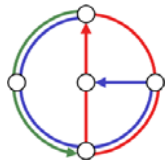


# Worst-Case Optimal and Average-Case Efficient Geometric Ad-Hoc Routing

*Fabian Kuhn*  
*Roger Wattenhofer*  
*Aaron Zollinger*

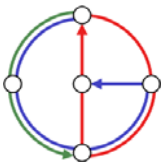
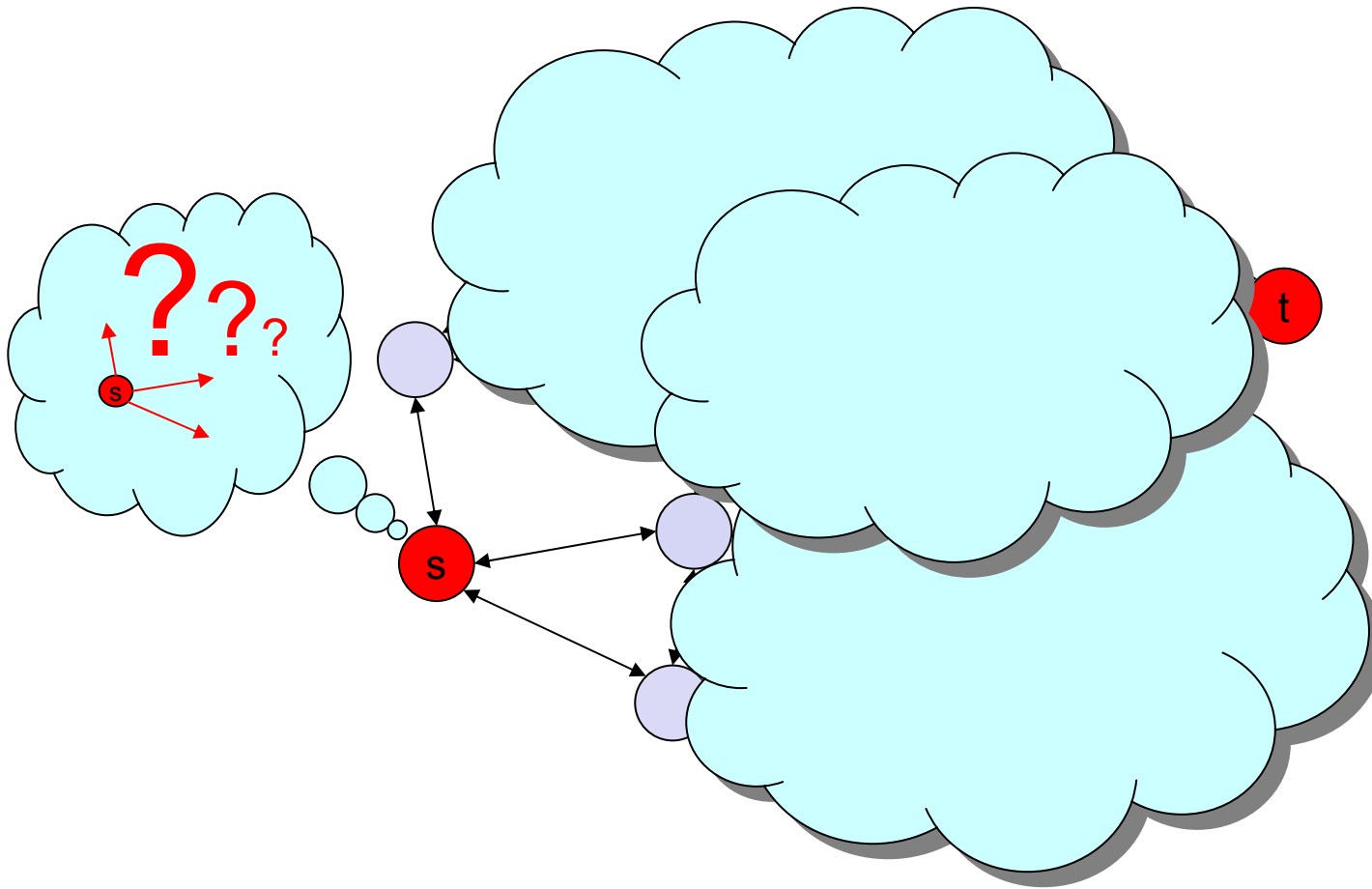


**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich



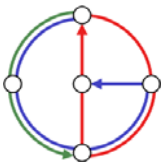
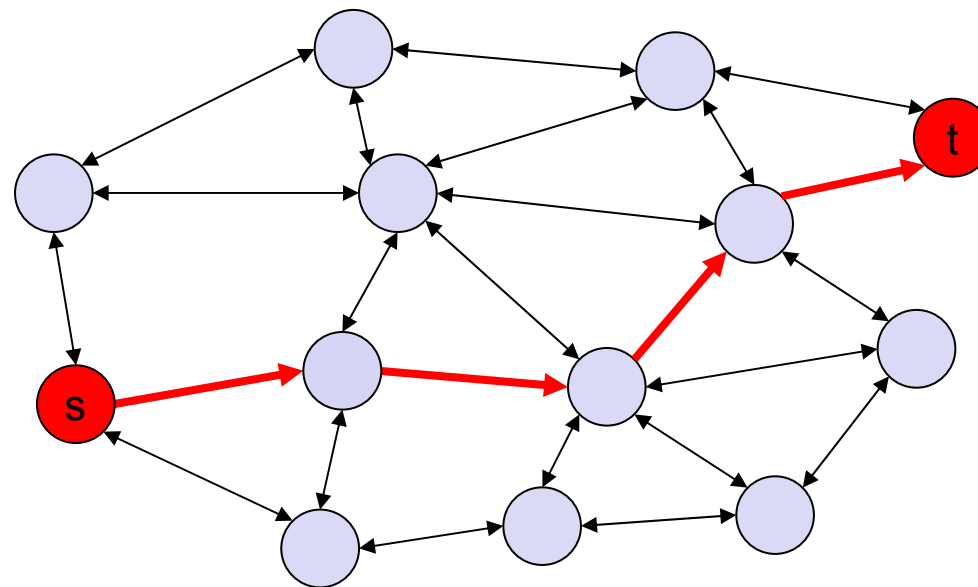
# Geometric Routing



# Greedy Routing



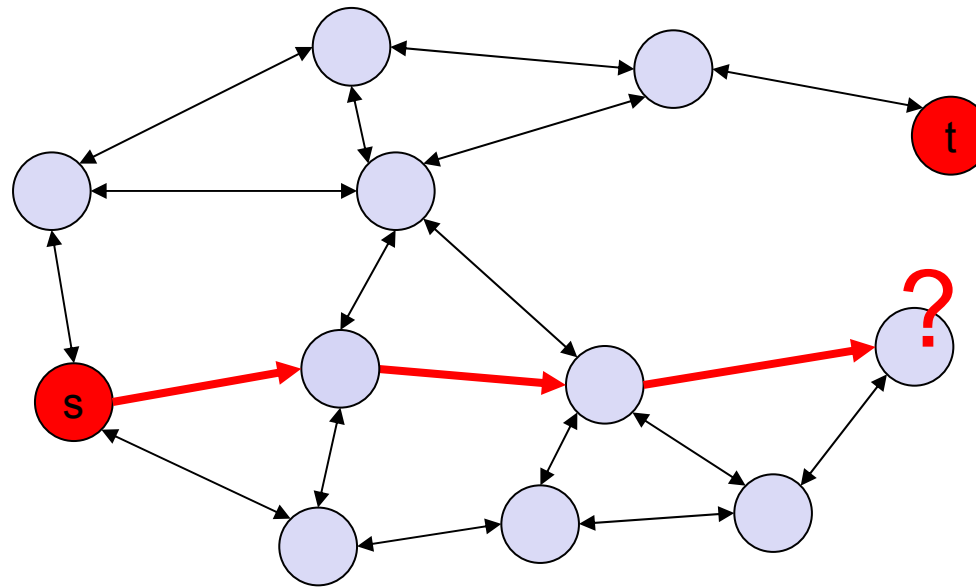
- Each node forwards message to “best” neighbor



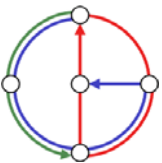
# Greedy Routing



- Each node forwards message to “best” neighbor



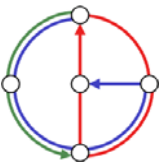
- But greedy routing may fail: message may get stuck in a “dead end”
- Needed: Correct geometric routing algorithm



# What is Geometric Routing?



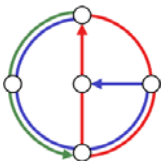
- A.k.a. location-based, position-based, geographic, etc.
- Each node knows its own position and position of neighbors
- Source knows the position of the destination
- **No routing tables stored in nodes!**
- Geometric routing is important:
  - GPS/Galileo, local positioning algorithm, overlay P2P network, Geocasting
  - Most importantly: **Learn about general ad-hoc routing**



# Related Work in Geometric Routing



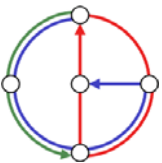
Kleinrock et al.	Various 1975ff	MFR et al.	Geometric Routing <b>proposed</b>
Kranakis, Singh, Urrutia	CCCG 1999	Face Routing	First <b>correct</b> algorithm
Bose, Morin, Stojmenovic, Urrutia	DialM 1999	GFG	First average-case <b>efficient</b> algorithm (simulation but no proof)
Karp, Kung	MobiCom 2000	GPSR	A <b>new name</b> for GFG
Kuhn, Wattenhofer, Zollinger	DialM 2002	AFR	First <b>worst-case</b> analysis. Tight $\Omega(c^2)$ bound.
Kuhn, Wattenhofer, Zollinger	MobiHoc 2003	GOAFR	Worst-case optimal <b>and</b> average- case efficient, percolation theory



# Overview



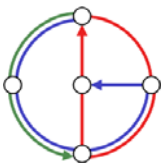
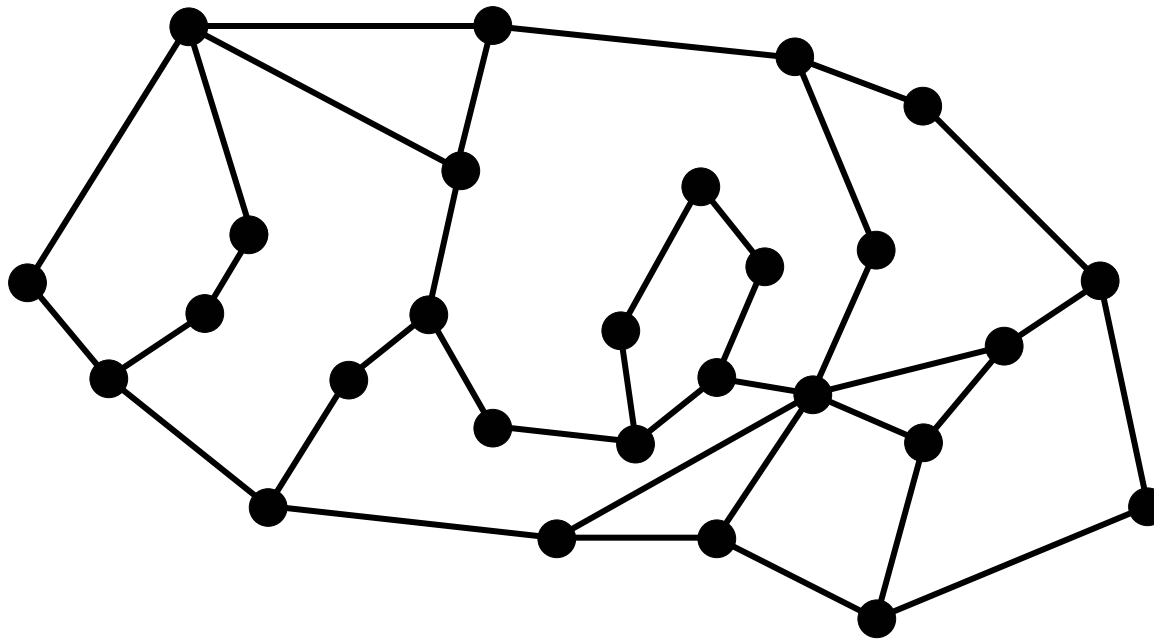
- Introduction
  - What is Geometric Routing?
  - Greedy Routing
- **Correct Geometric Routing: Face Routing**
- Efficient Geometric Routing
  - Adaptively Bound Searchable Area
  - Lower Bound, Worst-Case Optimality
  - Average-Case Efficiency
  - Critical Density
  - GOAFR
- Conclusions



# Face Routing



- Based on ideas by [Kranakis, Singh, Urrutia CCCG 1999]
- Here simplified (and actually improved)



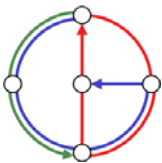
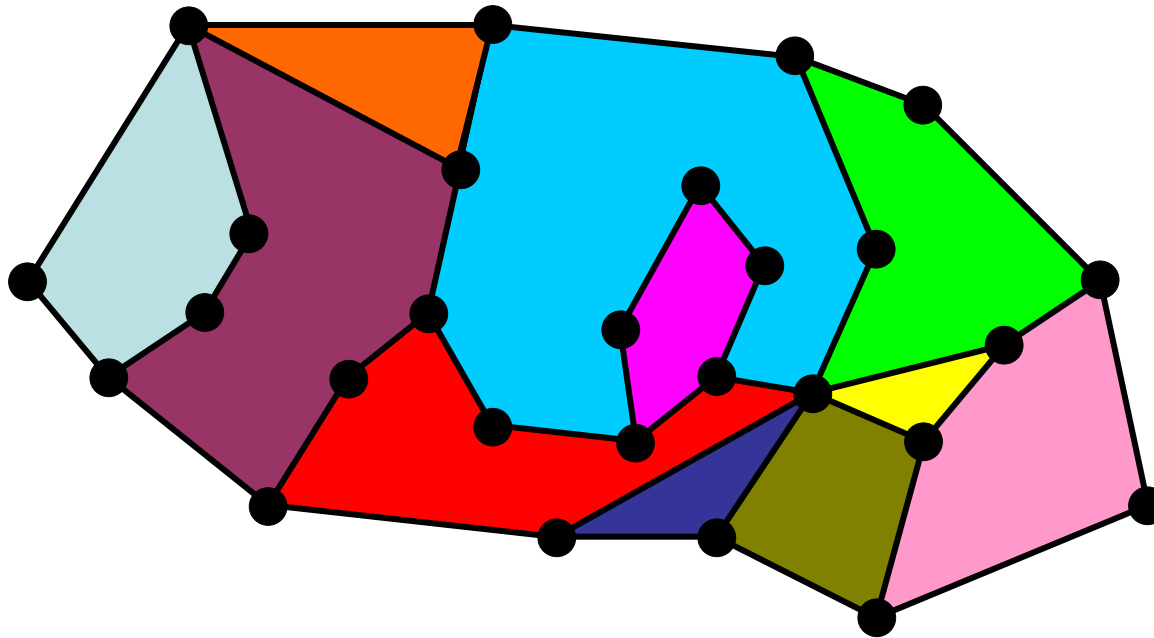


# Face Routing

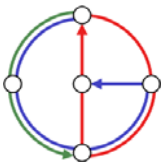
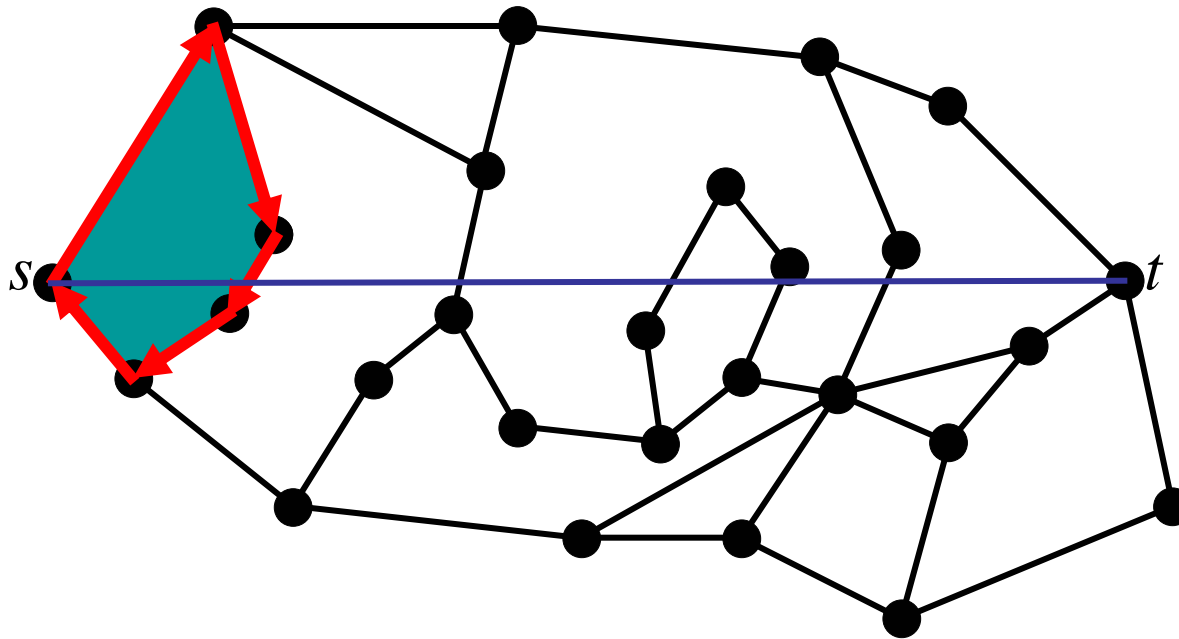


- Remark: Planar graph can easily (and locally!) be **computed** with the Gabriel Graph, for example

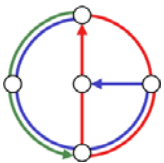
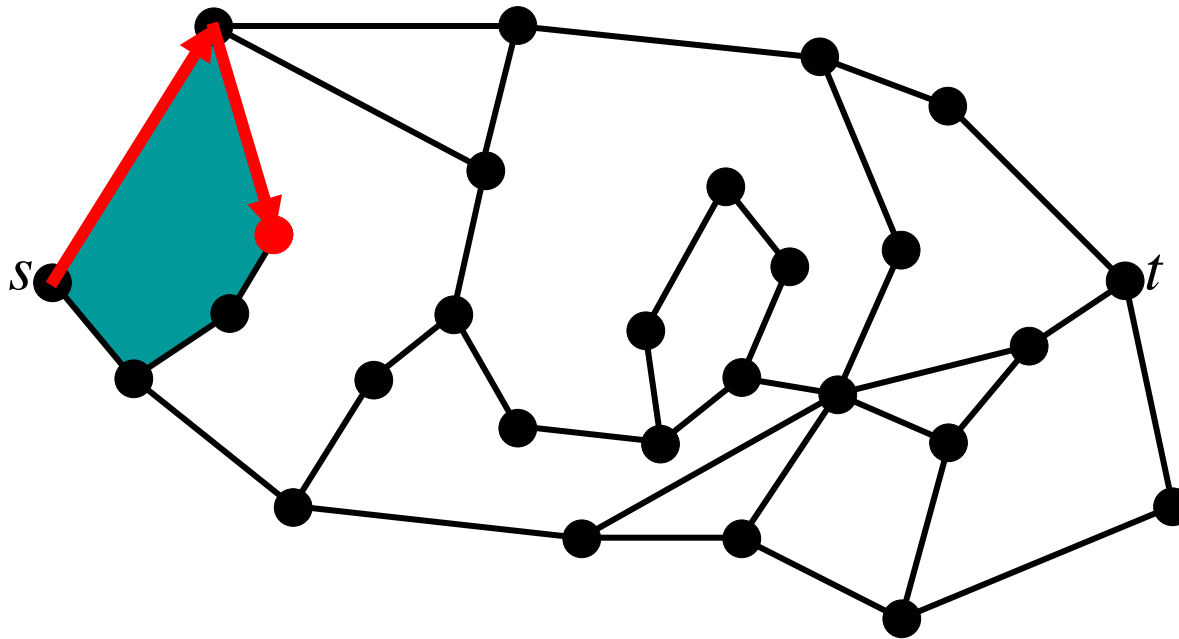
Planarity is NOT an assumption



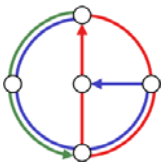
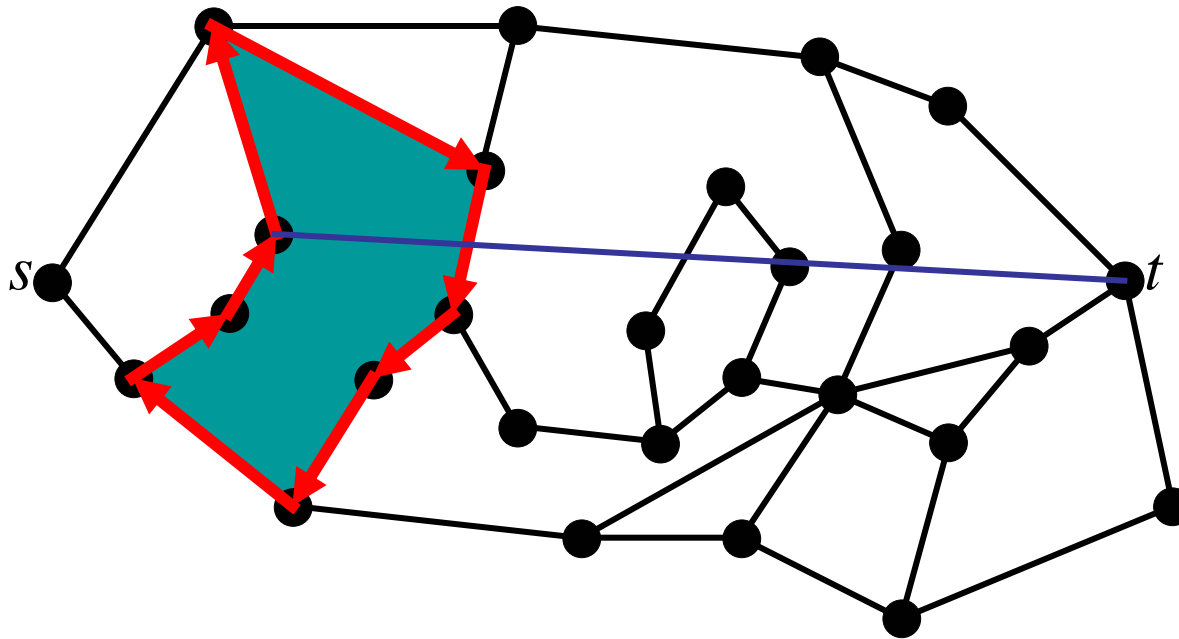
# Face Routing



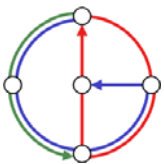
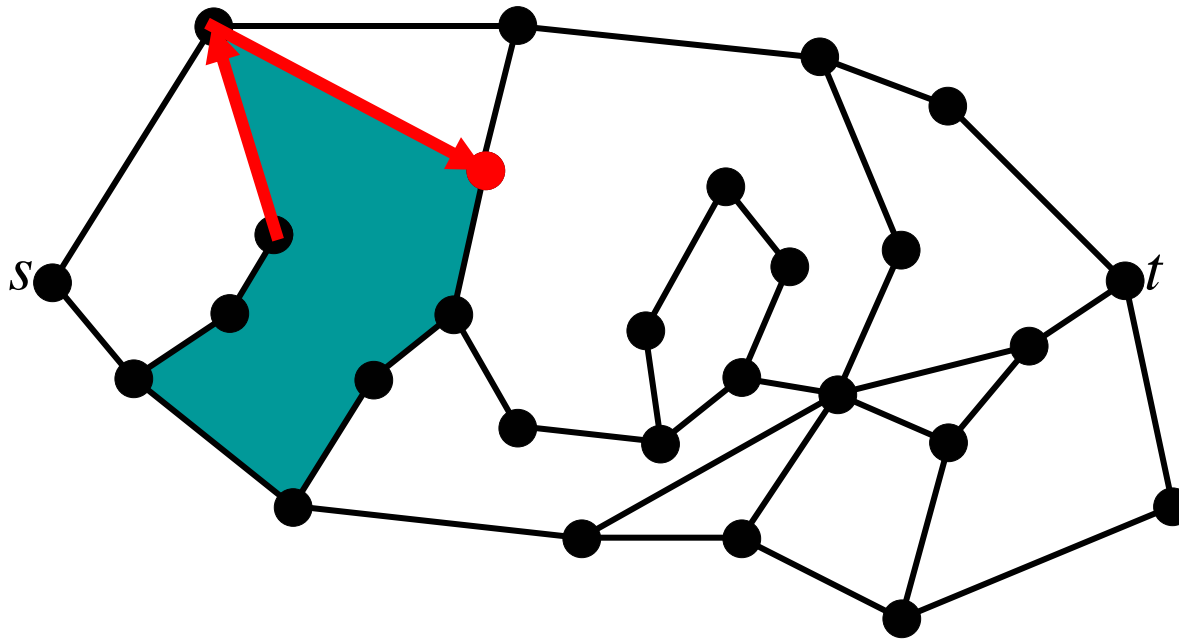
# Face Routing



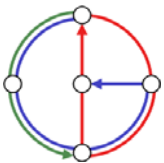
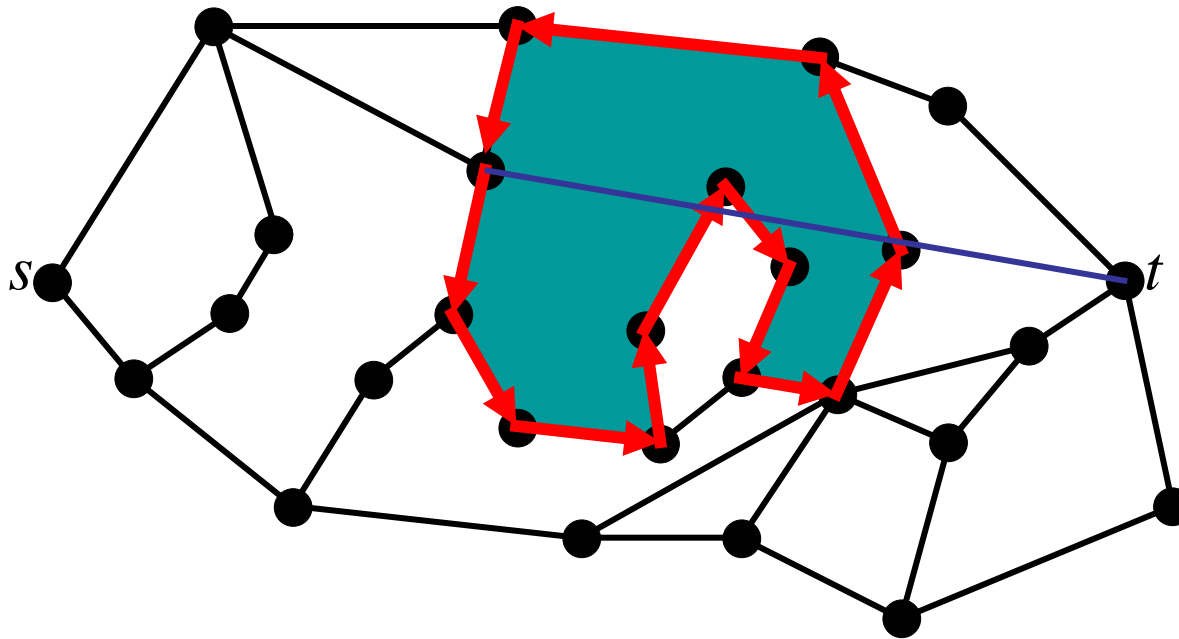
# Face Routing



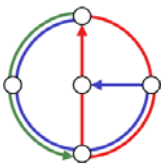
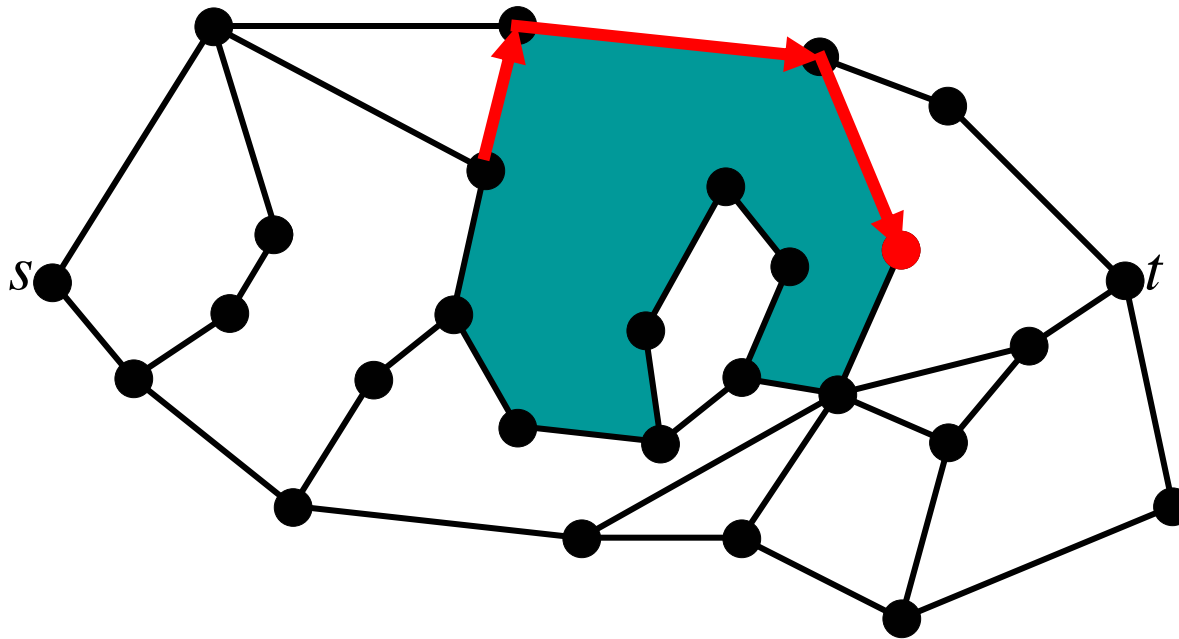
# Face Routing



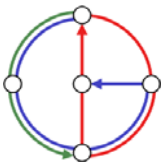
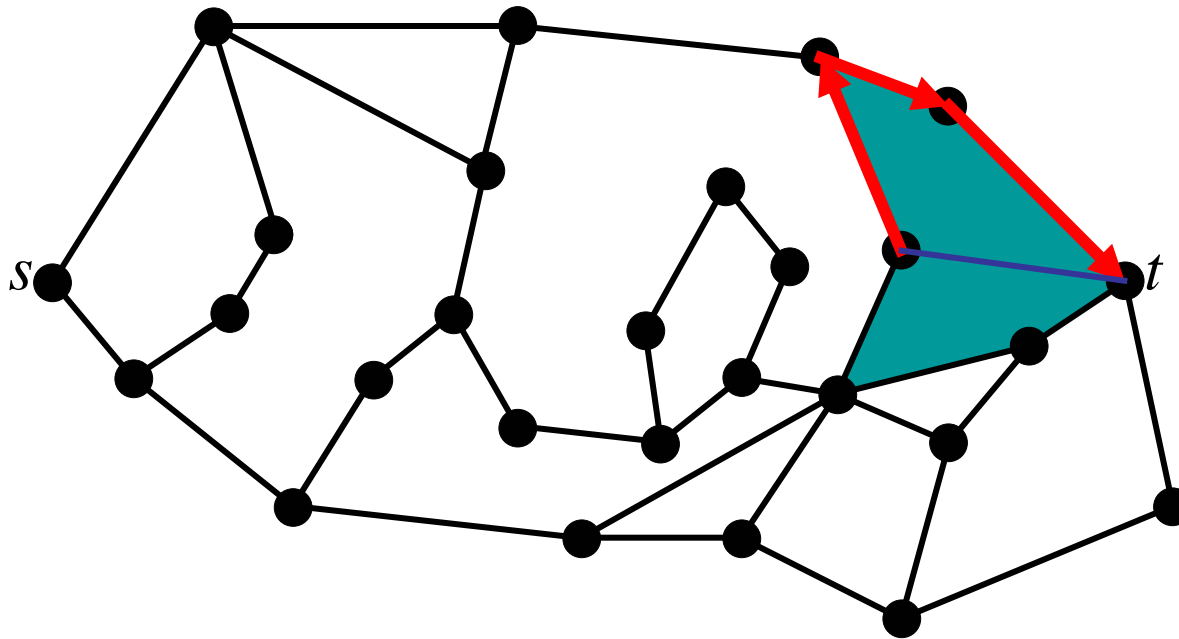
# Face Routing



# Face Routing



# Face Routing

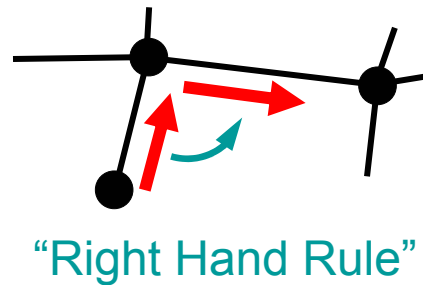
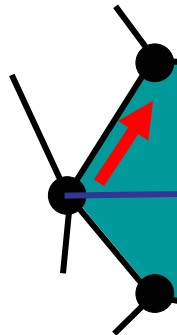




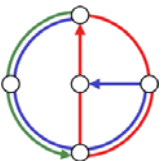
# Face Routing Properties



- All necessary information is stored in the message
  - Source and destination positions
  - Point of transition to next face
- Completely local:
  - Knowledge about direct neighbors' positions sufficient
  - Faces are **implicit**



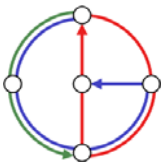
- **Planarity** of graph is **computed** locally (not an assumption)
  - Computation for instance with Gabriel Graph



# Overview



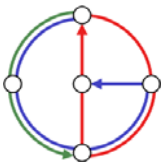
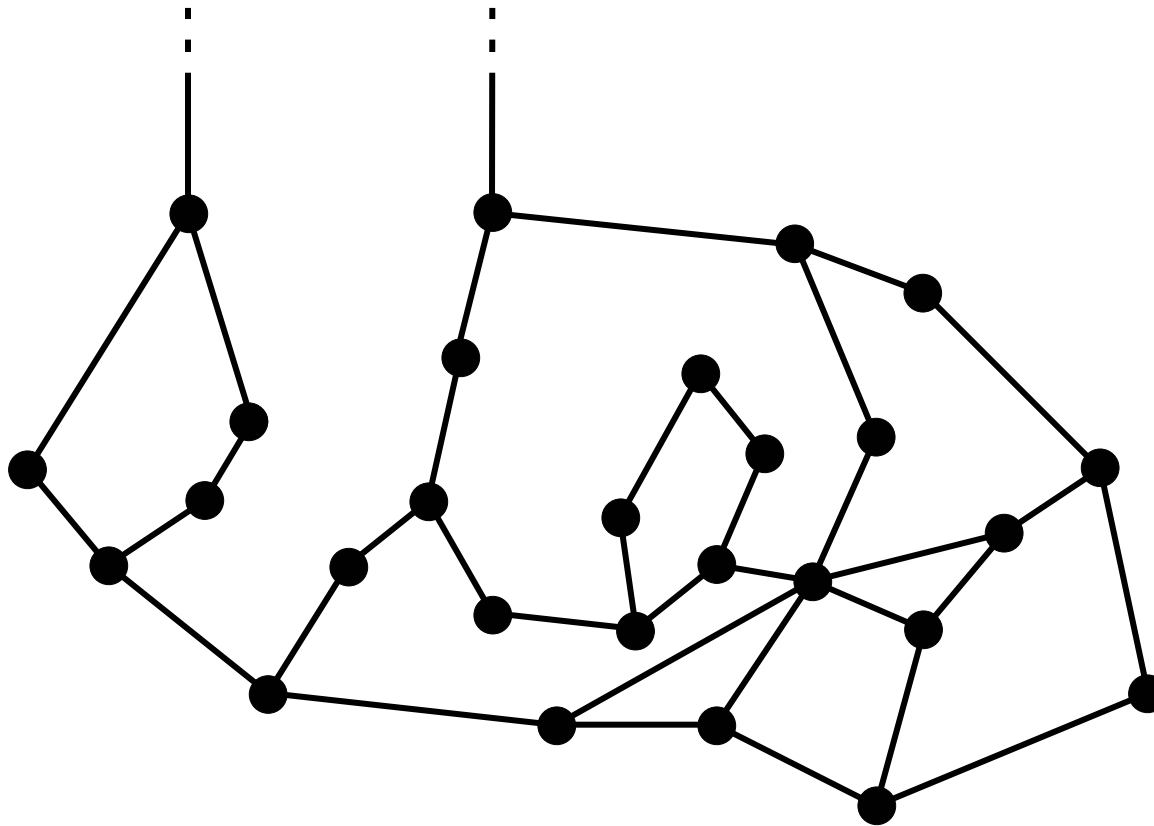
- Introduction
  - What is Geometric Routing?
  - Greedy Routing
- Correct Geometric Routing: Face Routing
- **Efficient Geometric Routing**
  - **Adaptively Bound Searchable Area**
  - **Lower Bound, Worst-Case Optimality**
  - Average-Case Efficiency
  - Critical Density
  - GOAFR
- Conclusions



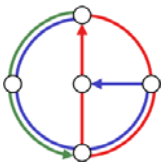
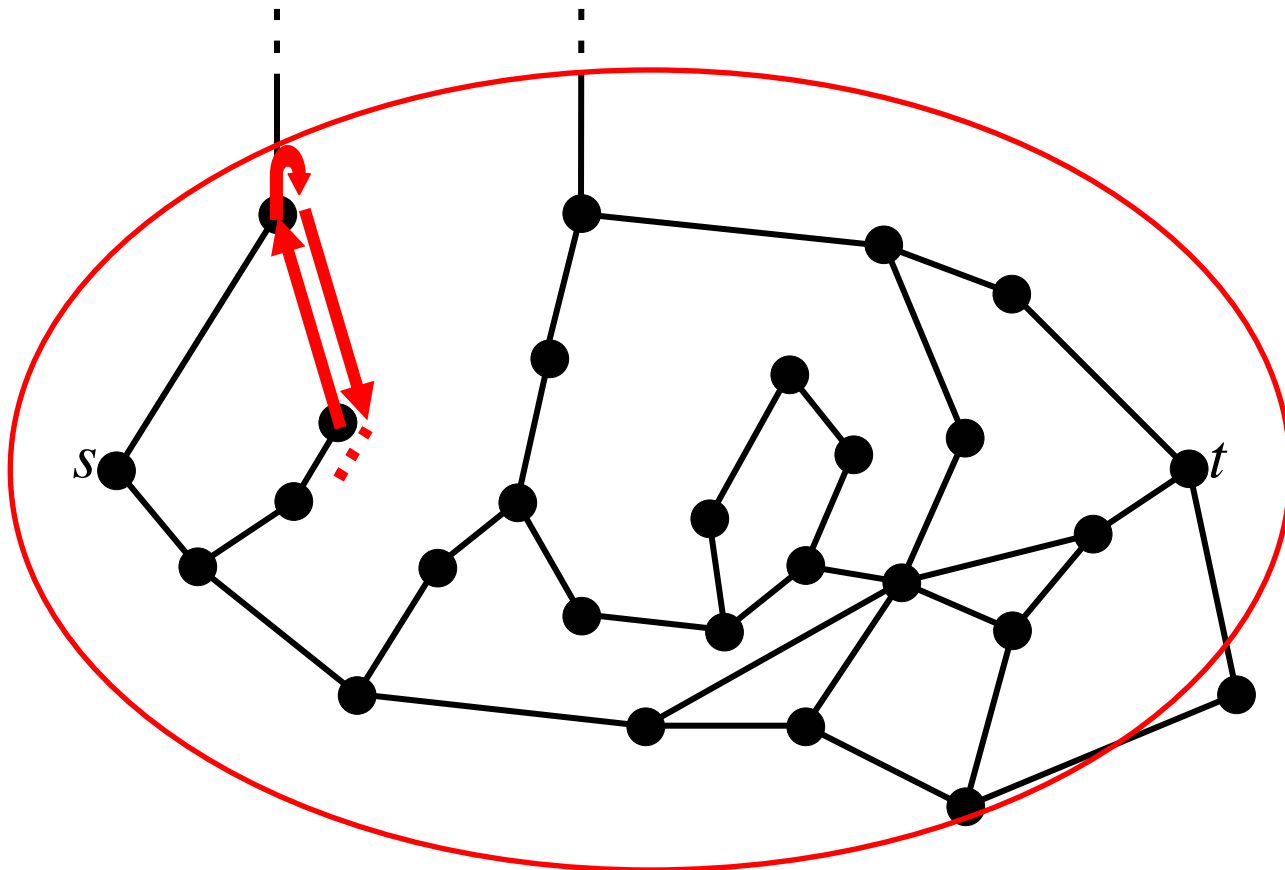
# Face Routing



- Theorem: Face Routing reaches destination in  $O(n)$  steps
- But: Can be very bad compared to the optimal route



# Bounding Searchable Area



# Adaptively Bound Searchable Area



What is the correct size of the bounding area?

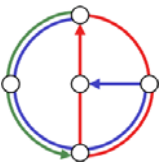
- Start with a small searchable area
- Grow area each time you cannot reach the destination
- In other words, **adapt** area size whenever it is too small

→ Adaptive Face Routing AFR

Theorem: AFR Algorithm finds destination after  $O(c^2)$  steps, where  $c$  is the cost of the optimal path from source to destination.

Theorem: AFR Algorithm is asymptotically worst-case optimal.

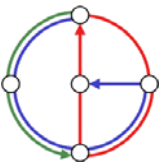
[Kuhn, Wattenhofer, Zollinger DIALM 2002]



# Overview

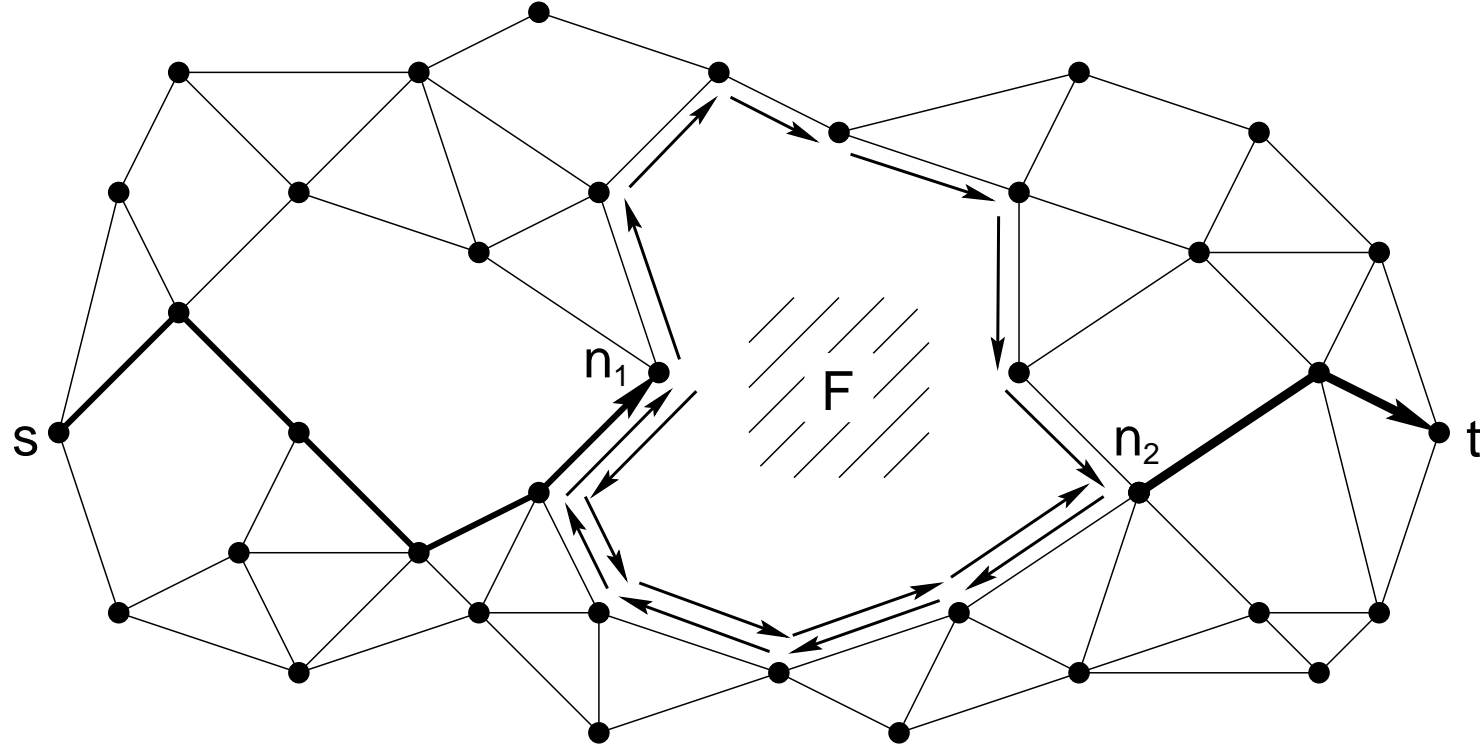


- Introduction
  - What is Geometric Routing?
  - Greedy Routing
- Correct Geometric Routing: Face Routing
- Efficient Geometric Routing
  - Adaptively Bound Searchable Area
  - Lower Bound, Worst-Case Optimality
  - Average-Case Efficiency
  - Critical Density
  - GOAFR
- Conclusions



# GOAFR – Greedy Other Adaptive Face Routing

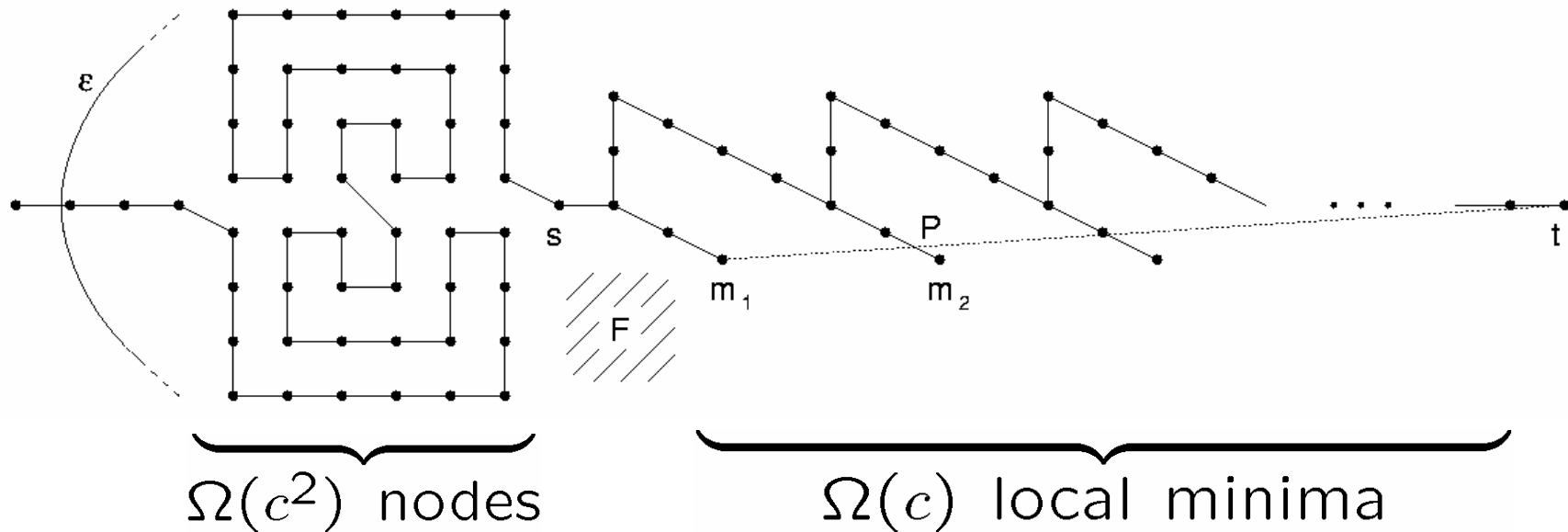
- AFR Algorithm is not very efficient (especially in dense graphs)
- Combine **G**reedy and (**O**ther **A**daptive) **F**ace **R**outing
  - Route greedily as long as possible
  - Overcome “dead ends” by use of face routing
  - Then route greedily again
- Similar as GFG/GPSR, but **adaptive**



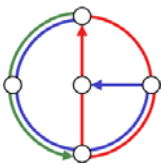
# Early Fallback to Greedy Routing?



- We could fall back to greedy routing as soon as we are closer to  $t$  than the local minimum
- But:



- “Maze” with  $\Omega(c^2)$  edges is traversed  $\Omega(c)$  times  $\rightarrow \Omega(c^3)$  steps






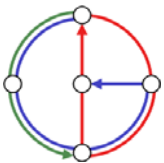
# GOAFR Is Worst-Case Optimal



- GOAFR traverses complete face boundary:

Theorem: GOAFR is asymptotically worst-case optimal.

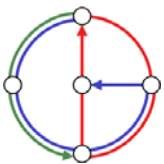
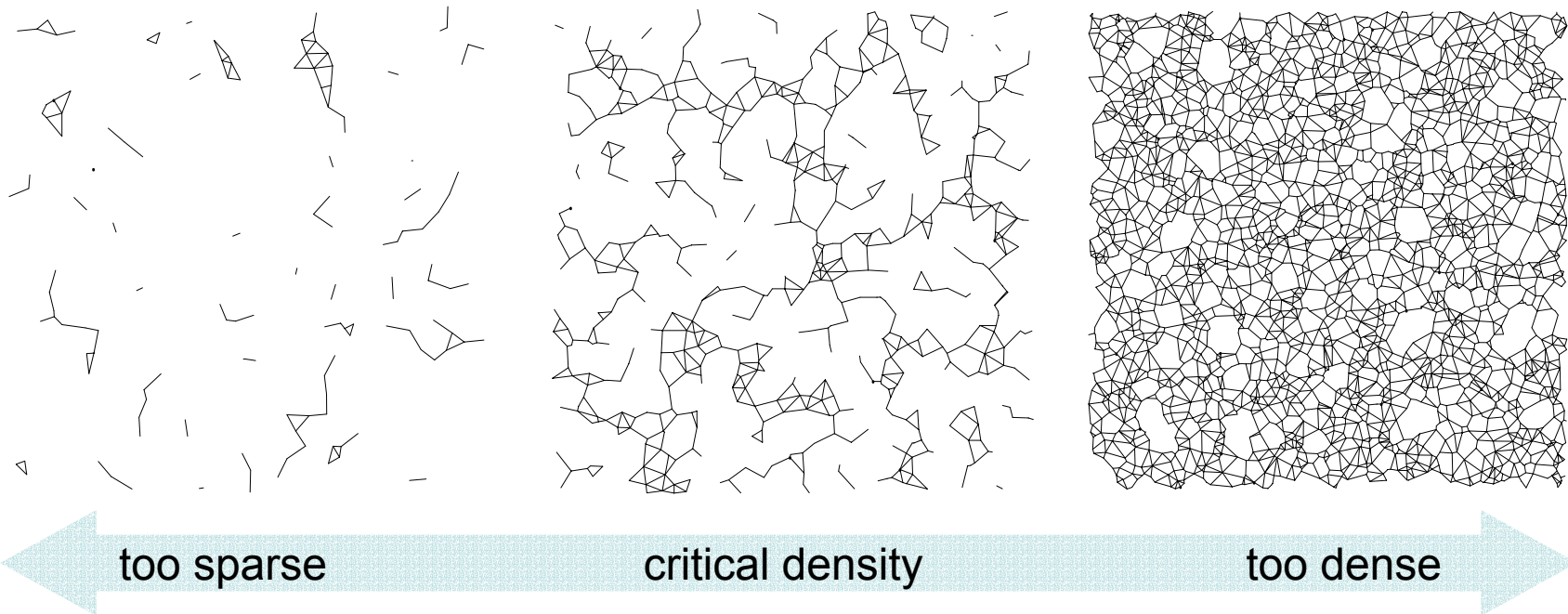
- Remark: GFG/GPSR is not
  - Searchable area not bounded
  - Immediate fallback to greedy routing 
- GOAFR's **average-case** efficiency?



# Average Case



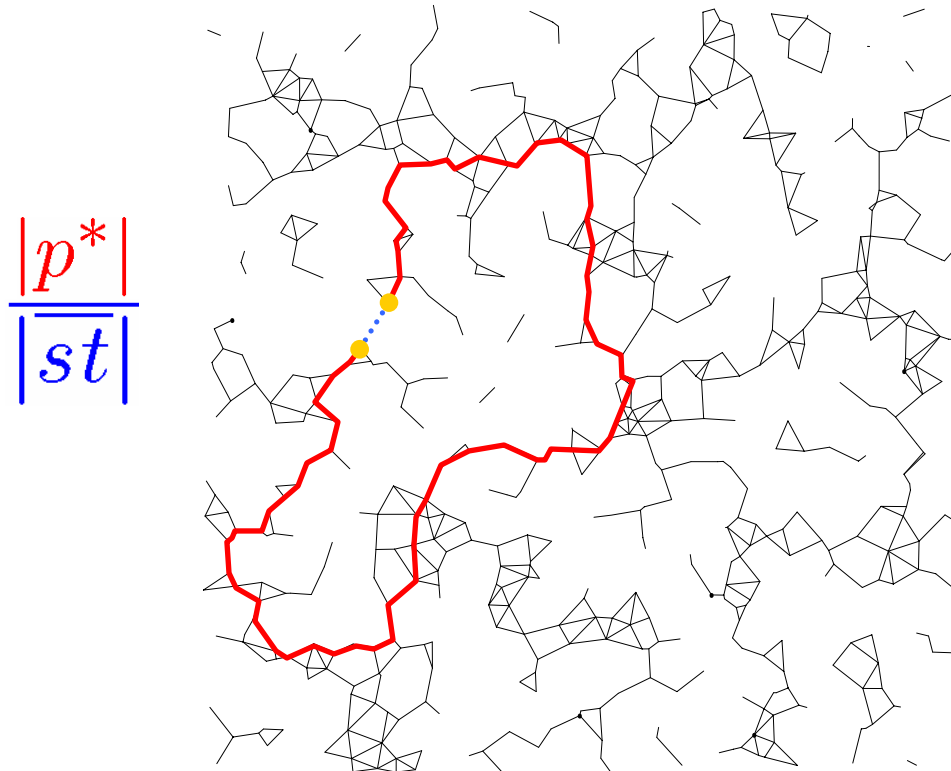
- Not interesting when graph not dense enough
- Not interesting when graph is too dense
- **Critical density range** (“percolation”)
  - Shortest path is significantly longer than Euclidean distance



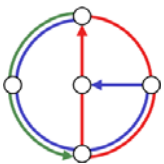
# Critical Density: Shortest Path vs. Euclidean Distance



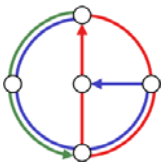
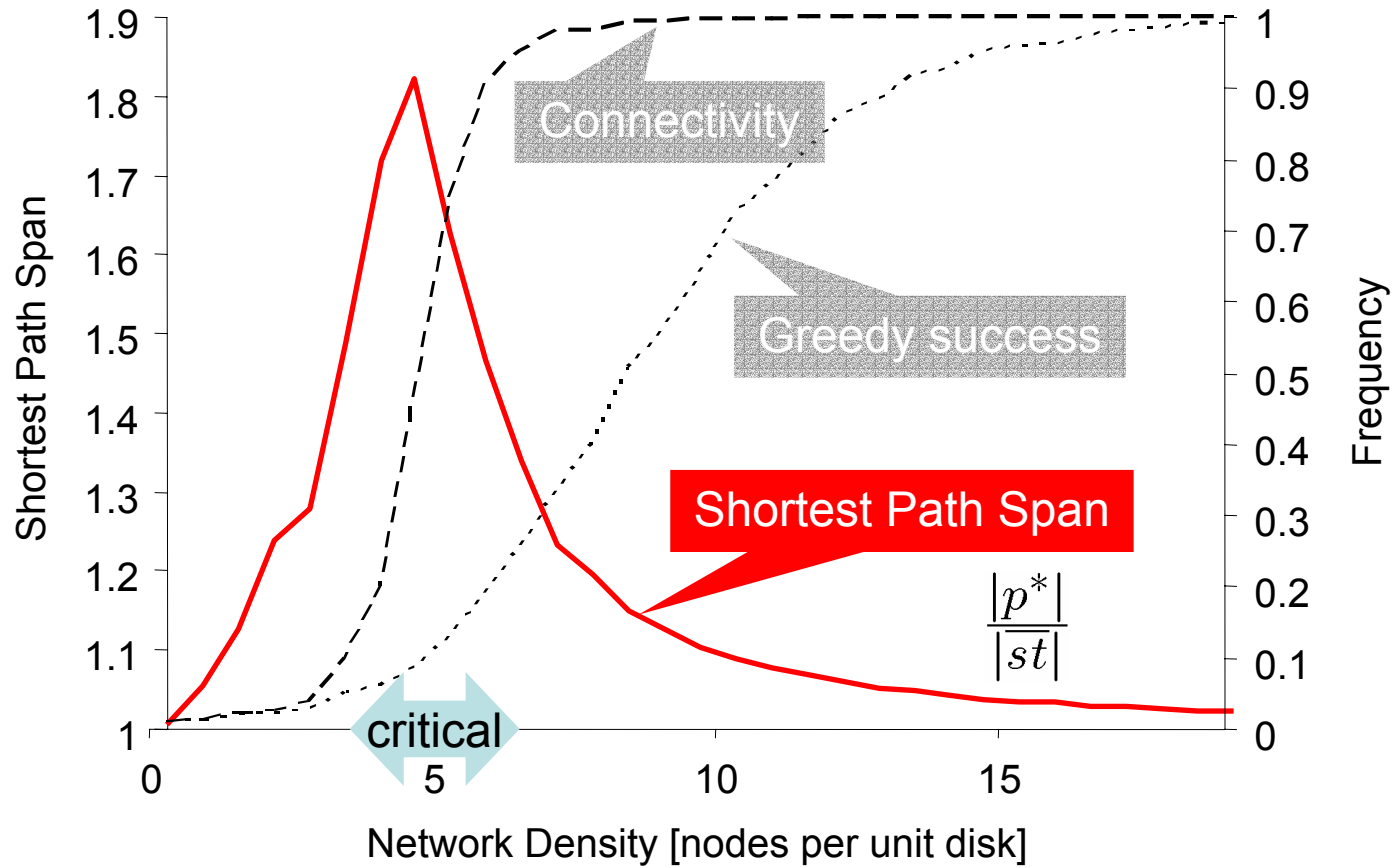
- Shortest path is significantly longer than Euclidean distance



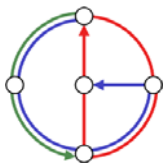
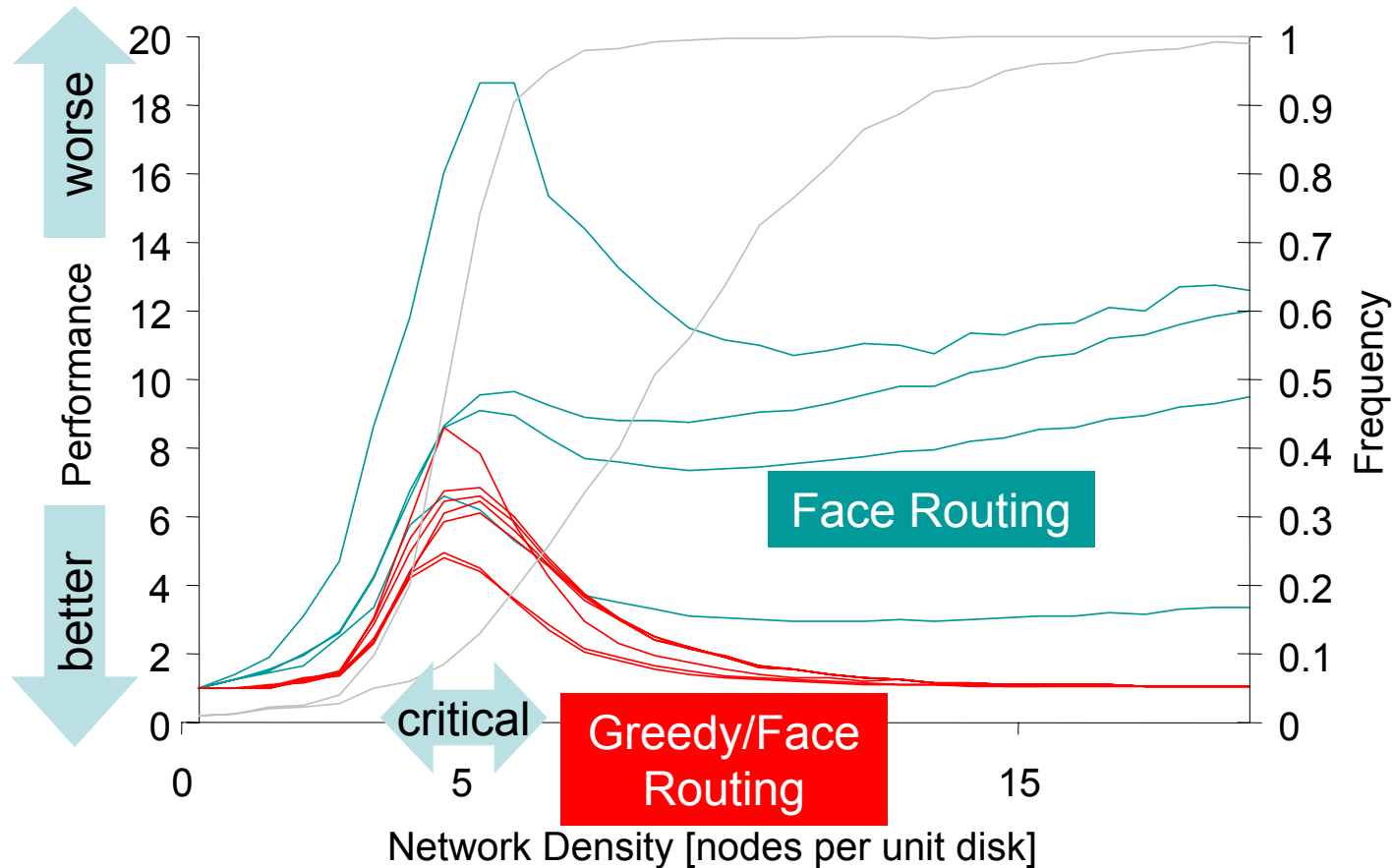
- Critical density range mandatory for the simulation of **any** routing algorithm (not only geometric)



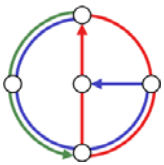
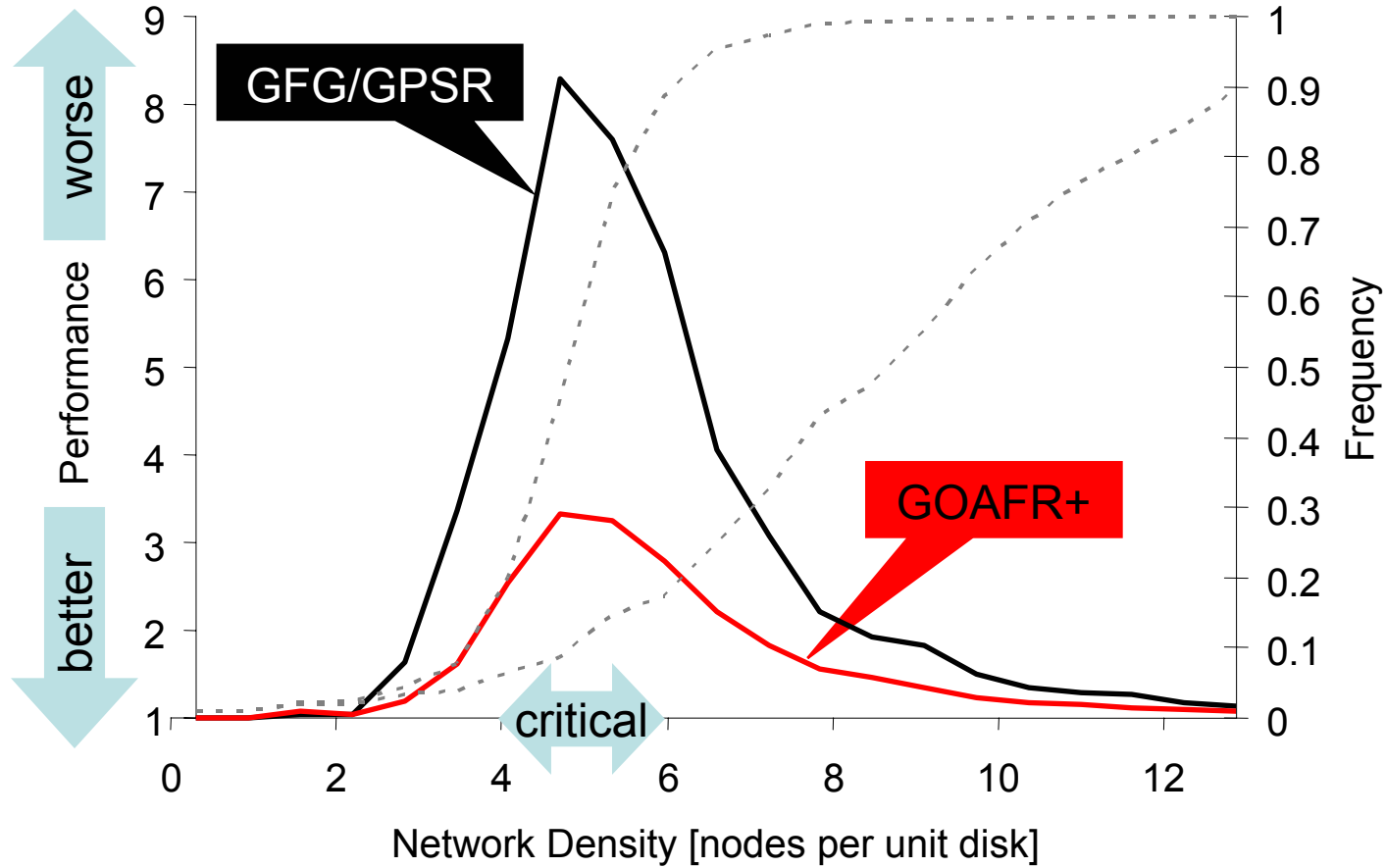
# Randomly Generated Graphs: Critical Density Range



# Average-Case Performance: Face vs. Greedy/Face



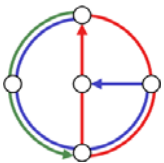
# Simulation on Randomly Generated Graphs



# Conclusion



	Correct Routing	Worst-Case Optimal	Avg-Case Efficient	Comprehensive Simulation
Greedy Routing (MFR)			(✓)	
Face Routing	✓			
GFG/GPSR	✓		✓	
AFR	✓	✓		
GOAFR	✓	✓	✓	✓





Questions?  
Comments?  
Demo?