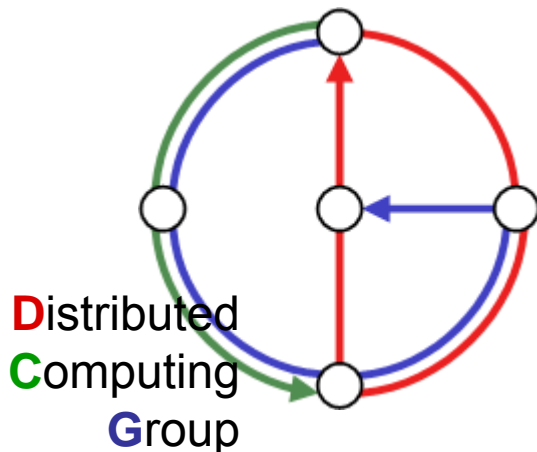


Word of Mouth: Rumor Dissemination in Social Networks

Jan Kostka

Yvonne Anne Oswald

Roger Wattenhofer



ETH

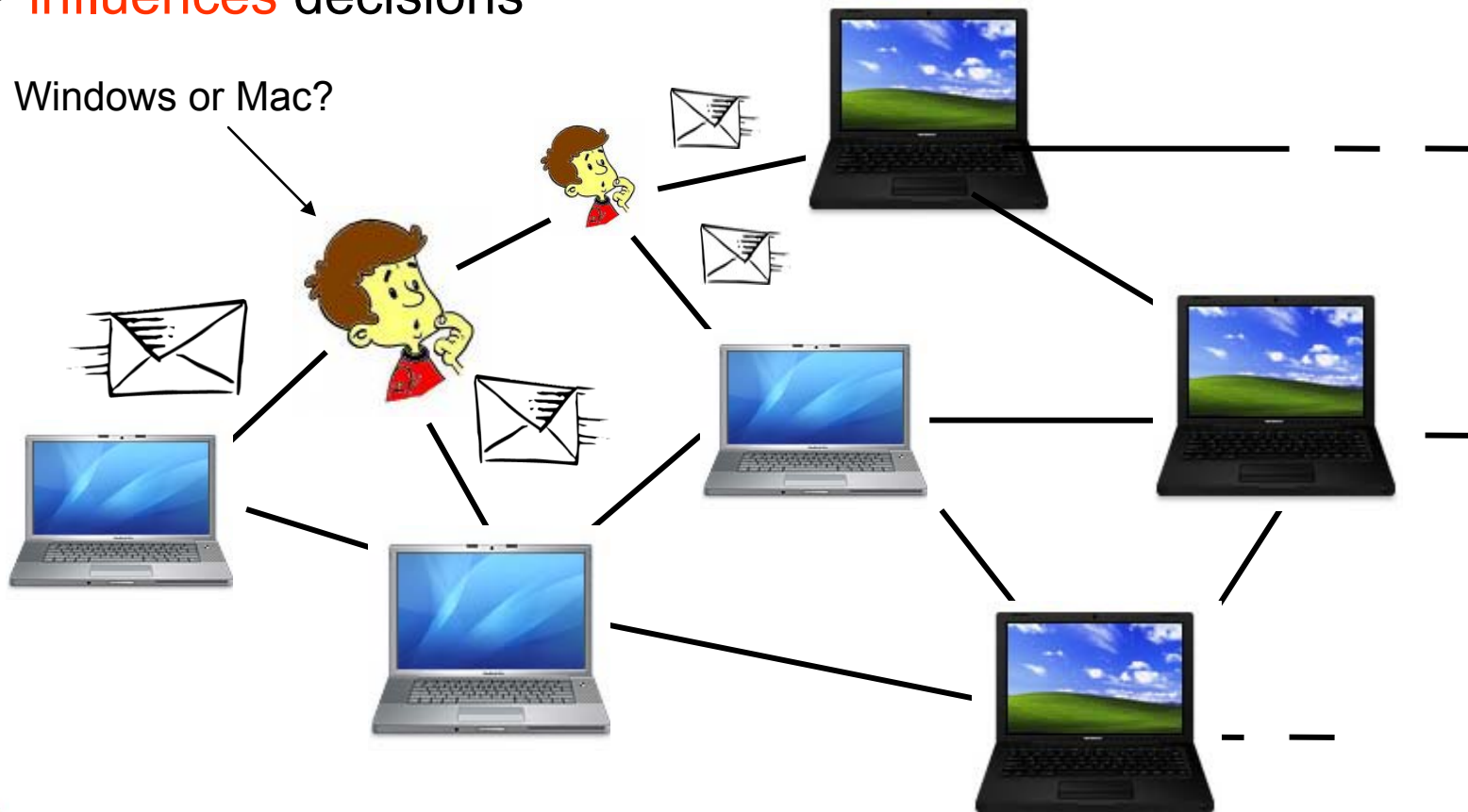
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Introduction

social networks everywhere: facebook, co-authors, email

=> effects **dissemination** of information

=> **influences** decisions



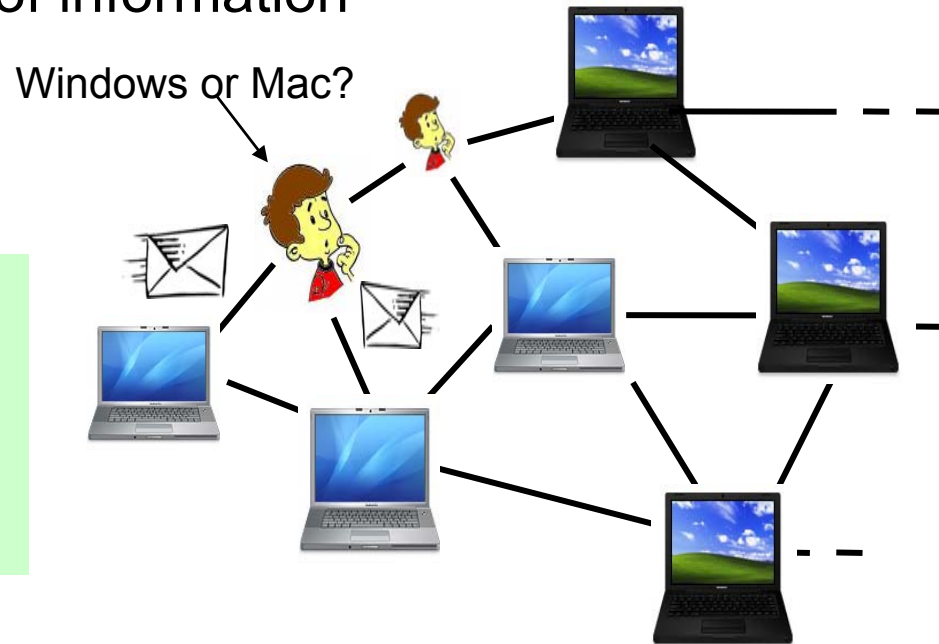
Introduction

social networks everywhere: facebook, co-authors, email

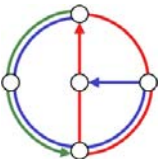
=> effects **dissemination** of information

=> **influences** decisions

- viral marketing
- competing theses, theories
- virus vs immunisation



GOAL: select **optimal** initiator set,
to convince as many nodes as possible



Related Work

1 rumour

- epidemics, physical processes:
sophisticated propagation models + simulation
- Kempe et al. [KDD03] :
selecting optimal initiators is NP-hard
greedy hill climbing algorithm: $(1-1/e)$ -approximation

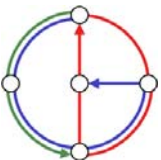


2 competing rumours

- Bharati et al.[WINE07], Carnes et al.[ICEC07]
2nd player: selecting optimal initiators is NP-hard
hill climbing works as well



What about the 1st player?



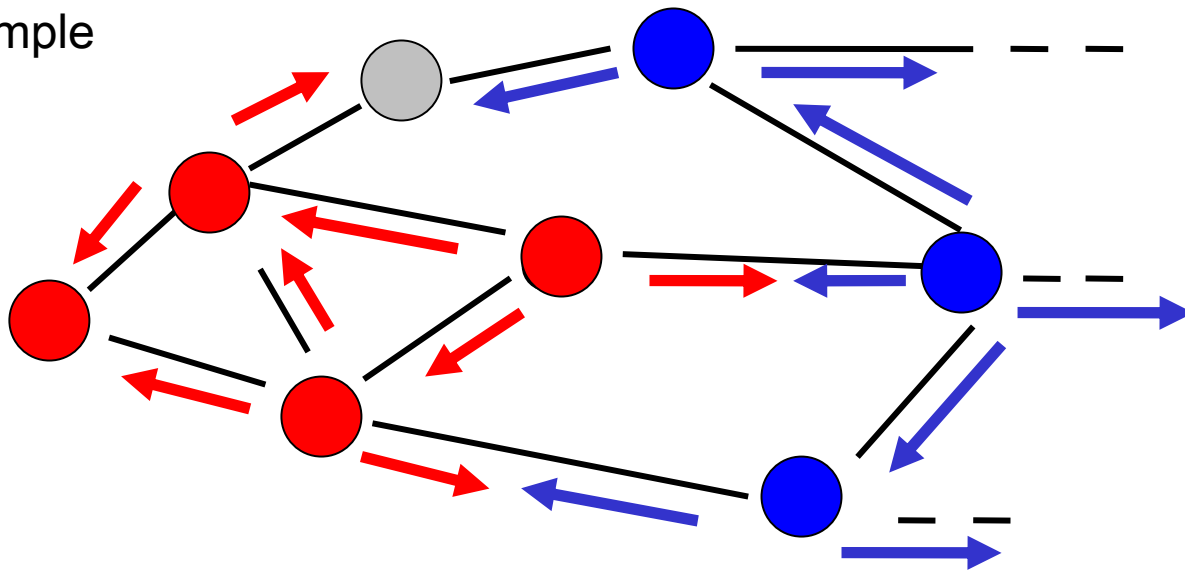
Basic Model



- **strategy:** select set of nodes to initiate the rumour
- **rumour propagation:**
 - accept first rumour encountered
 - forward rumour to all adjacent nodes

Payoff:
convinced nodes

Example



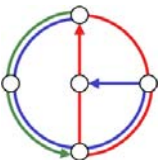
1st player:

4 nodes

2nd player:

3 nodes

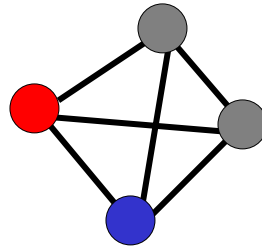
variations: more players, payoff definition, propagation model (cascade, threshold, ...), weighted or directed edges, ...



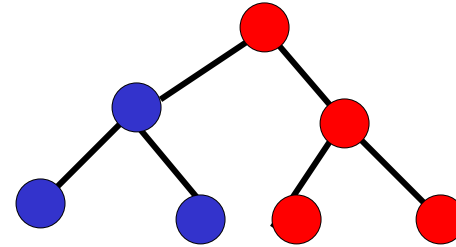
Warm-Up: 1 vs 1



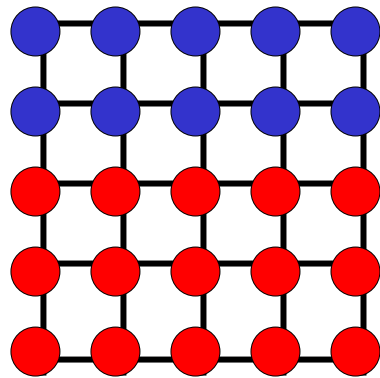
- Complete graph



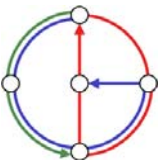
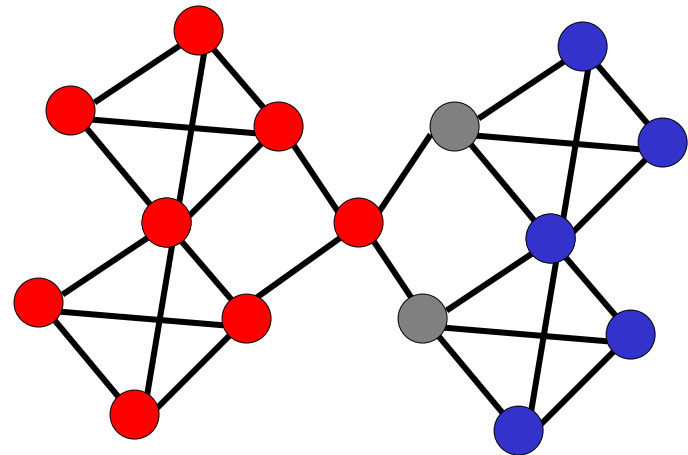
- Trees



- Grid



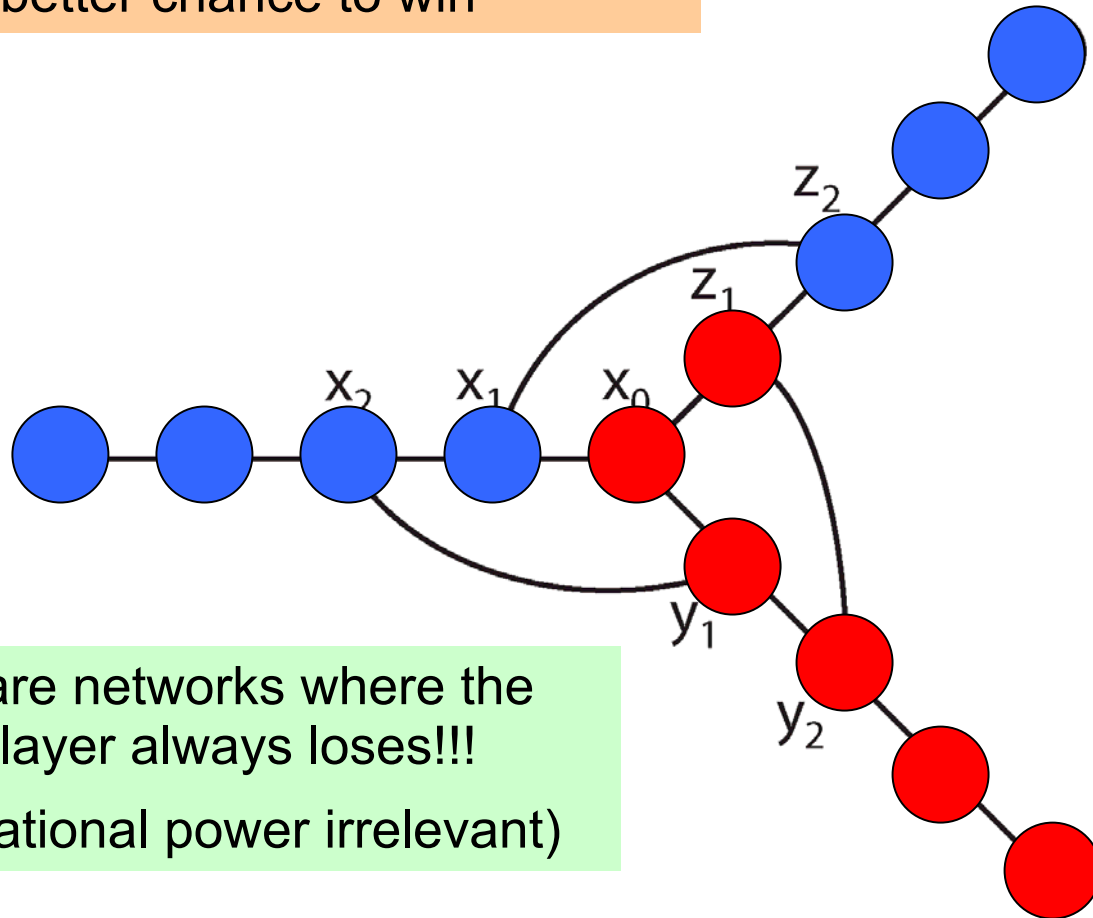
- Bottleneck



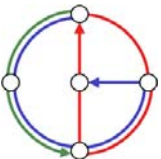
He who laughs last, laughs best?



Intuition: 1st player has more choice
⇒ better chance to win



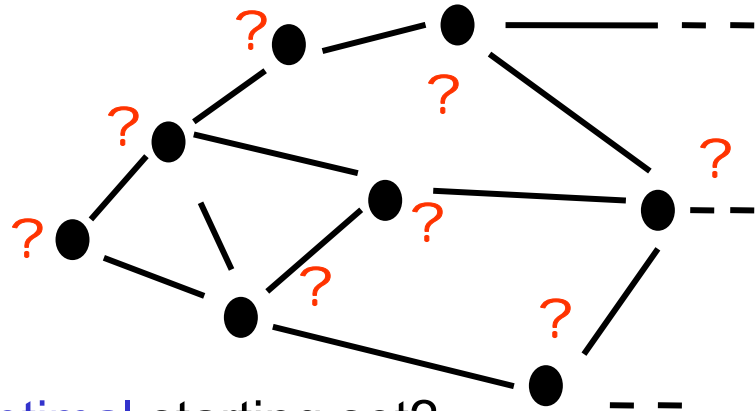
There are networks where the
1st player always loses!!!
(computational power irrelevant)



How hard is it to compute the optimal strategy?

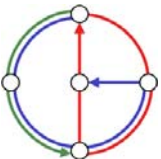
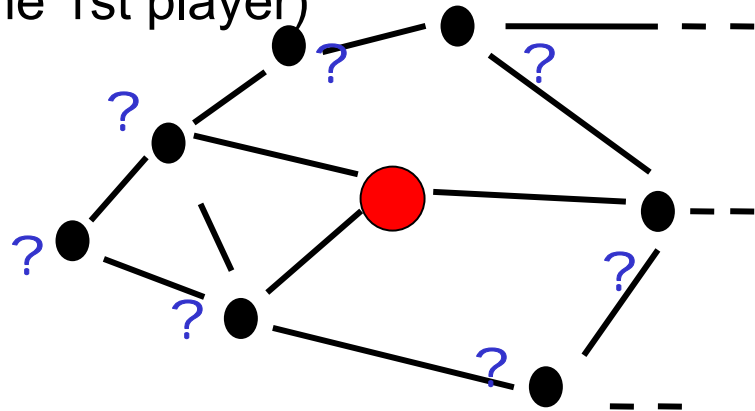
Centroid Problem

1st player: how do I choose the **optimal** starting set?
(knowing how many nodes the second player can select)



Medianoid Problem

2nd player: how do I choose my **optimal** starting set?
(knowing the nodes selected by the 1st player)



NP-hardness of Medianoid Problem



Theorem. *The (r|p)-medianoid problem is NP-hard.*

Proof:

Reduce Dominating Set (DS) problem to (r|1)-medianoid problem.

Idea: show that
 $\exists Y_r$ s.t. 2nd player wins
at least $|V|+r$ nodes

\Leftrightarrow

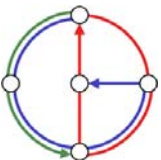
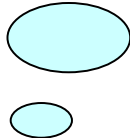
\exists DS with r nodes

1st player chooses x_1
2nd player selects Y_r

payoff 2nd player:
nodes closer to Y_r
than to X_p .

$G(V,E)$
 $\cup \{v, c\}$

DS



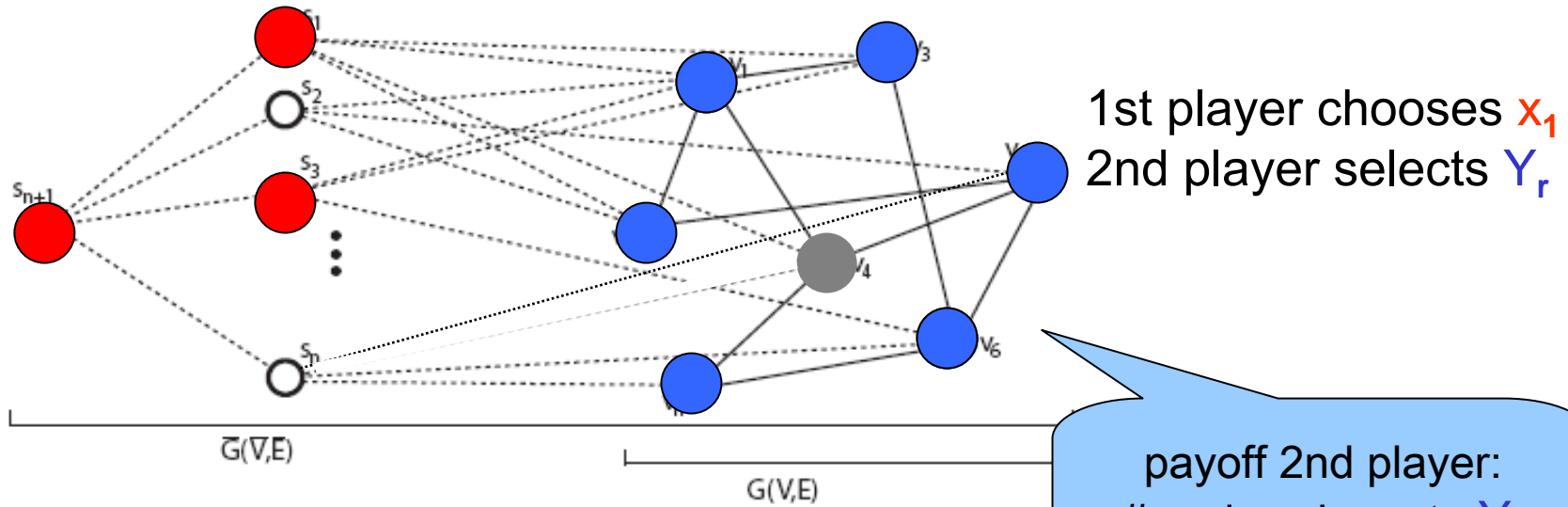
NP-hardness of Medianoid Problem



Theorem. *The $(r|p)$ -medianoid problem is NP-hard.*

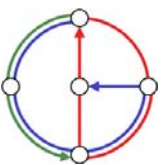
Proof:

Reduce Dominating Set (DS) problem to $(r|1)$ -medianoid problem.



$Y_r \neq DS$

payoff 2nd player:
nodes closer to Y_r
than to X_p .



NP-hardness of Centroid Problem

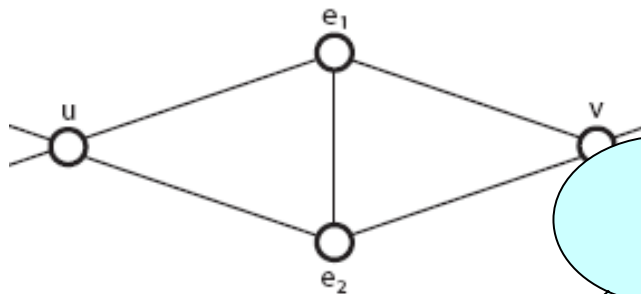


Theorem. *The $(r|p)$ -centroid problem is NP-hard.*

Proof:

Reduce Vertex Cover (VC) problem to $(1|p)$ -centroid problem.

Given graph $G(V,E)$, replace each edge with “diamond structure”



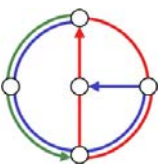
Idea: show that
 $\exists X_p$ s.t. less than 2
nodes closer to $Y_1(X_p)$

\Leftrightarrow

\exists VC with p nodes

if x_i on every diamond, we are ok
if no x_i on diamond, contradiction

1st player chooses X_p
2nd player selects $Y_1(X_p)$



NP-hardness of **Approximating the Centroid** Problem

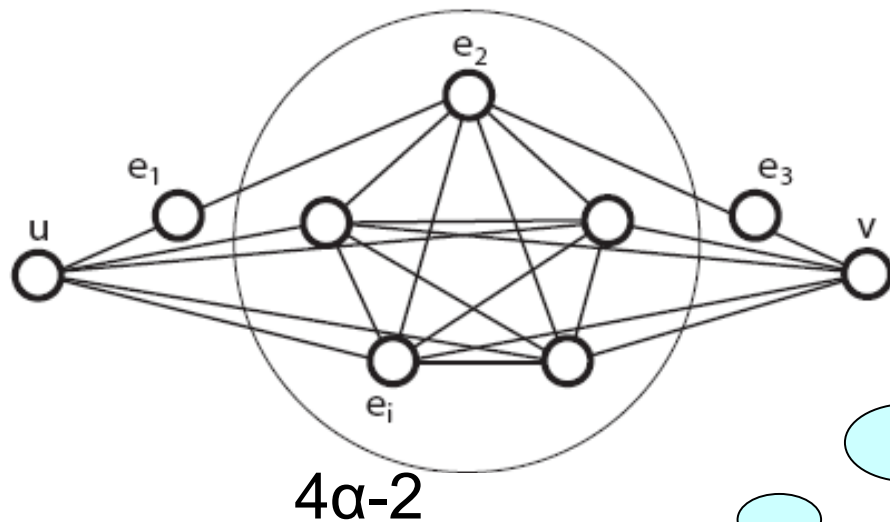


Theorem. Computing an α -approximation of the $(r|p)$ -centroid problem is NP-hard.

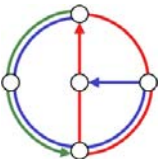
Proof:

Reduce Vertex Cover (VC) problem to $(1|p)$ -centroid problem.

Given graph $G(V,E)$, replace each edge with “clique structure”

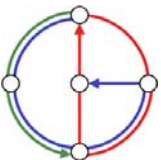
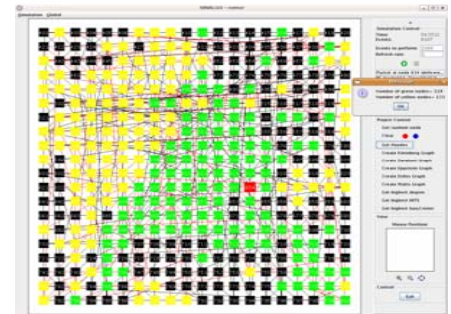


same idea, a little bit more complicated



More findings...

- relationship **Condorcet vertex** – centroid
- characterize weaknesses of **heuristics** for centroid
 - small radius
 - high degrees
 - midpoint of spanning tree
- (not in paper) **simulation of strategies** in random graphs:
Kleinberg, Watts, Epstein model



The End!



**THANK
YOU!**

Questions? Comments?

