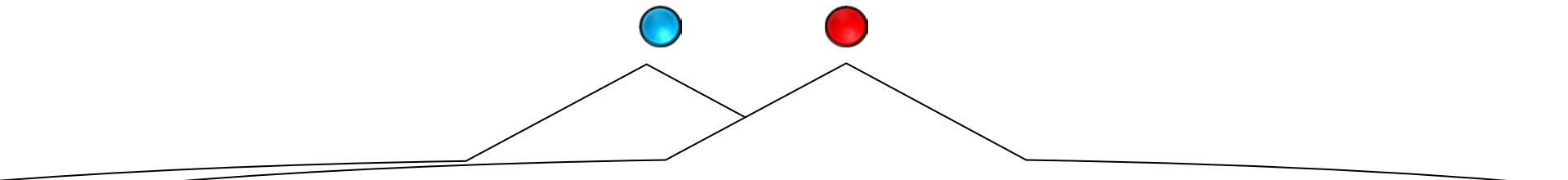
$k := \# \text{ information}$

Assume: k known

Size: $\sqrt{n \log n}$



Unique Subset



Send on random channel $\in \{1, \dots, 2^{\frac{\log n}{2k}}\}$.

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

Size: $\sqrt{n \log n}$

○ $map: \{1, \dots, \sqrt{n \log n}\} \rightarrow \text{subsets of } \left\{1, \dots, 2^{\frac{\log n}{2k}}\right\}$
○ of size at most k
○

○ Pr[at most half messages collide]
○ $> 1 - \frac{1}{n^c}$
○

Unique Subset

**Not too big ...
Not too small ...
Just right!**

Send on random channel $\in \left\{1, \dots, 2^{\frac{\log n}{2k}}\right\}$.

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

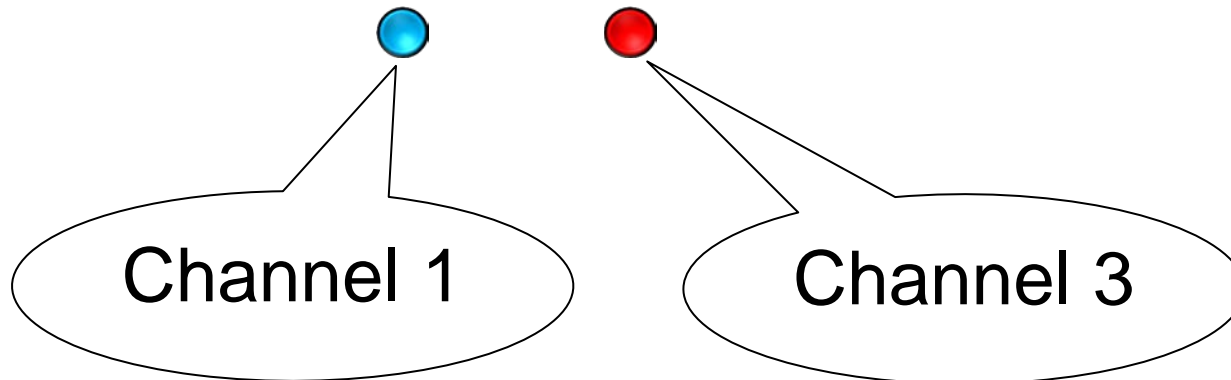
$k := \# \text{ information}$

Assume: k known

Example: 3 channels

- {1}
- {2}
- {3}
- {1,2}
- {1,3}
- {2,3}

Unique Subset



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

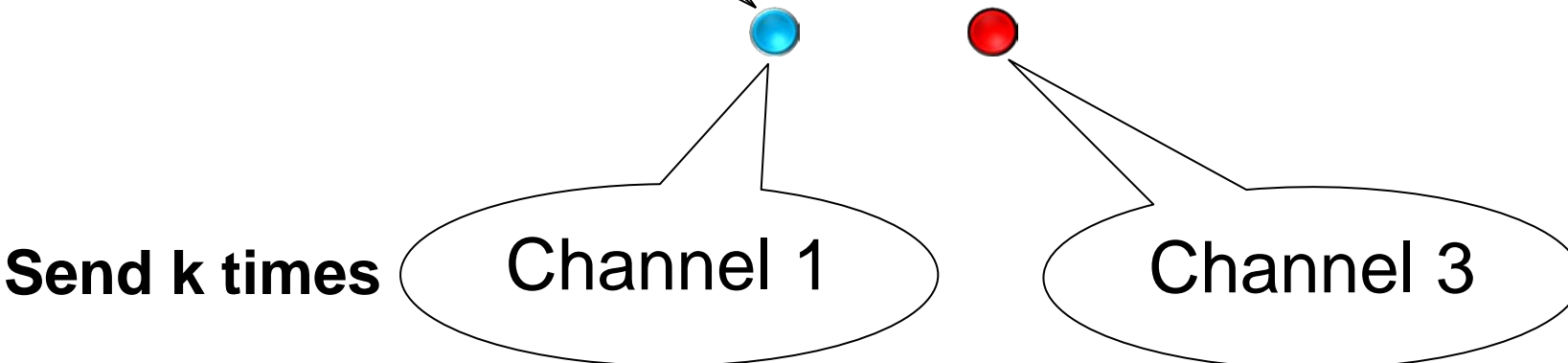
$k := \# \text{ information}$

Assume: k known

Example: 3 channels

- {1}
- {2}
- {3}
- {1,2}
- {1,3}
- {2,3}

Unique Subset



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

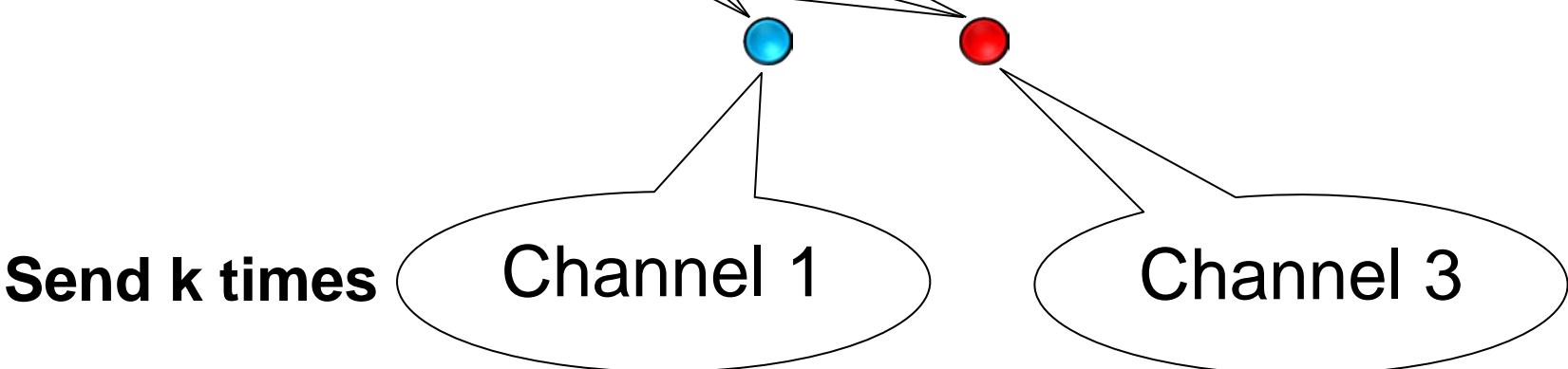
$k := \# \text{ information}$

Assume: k known

Example: 3 channels

- {1}
- {2}
- {3}
- {1,2}
- {1,3}
- {2,3}

Unique Subset



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

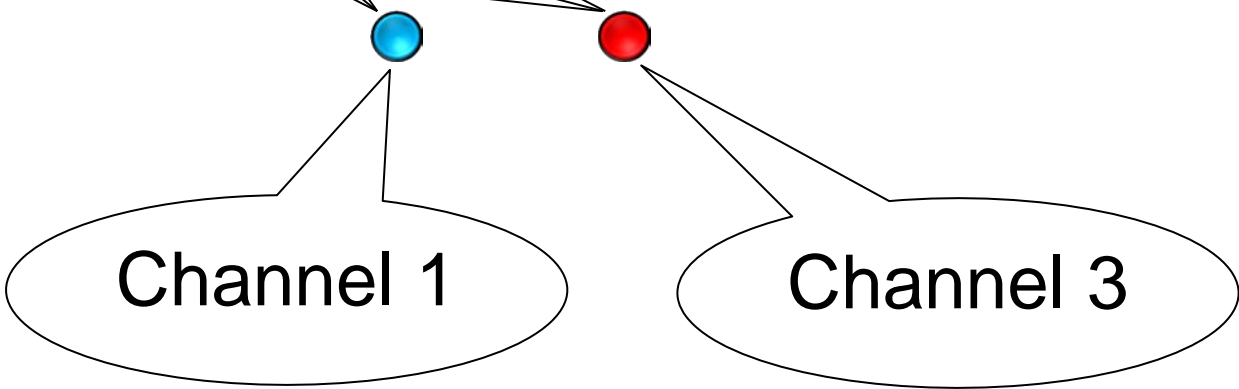
$k := \# \text{ information}$

Assume: k known

Example: 3 channels

- {1}
- {~~2~~}
- {3}
- {1,~~2~~}
- {1,3}
- {~~2~~,3}

Unique Subset



Send k times

Channel 1

Channel 3

Time-Optimal Information Exchange on Multiple Channels

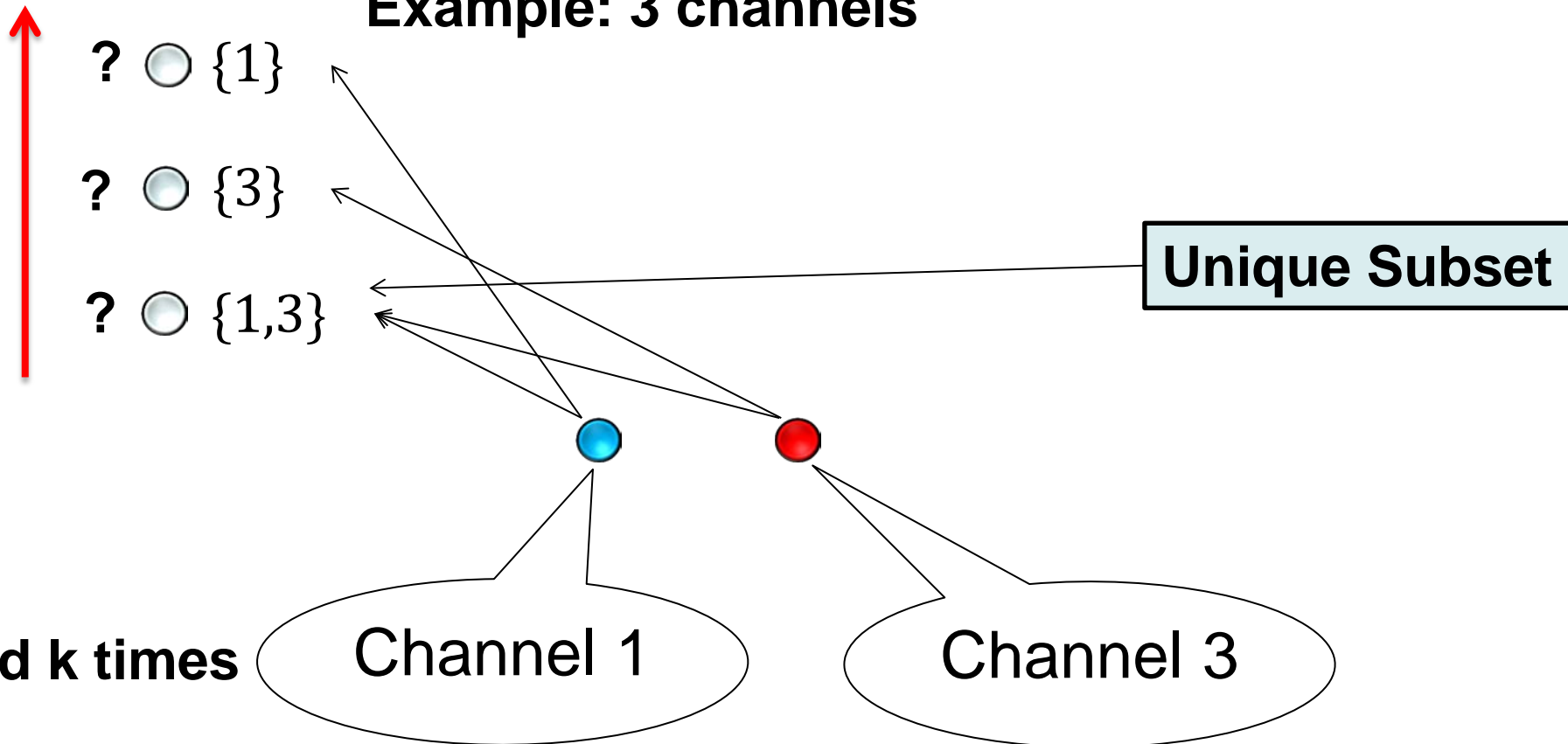


$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

Example: 3 channels



Time-Optimal Information Exchange on Multiple Channels

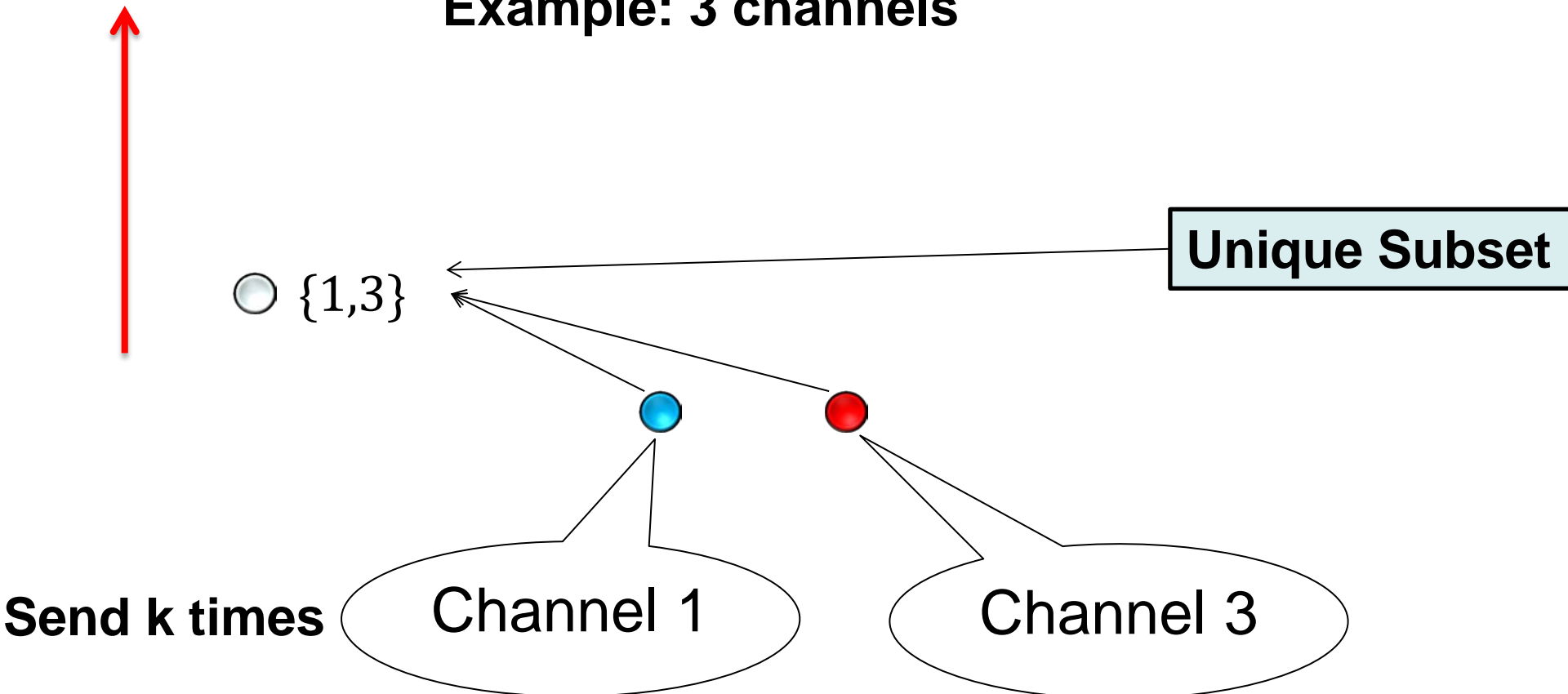


$n := \#$ nodes

$k := \#$ information

Assume: k known

Example: 3 channels

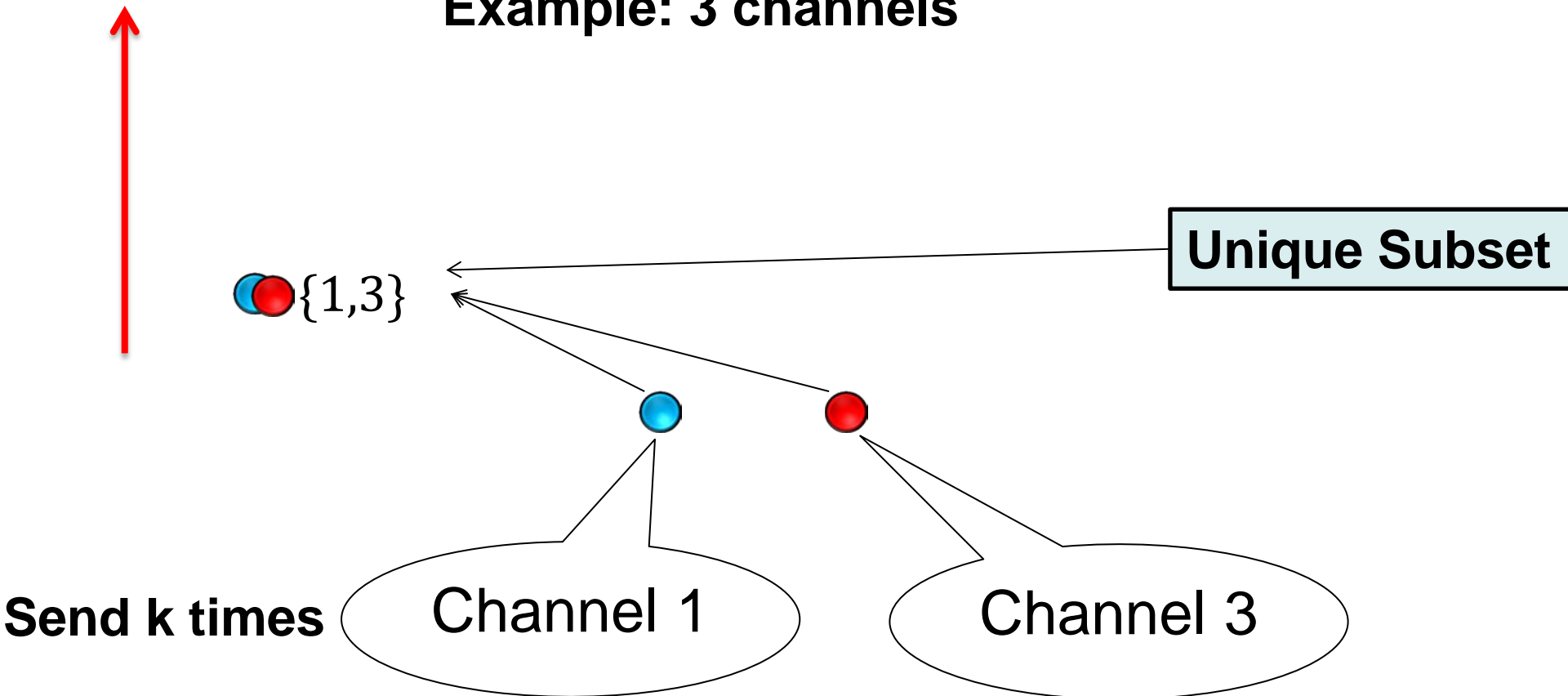


Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$
Assume: k known

Example: 3 channels



Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$
 $k := \# \text{ information}$
Assume: k known

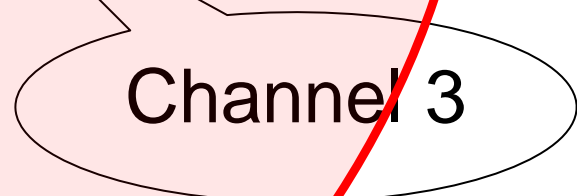
Example: 3 channels

 {1,3}

Unique Subset



Send k times



Channel 1

Channel 3

Time-Optimal Information Exchange on Multiple Channels



$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

Example: 3 channels

$$\Pr[\text{at most half messages collide}] > 1 - \frac{1}{n^c}$$

 {1,3}



Unique Subset

$O(k)$

Time-Optimal Information Exchange on Multiple Channels



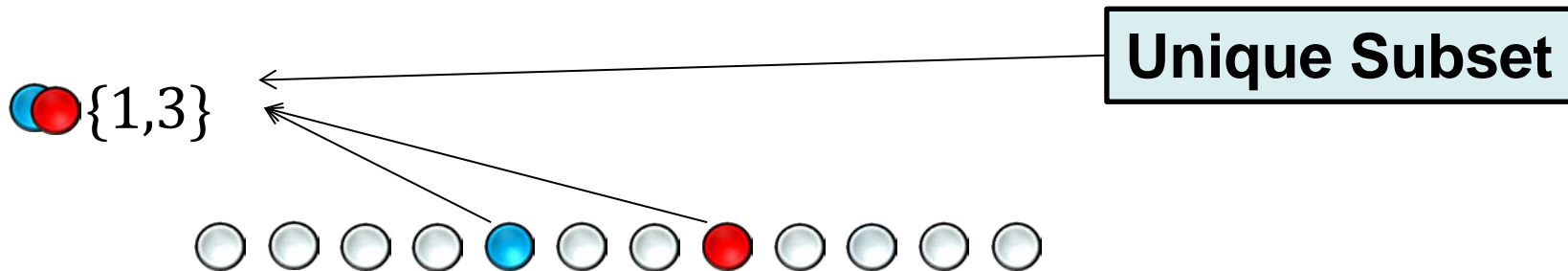
$n := \# \text{ nodes}$

$k := \# \text{ information}$

Assume: k known

Example: 3 channels

$$\Pr[\text{at most half messages collide}] > 1 - \frac{1}{n^c}$$



$$O(k + k/2 + k/4 \dots)$$

Time-Optimal Information Exchange on Multiple Channels



n := # nodes

k := # information

Assume: k known

Example: 3 channels

$\Pr[\text{at most half messages collide}] > 1 - \frac{1}{n^c}$

$$2^{\frac{\log n}{2k}} = \sqrt{n} \text{ channels}$$

Unique Subset

$O(k)$

$\Pr[\text{this works}] > 1 - \frac{1}{n^c}$

Time-Optimal Information Exchange on Multiple Channels



n := # nodes

k := # information

Assume: k known

Unique Subset

Balls into Bins

TREE

$$k < \sqrt{\log n}$$

$$\sqrt{\log n} \leq k < \log n$$

$$\log n \leq k$$

Time-Optimal Information Exchange on Multiple Channels



$n :=$ # nodes

$k :=$ # information

Assume: k ~~known~~ unknown

\sqrt{n} channels

2^k channels

n channels

n channels

Unique Subset

Balls into Bins

TREE

$$k < \sqrt{\log n}$$

$$\sqrt{\log n} \leq k < \log n$$

$$\log n \leq k$$

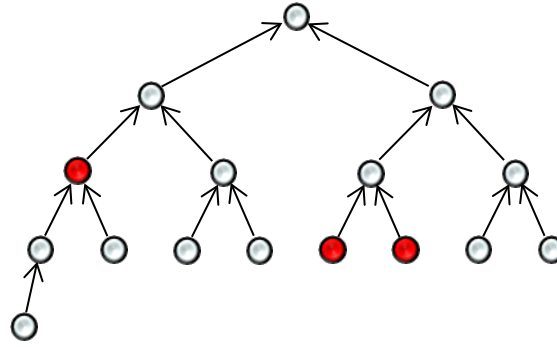
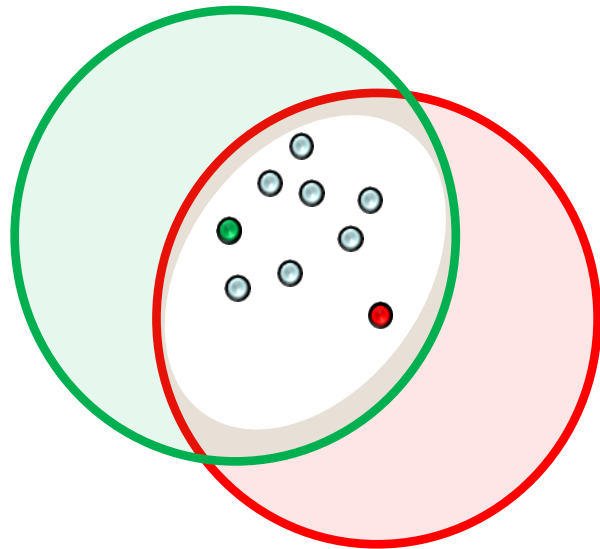
**Future directions: deterministic
less channels
lower bounds**

Time-Optimal Information Exchange on Multiple Channels

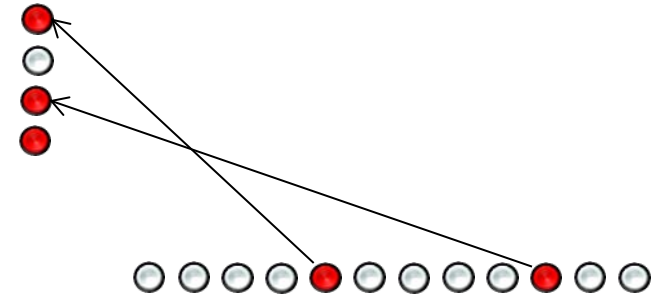


in Summary ...

Detect / Disseminate Information!



101 Mhz
117 Mhz
132 Mhz
...



○ {1,3}



$\Theta(k)$

Thank You!

Questions & Comments?



Stephan Holzer

Yvonne Anne Pignolet

Jasmin Smula

Roger Wattenhofer