Worst-Case Optimal and Average-Case Efficient Geometric Ad-Hoc Routing

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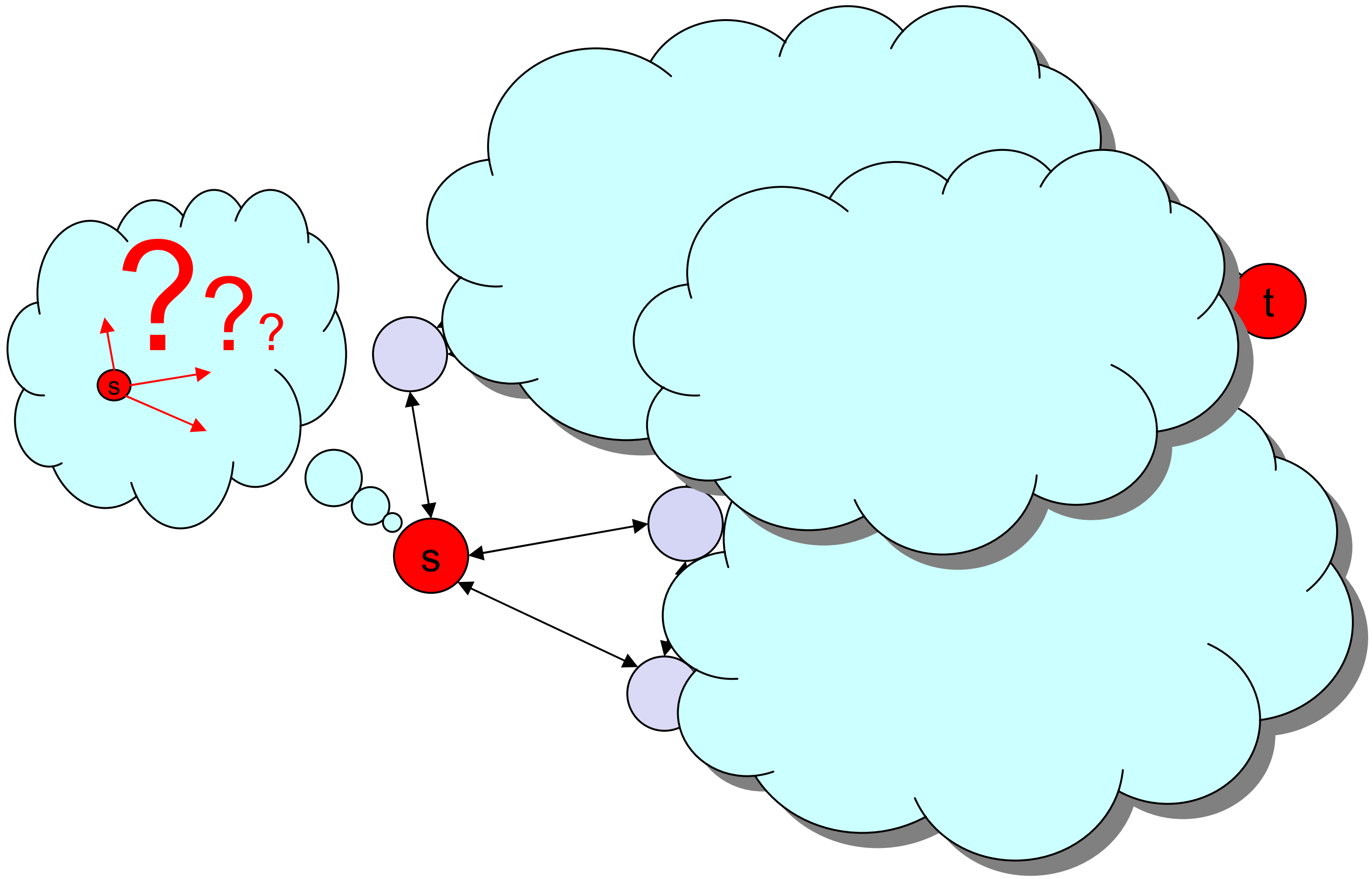
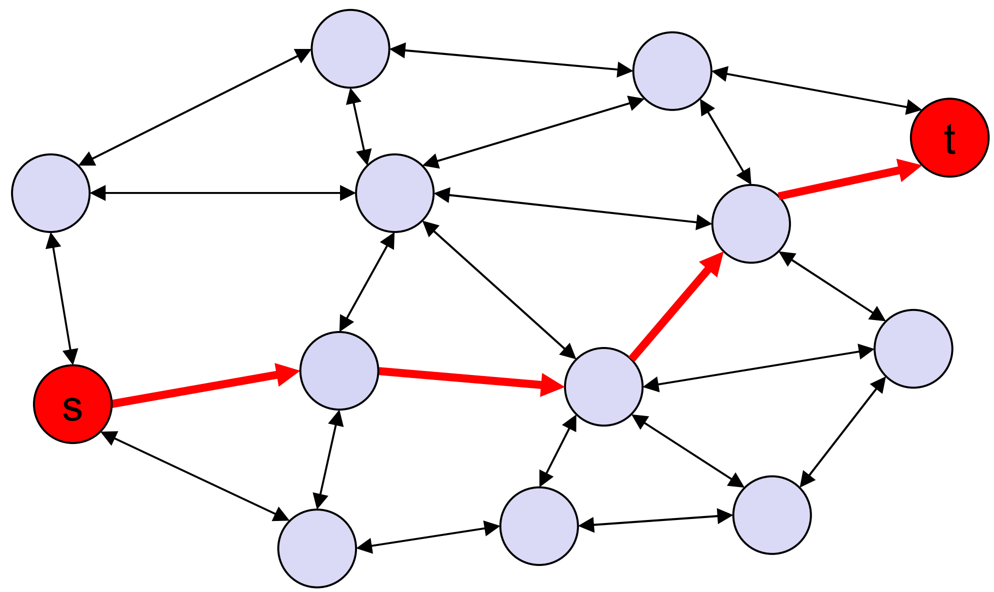
Geometric Routing



Greedy Routing

- Each node forwards message to "best" neighbor



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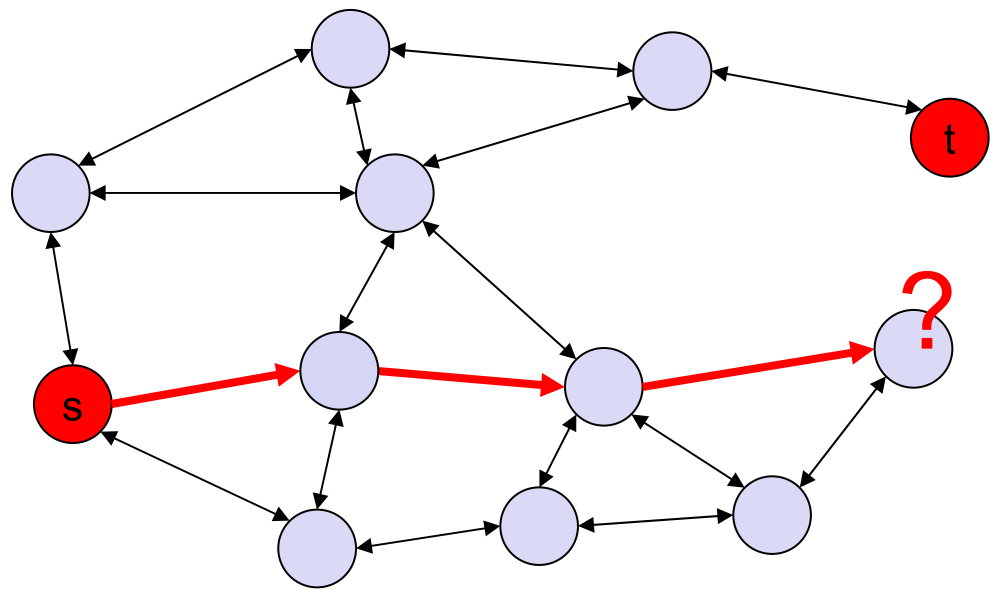


- But greedy routing may fail: message may get stuck in a “dead end”
- Needed: Correct geometric routing algorithm

What is Geometric Routing?

- A.k.a. location-based, position-based, geographic, etc.

- Each node knows its own position and position of neighbors
- Source knows the position of the destination
- No routing tables stored in nodes!

- Geometric routing is important:
  - GPS/Galileo, local positioning algorithm, overlay P2P network, Geocasting
  - Most importantly: Learn about general ad-hoc routing

Related Work in Geometric Routing

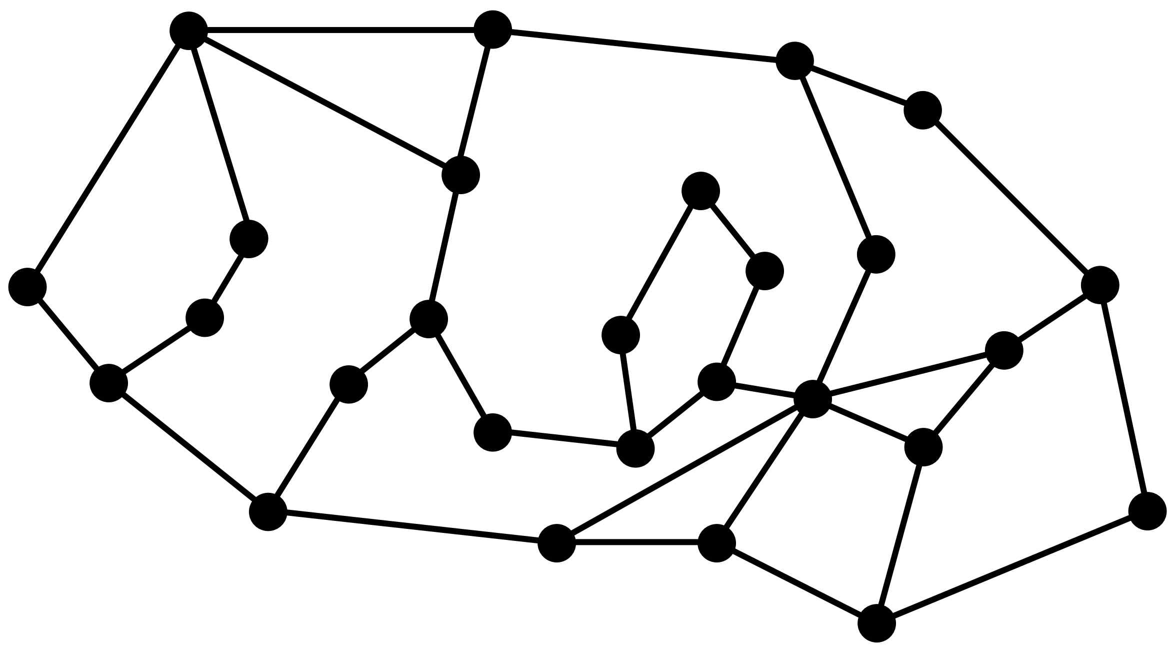
Kleinrock et al.	Various 1975ff	MFR et al.	Geometric Routing proposed
Kranakis, Singh, Urrutia	CCCG 1999	Face Routing	First correct algorithm
Bose, Morin, Stojmenovic, Urrutia	DialM 1999	GFG	First average-case efficient algorithm (simulation but no proof)
Karp, Kung	MobiCom 2000	GPSR	A new name for GFG
Kuhn, Wattenhofer, Zollinger	DialM 2002	AFR	First worst-case analysis. Tight $\Omega(c^2)$ bound.
Kuhn, Wattenhofer, Zollinger	MobiHoc 2003	GOAFR	Worst-case optimal and average-case efficient, percolation theory

Overview

- Introduction
  - What is Geometric Routing?
  - Greedy Routing
- Correct Geometric Routing: Face Routing
- Efficient Geometric Routing
  - Adaptively Bound Searchable Area
  - Lower Bound, Worst-Case Optimality
  - Average-Case Efficiency
  - Critical Density
  - GOAFR
- Conclusions

Face Routing

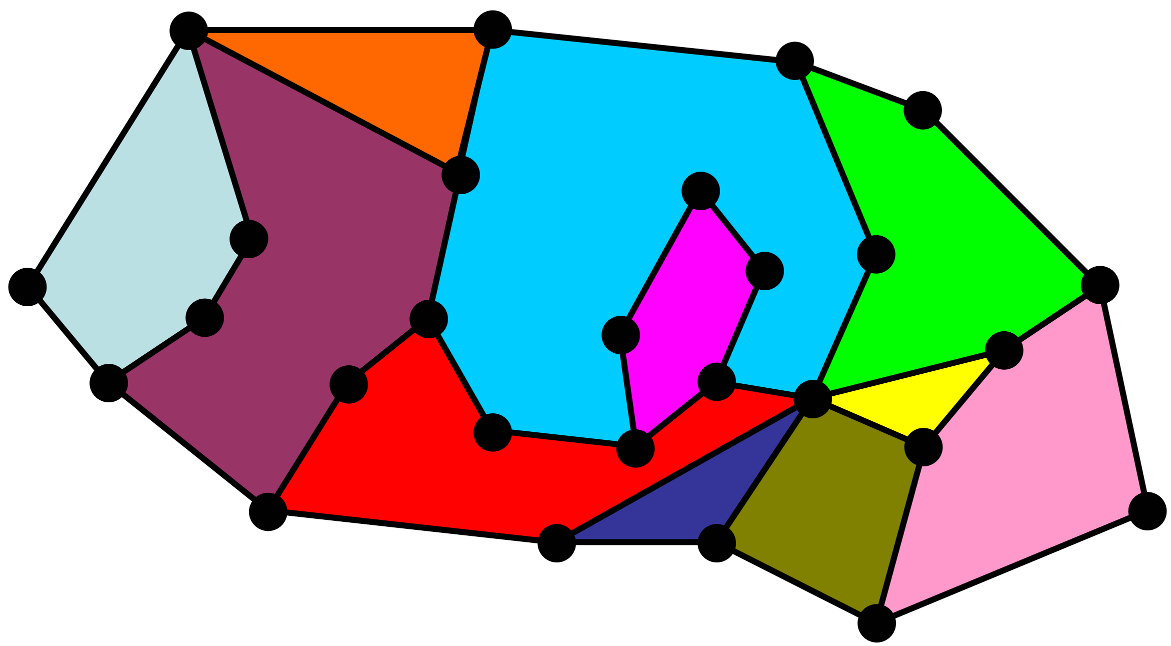
- Based on ideas by [Kranakis, Singh, Urrutia CCCG 1999]
- Here simplified (and actually improved)



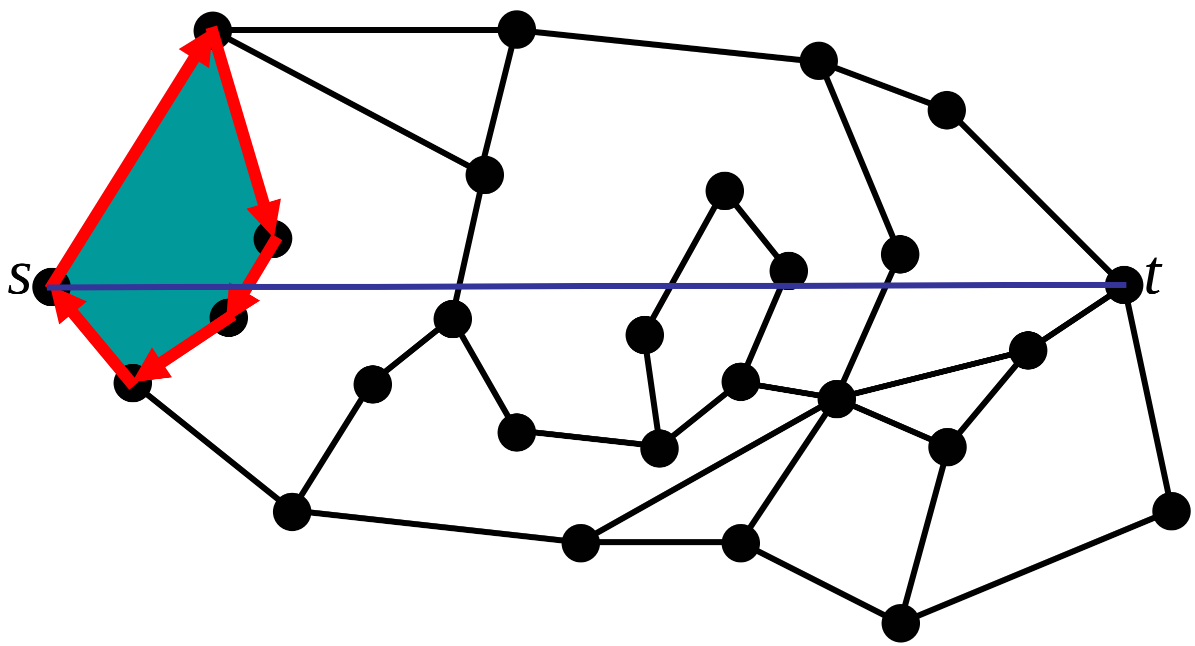
Face Routing

- Remark: Planar graph can easily (and locally!) be computed with the Gabriel Graph, for example

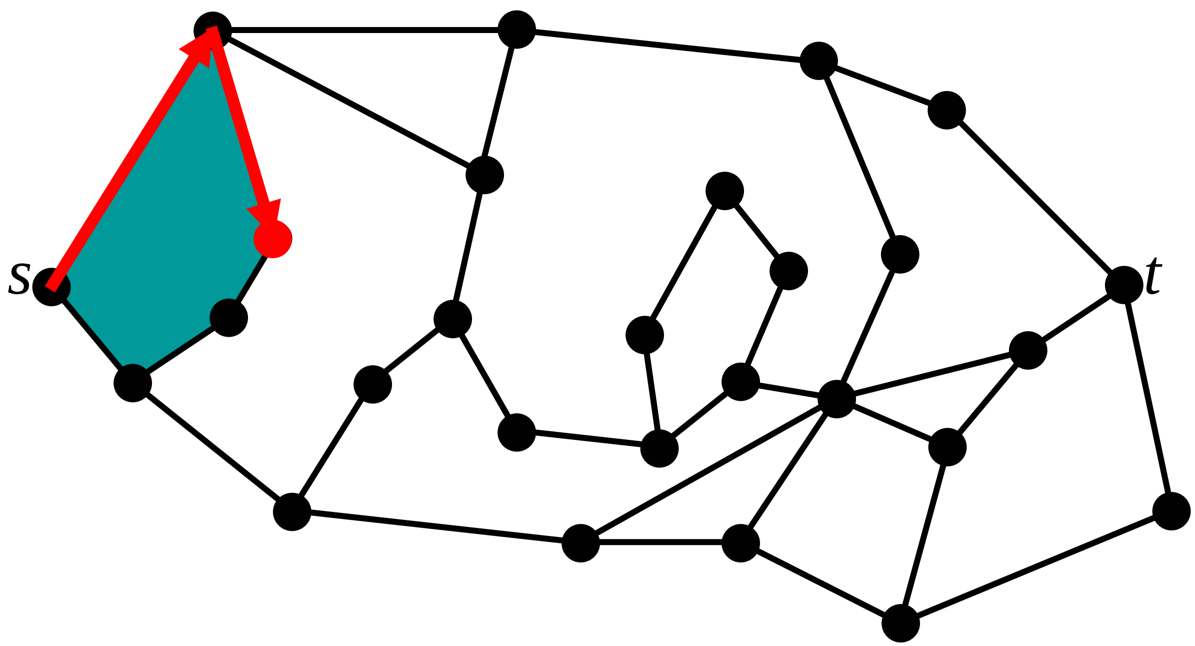
Planarity is NOT an assumption



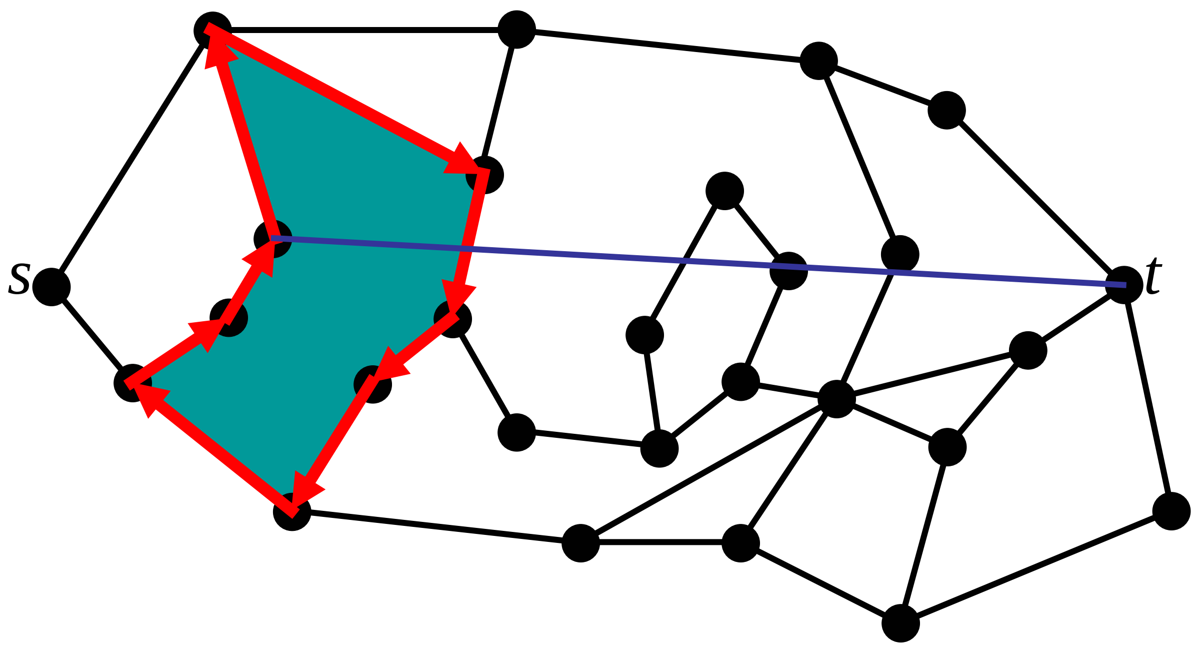
Face Routing



Face Routing



Face Routing



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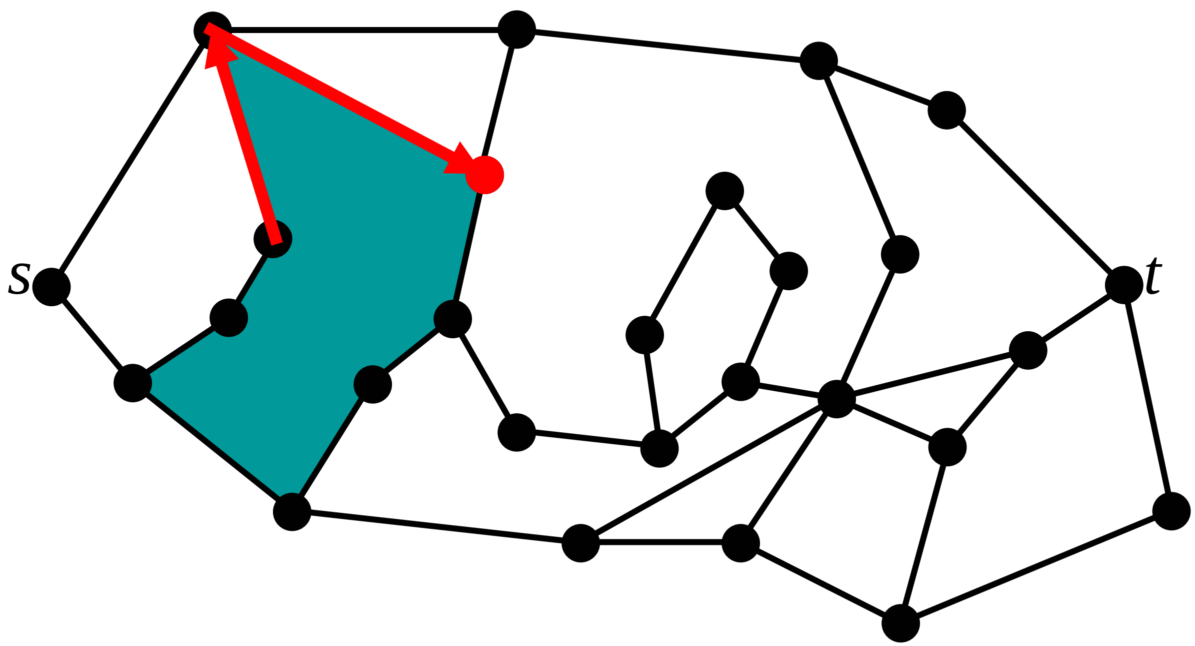

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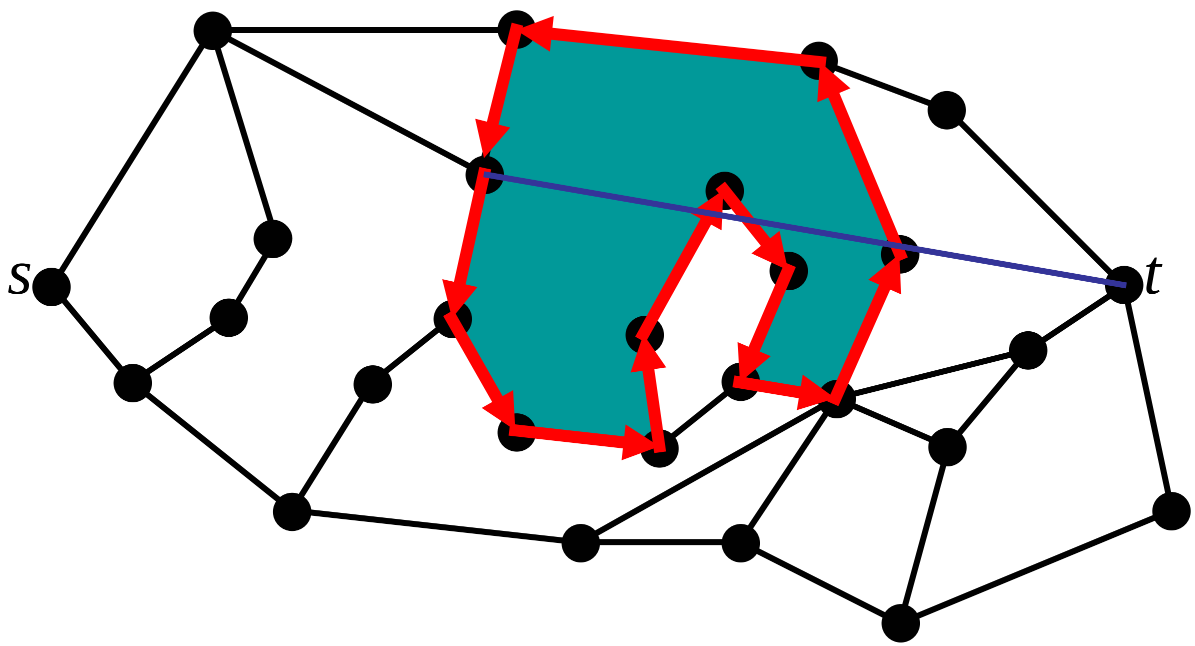

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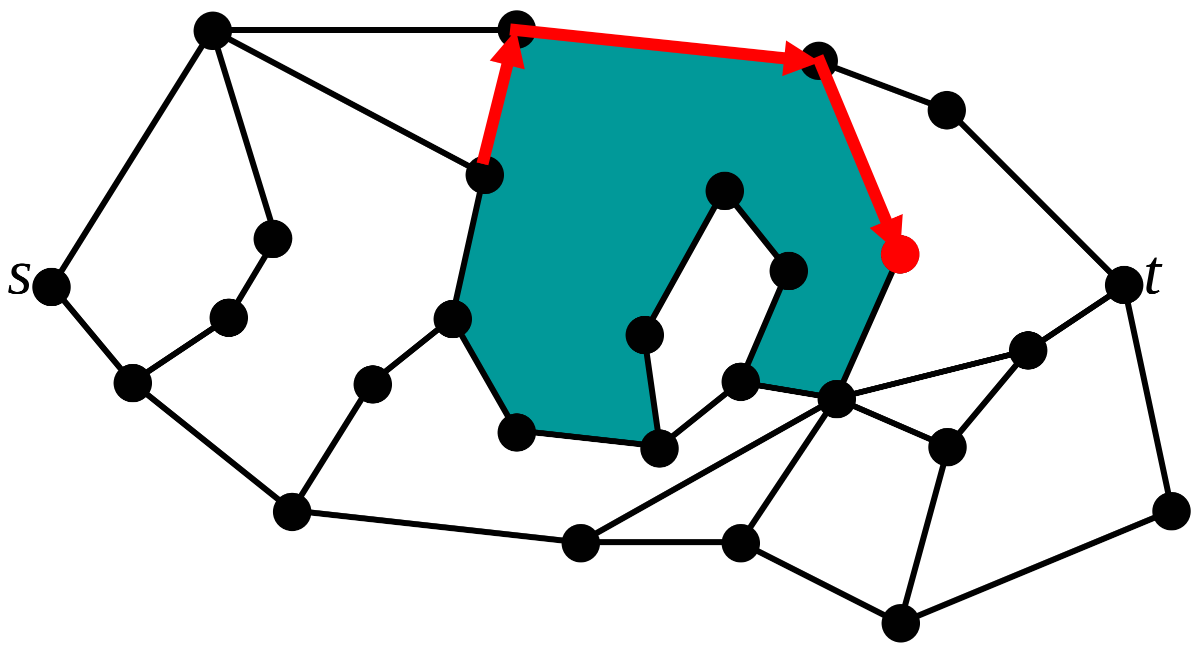

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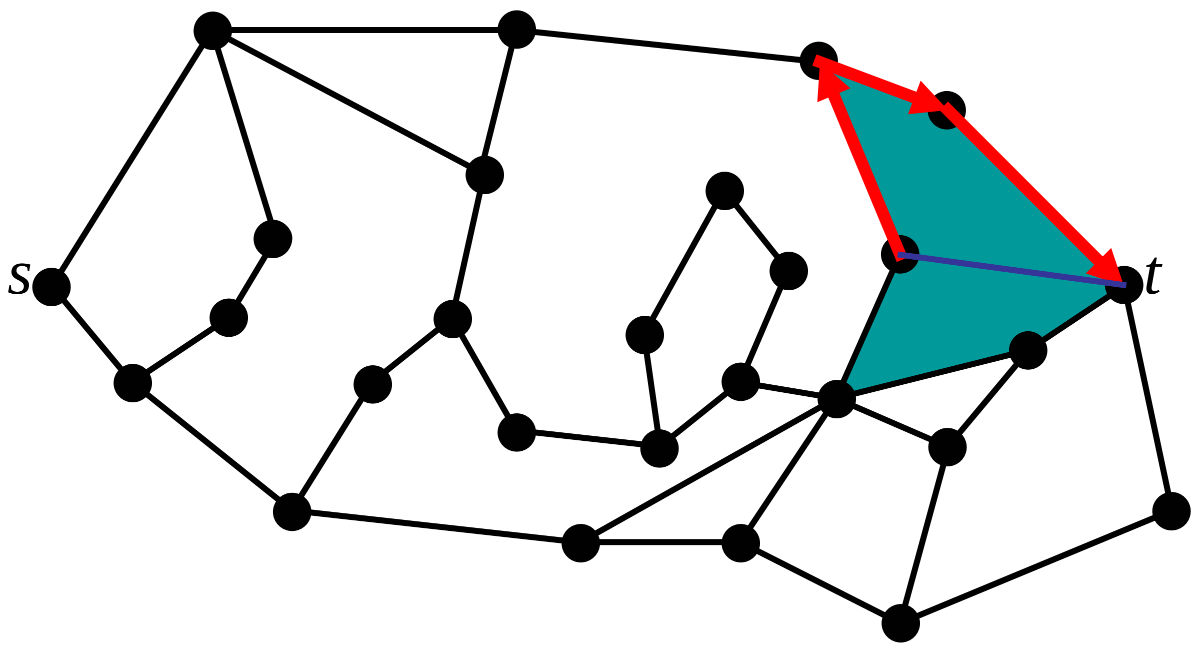
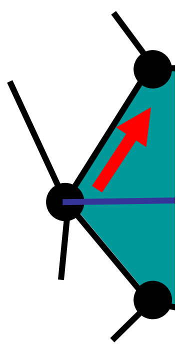
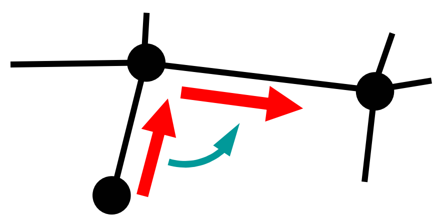


Face Routing Properties

- All necessary information is stored in the message
  - Source and destination positions
  - Point of transition to next face
- Completely local:
  - Knowledge about direct neighbors‘ positions sufficient
  - Faces are implicit





“Right Hand Rule”

- Planarity of graph is computed locally (not an assumption)
  - Computation for instance with Gabriel Graph

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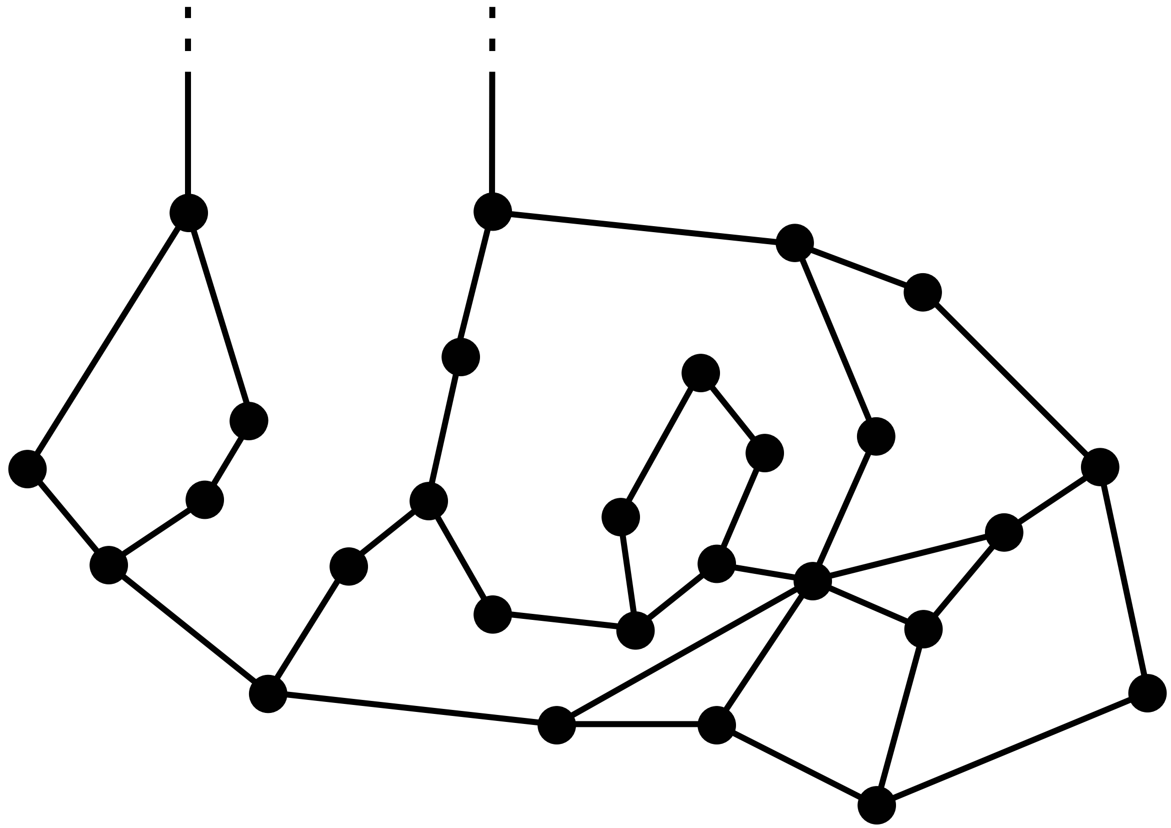
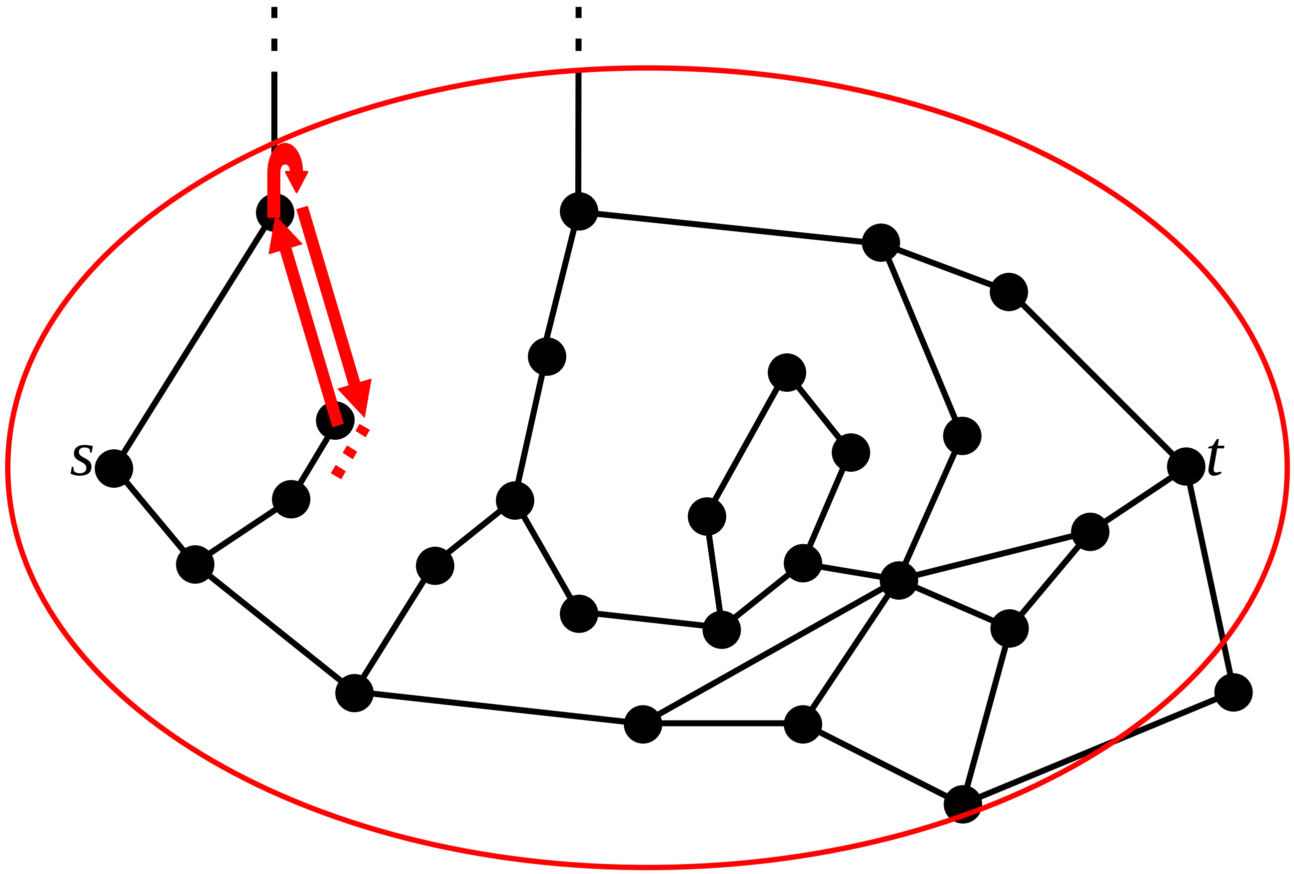
Face Routing

- Theorem: Face Routing reaches destination in O(n) steps
- But: Can be very bad compared to the optimal route



Bounding Searchable Area



Adaptively Bound Searchable Area

What is the correct size of the bounding area?

- Start with a small searchable area
- Grow area each time you cannot reach the destination
- In other words, adapt area size whenever it is too small

→ Adaptive Face Routing AFR

Theorem: AFR Algorithm finds destination after $O(c^2)$ steps, where c is the cost of the optimal path from source to destination.

Theorem: AFR Algorithm is asymptotically worst-case optimal.

[Kuhn, Wattenhofer, Zollinger DIALM 2002]

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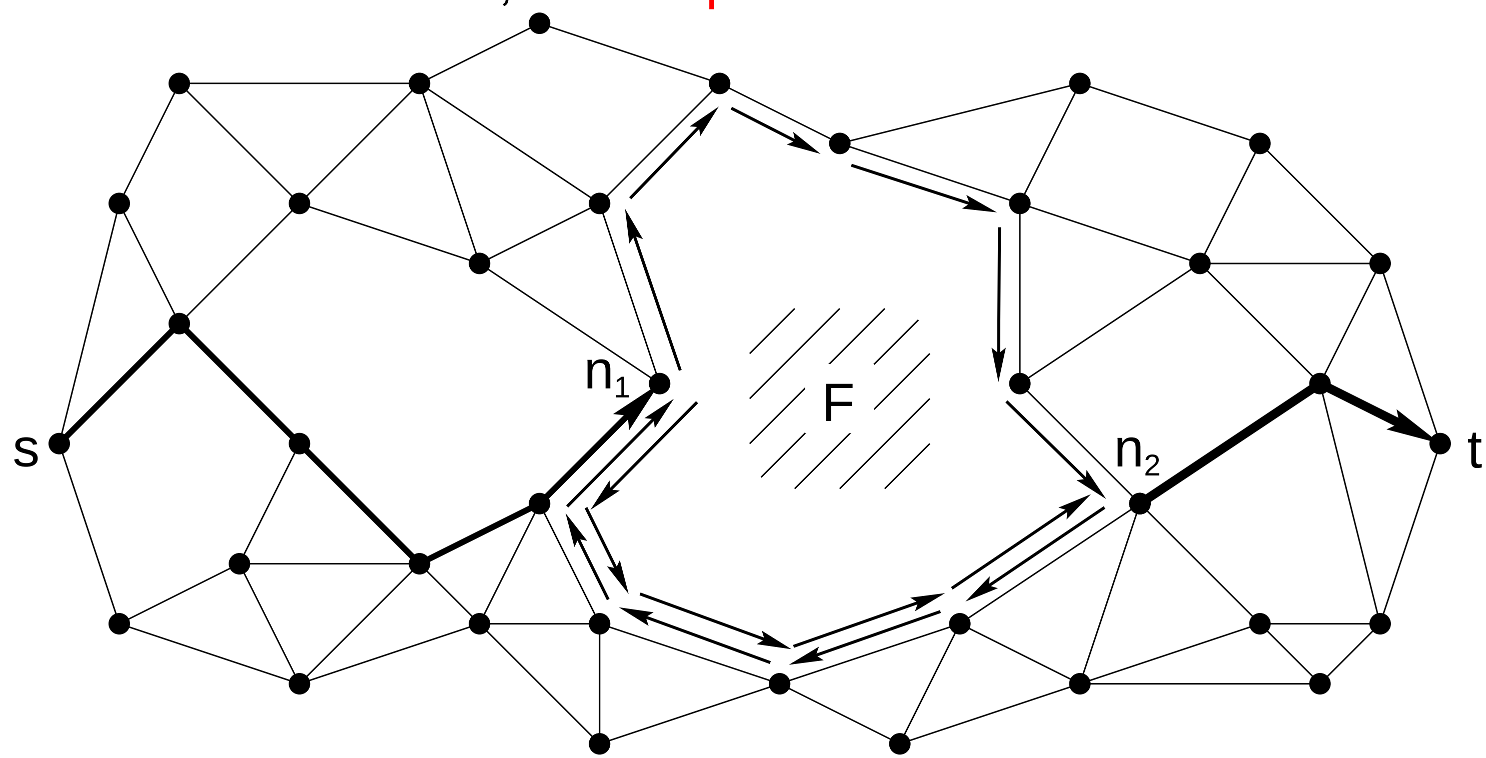
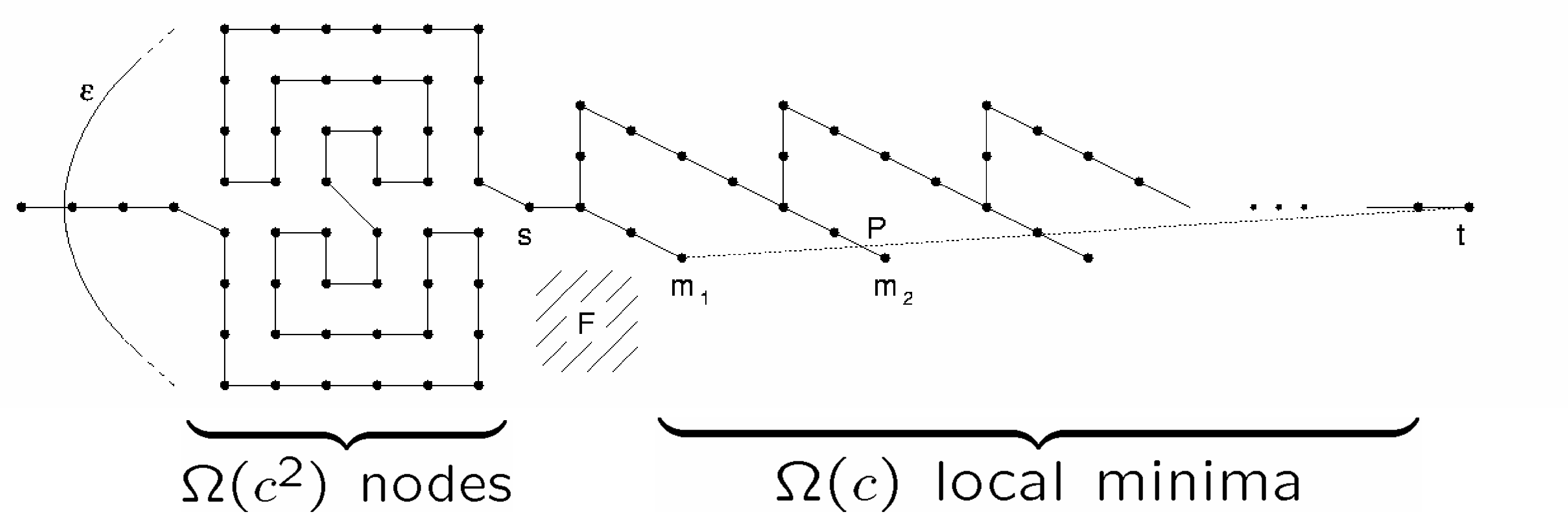
GOAFR – Greedy Other Adaptive Face Routing

- AFR Algorithm is not very efficient (especially in dense graphs)
- Combine Greedy and (Other Adaptive) Face Routing
  - Route greedily as long as possible
  - Overcome "dead ends" by use of face routing
  - Then route greedily again
- Similar as GFG/GPSR, but adaptive



Early Fallback to Greedy Routing?

- We could fall back to greedy routing as soon as we are closer to t than the local minimum
- But:



- "Maze" with $\Omega(c^2)$ edges is traversed $\Omega(c)$ times $\rightarrow$ $\Omega(c^3)$ steps

GOAFR Is Worst-Case Optimal

- GOAFR traverses complete face boundary:

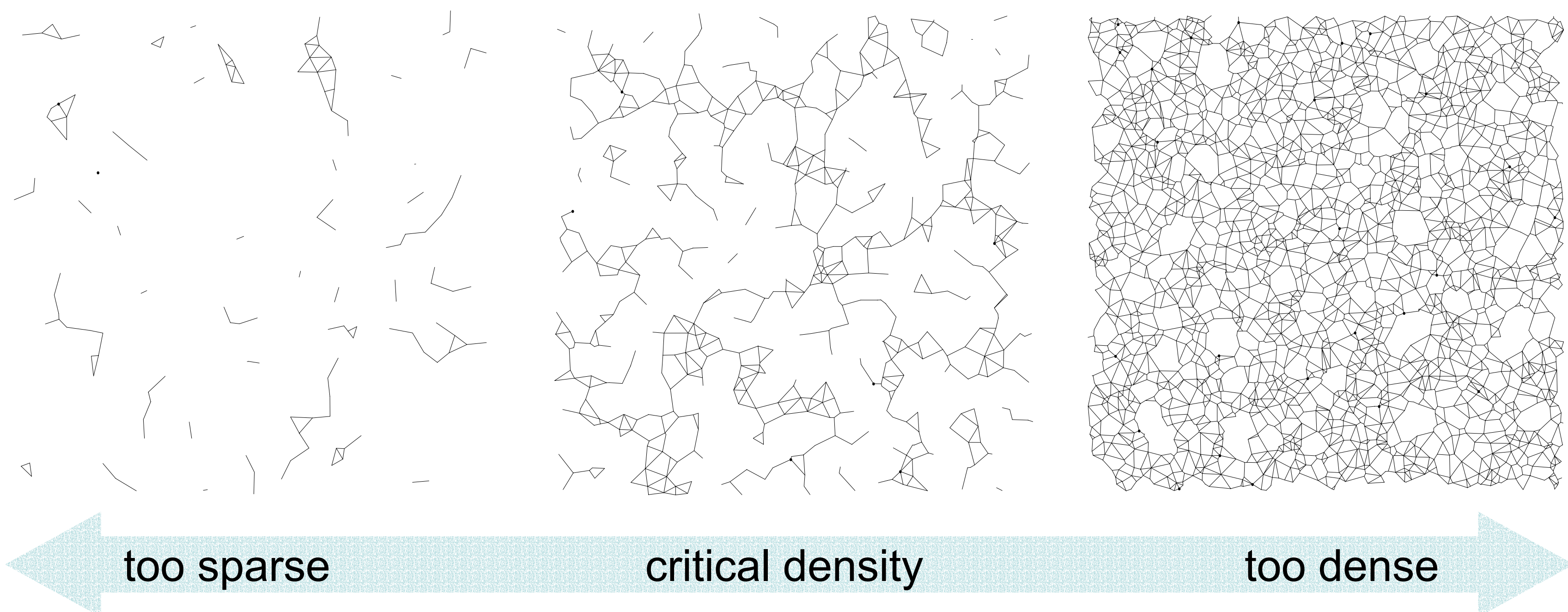
Theorem: GOAFR is asymptotically worst-case optimal.

- Remark: GFG/GPSR is not
  - Searchable area not bounded
  - Immediate fallback to greedy routing

- GOAFR's average-case efficiency?

Average Case

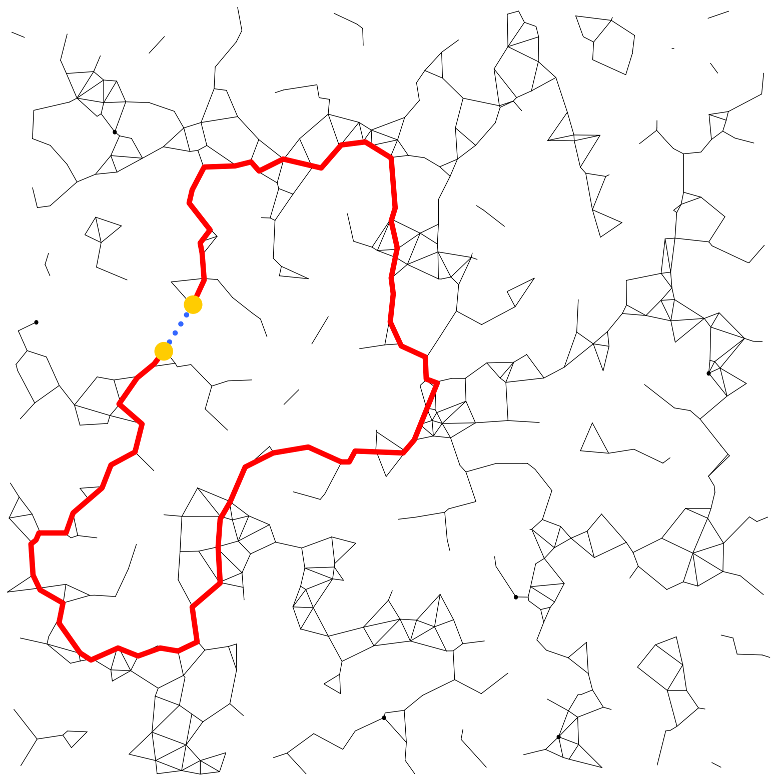
- Not interesting when graph not dense enough
- Not interesting when graph is too dense
- Critical density range ("percolation")
  - Shortest path is significantly longer than Euclidean distance



Critical Density: Shortest Path vs. Euclidean Distance

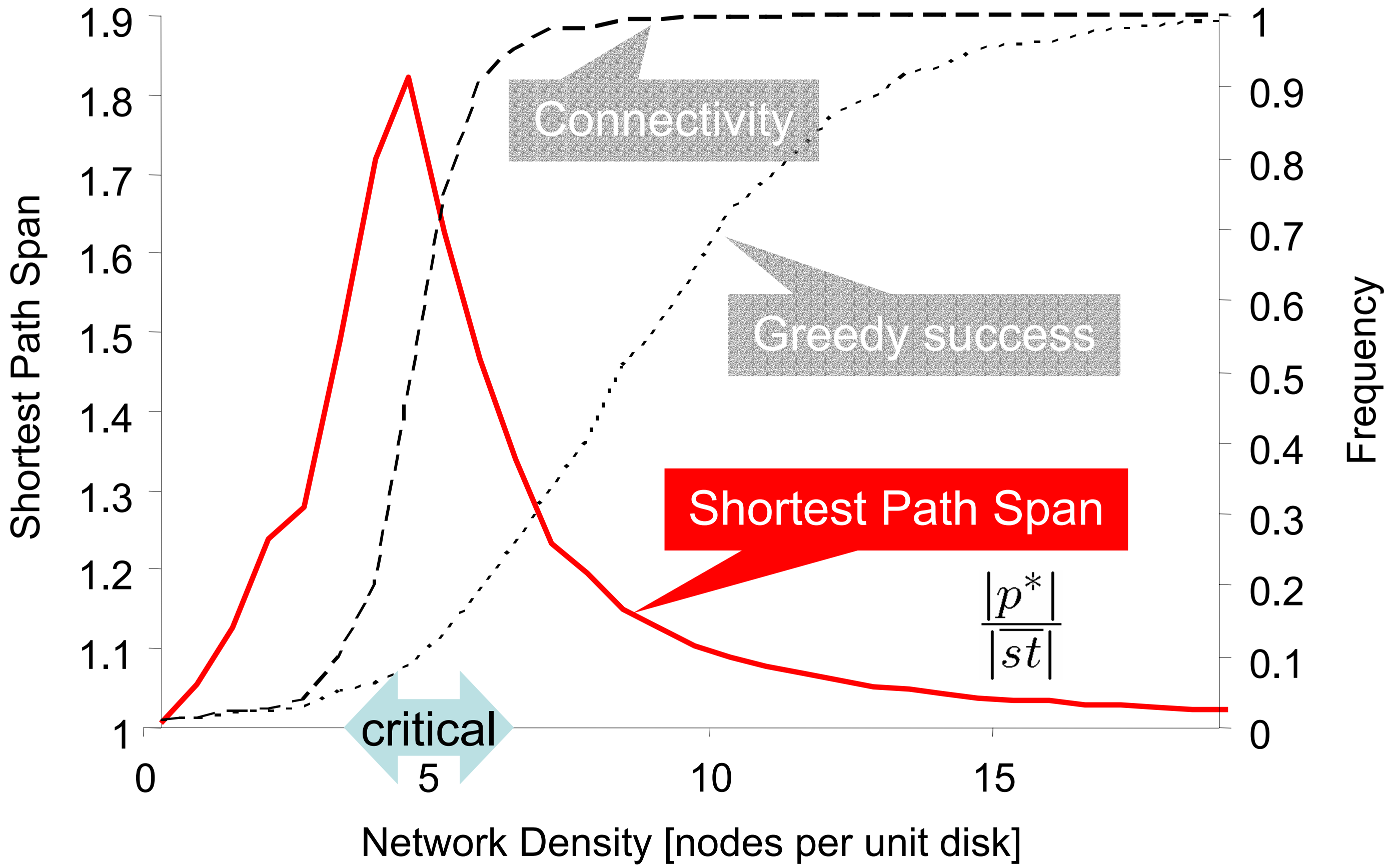
- Shortest path is significantly longer than Euclidean distance

$$\frac{|p^*|}{|\overline{st}|}$$

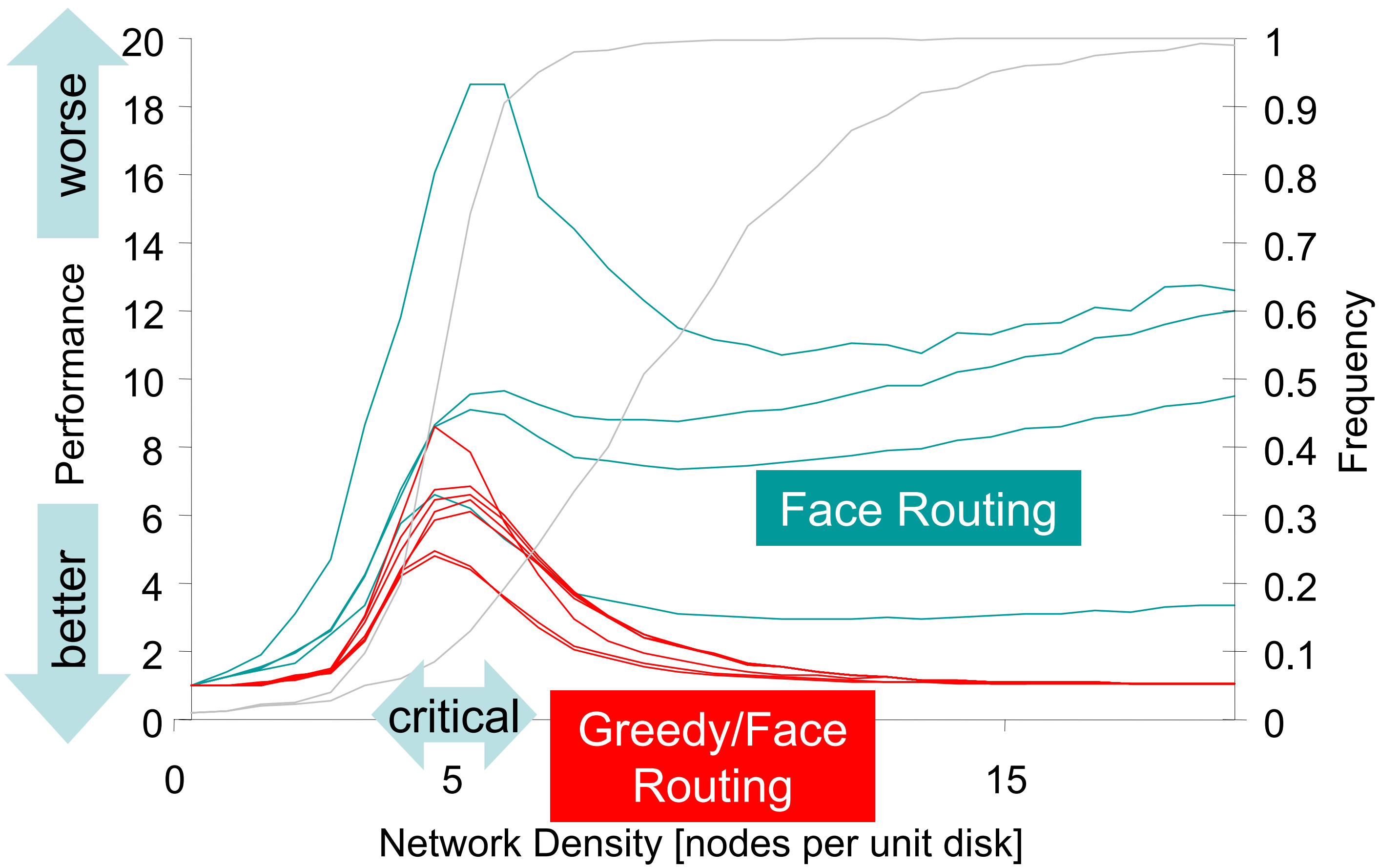


- Critical density range mandatory for the simulation of any routing algorithm (not only geometric)

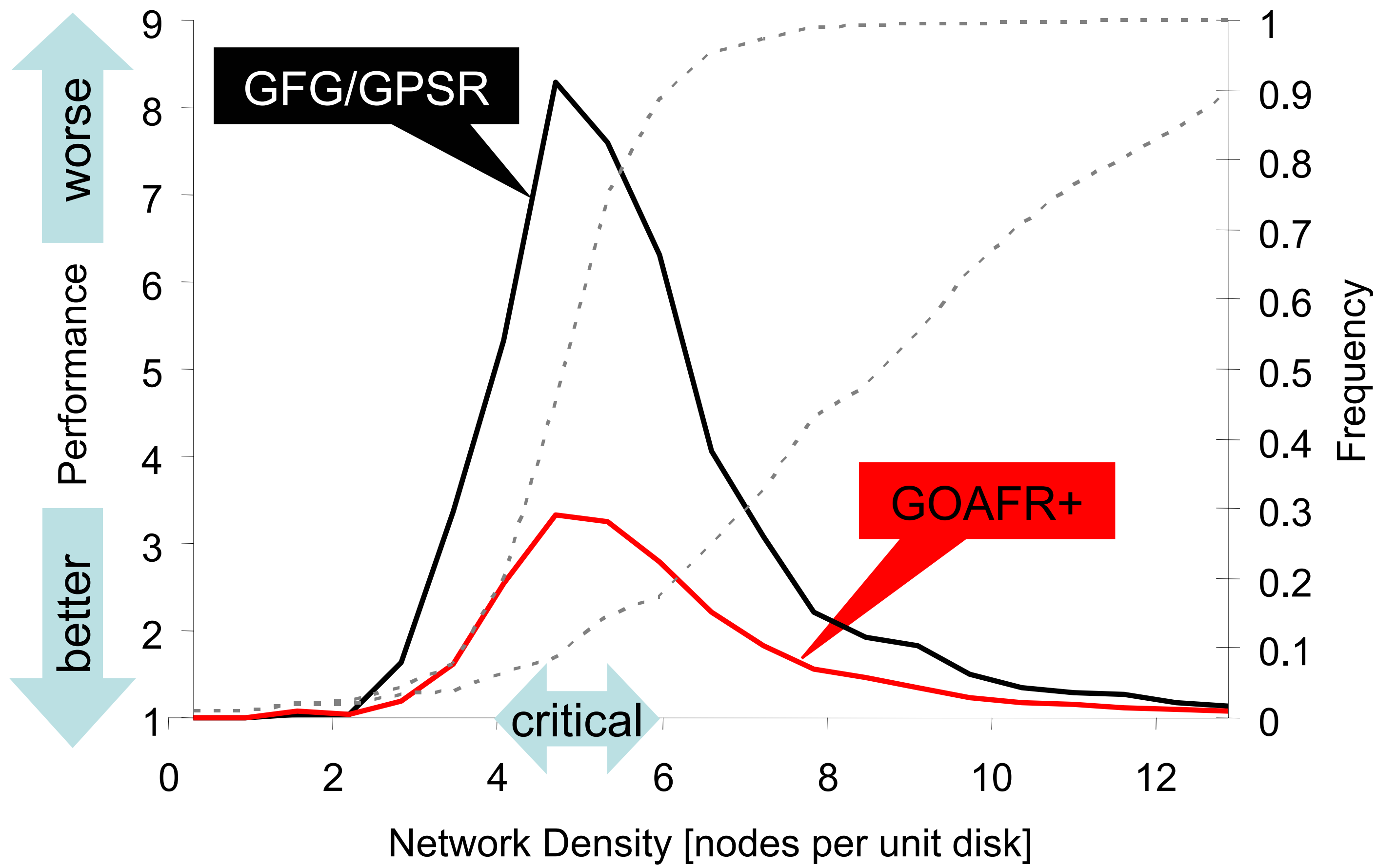
Randomly Generated Graphs: Critical Density Range



Average-Case Performance: Face vs. Greedy/Face



Simulation on Randomly Generated Graphs



Conclusion

	Correct Routing	Worst-Case Optimal	Avg-Case Efficient	Comprehensive Simulation
Greedy Routing (MFR)			(✓)	
Face Routing	✓			
GFG/GPSR	✓		✓	
AFR	✓	✓		
GOAFR	✓	✓	✓	✓

Questions?
Comments?
Demo?