Eidgenössische Technische Hochschule Zürich
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Distributed Computing

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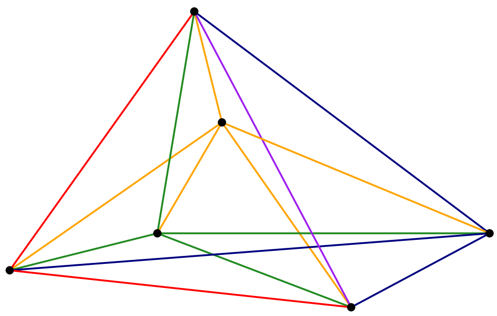
BA/MA/SA:

Geometric Edge-Coloring

A *geometric graph* is a graph drawn on a finite point set in the plane using only straight lines as edges. Consider the following problem:

> Given a complete geometric graph, how many plane (i.e. crossing-free) subgraphs, so-called *layers*, are required to form a partition of its edge set?



For illustration, we will simply color each layer with one color. The problem can thus be reformulated as follows: Given a complete geometric graph on n vertices, how many colors are required to color all the edges such that no two edges of the same color cross?

We can establish a simple linear upper bound using $n-1$ colors as follows: Start with any vertex and color all its incident edges in one color. On the remaining uncolored graph, pick another vertex and color all its incident uncolored edges in another color; then, repeat until the whole graph is colored. This construction requires one color per vertex, but as the last vertex will have no incident uncolored edges, a total number of $n-1$ colors will suffice. This construction is illustrated in the figure at the side, coloring the vertices in the order:

orange - green - blue - red - purple.

The problem at hand was initially raised by Araujo et al. [1]. Later, Bose, Hurtado, Rivera-Campo and Wood [2] stated this problem in the following open question:

> Is there an $\epsilon > 0$, such that every complete geometric graph on n vertices can be partitioned into at most $(1-\epsilon) \cdot n$ plane subgraphs?

We want to tackle this open problem in computational geometry by studying corner cases and generalizations of the problem.

Requirements: Interest and previous knowledge in computational geometry are advantageous.

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References:

[1] Gabriela Araujo, Adrian Dumitrescu, Ferran Hurtado, Marc Noy, and Jorge Urrutia. On the chromatic number of some geometric type kneser graphs. *Computational Geometry*, 32(1):59–69, 2005.

[2] Prosenjit Bose, Ferran Hurtado, Eduardo Rivera-Campo, and David R. Wood. Partitions of complete geometric graphs into plane trees. *Computational Geometry*, 34(2):116–125, 2006.