

On Leveraging Partial Paths in Partially-Connected Networks

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Abstract—Mobile wireless network research focuses on scenarios at the extremes of the network connectivity continuum where the probability of all nodes being connected is either close to unity, assuming connected paths between all nodes (mobile ad hoc networks), or it is close to zero, assuming no multi-hop paths exist at all (delay-tolerant networks). In this paper, we argue that a sizable fraction of networks lies between these extremes and is characterized by the existence of *partial paths*, i.e., multi-hop path segments that allow forwarding data closer to the destination even when no end-to-end path is available. A fundamental issue in such networks is dealing with disruptions of end-to-end paths. Under a stochastic model, we compare the performance of the established end-to-end retransmission (ignoring partial paths), against a forwarding mechanism that leverages partial paths to forward data closer to the destination even during disruption periods. Perhaps surprisingly, the alternative mechanism is not necessarily superior. However, under a stochastic monotonicity condition between current vs. future path length, which we demonstrate to hold in typical network models, we manage to prove superiority of the alternative mechanism in stochastic dominance terms. We believe that this study could serve as a foundation to design more efficient data transfer protocols for partially-connected networks, which could potentially help reducing the gap between applications that can be supported over disconnected networks and those requiring full connectivity.

I. INTRODUCTION

More and more people nowadays carry a device with wireless networking capabilities. The majority of laptops, organizers, and other portable devices provide wireless networking; whereas in the cellular phone market this feature is trickling down from smart phones to the mainstream. Such devices traditionally need infrastructure networks to communicate with each other, using protocol optimizations of the TCP/IP suite to make up for the additional challenges of wireless environments [1]. Nevertheless, their increasing ubiquity creates opportunities for networking such devices “on the fly” or in “ad hoc” mode, bypassing or extending infrastructure, for applications ranging from social networks to multi-player gaming.

However, operating such a network of mobile nodes in ad hoc mode (traditionally called a *MANET*, for Mobile Ad Hoc Network) presents a number of challenges for transport and routing protocols. Initially, it was commonly assumed that MANETs are always connected, i.e. *each node has an end-to-end path to every other node with probability one*; various mechanisms were proposed to discover and maintain such paths, and traditional transport mechanisms (or modifications)

were assumed to provide support for all known applications. Yet, frequent path changes resulting from node mobility, wireless propagation effects, nodes powering down, etc. induce a significant overhead and performance penalty for these protocols as opposed to the infrastructure-based counterparts. What is more, it has been recently recognized that, when network density becomes lower, these networks experience frequent disconnections, implying that no end-to-end paths exist most of the time, or *connectivity occurs with probability zero*. At this end of the connectivity continuum, nodes are assumed to be relatively isolated; in occasional node encounters (“contacts”), forwarding decisions are made *speculatively*, trying to predict future contacts with the destination (e.g., based on mobility patterns, social relationships, etc.), but with no knowledge about end-to-end paths. As a result, only few asynchronous applications with high tolerance in delay can be supported under this network model, commonly referred to as *DTN* (Delay Tolerant Networking).

In this work we argue that there is a sizable region in between the two extremes of the network connectivity continuum, where the network is not connected (and thus MANET protocols suffer), yet more optimistic assumptions are in order than commonly made in DTNs. In a *partially-connected network*, nodes may not always be able to reach all other nodes, still they can reach a subset of them. If this subset is sufficiently variable over time, e.g., due to node mobility, then there will be multi-hop path segments that can be used to forward data closer to the destination node, even during periods where the full path is disrupted. We refer to such path segments as *partial paths*.

Partial paths correspond to additional transmission opportunities that can be exploited by known network data forwarding mechanisms such as hop-by-hop transport protocols [2], [3] and route salvaging mechanisms (for example, [4]) proposed in the context of MANET routing, in order to achieve more efficient data transport over these networks. This is in stark contrast to most algorithms proposed for DTNs [5] that only exploit single-hop paths at a time. Specifically, given a path breaks after the message has traversed a partial segment of it, the crucial question we are interested in answering is whether to try forwarding data from the intermediate node before the breakage (*intermediate forwarding*) or from the source node (*source forwarding*). Whereas the superiority of the former might be “intuitively” clear in a network with

fixed topology, it is not obvious in a network whose topology varies stochastically due to frequent link failures and node movements. Our work, therefore, intentionally focuses on the concept of partial paths and these two basic mechanisms rather than on particular protocols. Specifically, we argue for the existence of partial paths, and *analytically explore* what are the conditions under which “intermediate forwarding” or “local recovery” techniques like the above would result in more efficient data forwarding in partially-connected networks.

The main contributions of this paper are as follows. First, we demonstrate in Sec. II that partial paths do exist under a wide range of network conditions. Further, when a path breaks at an intermediate point, either an alternate partial path from this point towards the destination exists or the intermediate hop will have to “wait” until a new partial path for the remaining distance arises. In this case, performance will depend on the alternate route length (hop count) statistics and its relation to the primary route length. Using the network model introduced in Sec. III, we show in Sec. IV that—counter to intuition—if the alternate route length is positively correlated with the primary route length; or even if the expected value of the alternate route length is monotonically increasing in the primary route length, superiority does not necessarily hold. Nonetheless, we are able to show that if the length of the alternate route is *stochastically monotonic* in the length of the primary route, then the time to deliver a packet under source forwarding stochastically dominates that under intermediate forwarding. In Sec. V, we introduce the concepts of stochastic dominance and stochastic monotonicity, which we then use to prove this claim. In Sec. VI we provide strong evidence that the alternate route length is indeed stochastically monotonic in the primary route length—leading to the desired result of superiority. Section VII discusses related work, and Sec. VIII concludes the paper.

II. UNDER WHICH CONDITIONS ARE NETWORKS PARTIALLY-CONNECTED?

In this section, we shed more light into the existence of “partially-connected” networks. Specifically, we are interested in identifying whether scenarios exist where networks are disconnected (and thus MANET approaches would not be applicable or efficient), but still have sufficiently large connected components with partial paths of more than one hops (that DTN approaches would ignore).

A. Partial Paths in Stochastic Network Models

We have looked into the network connectivity dynamics using Monte Carlo simulations. We spread N nodes uniformly over a toroidal area A and add links between pairs of them according to the geometric link model, i.e., a link between two nodes exists as long as their distance is smaller than the node transmission range r_0 . We vary A to obtain the desired network density, expressed by the expected node degree d as $d = \frac{(N-1) \cdot \pi r_0^2}{A}$. For each $\{N, A\}$ tuple, simulations are repeated $M = 10^5$ times with different seeds, each time yielding a different snapshot of node distribution in A .

It is well-known by percolation results [6] that, for asymptotically large networks, connectivity exhibits a sharp phase-transition with respect to node density. Either almost all nodes are connected into a large cluster (*connected regime*) or all nodes are isolated into many much smaller clusters (*disconnected regime*). Nevertheless, for finite N values, we argue that this phase transition is less sharp. Specifically, a non-negligible range of network density values exists where sizable clusters are formed; each node can reach a non-negligible subset of other nodes (but not all) using partial paths of multiple connected hops. To demonstrate this, we estimate and plot the following three quantities:

- *Connectivity probability, C* : it is the probability that each node can reach every other node via a connected path, estimated as the ratio of the connected topologies over the full set size M of random topologies [7].
- *Reachability, R* : For each random topology, it equals the fraction of connected node pairs [8].¹
- *Shortest path length, P* : For each random topology, it denotes the average length of the shortest path in number of hops, considering all connected node pairs.

The plotted values for R and P are their averages over all M random topologies.

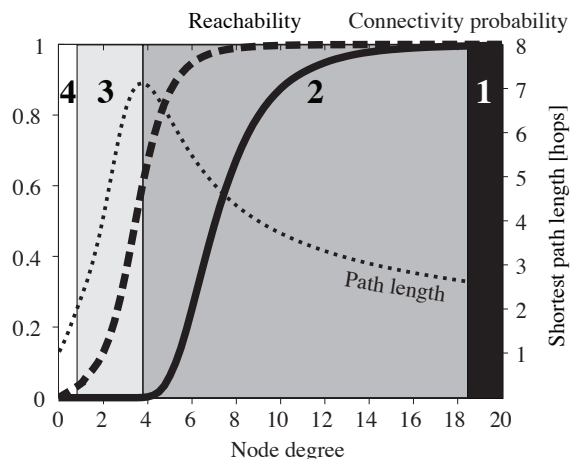


Fig. 1: Partially-connected networks (grey areas) are situated between connected (black) and disconnected networks (white). Model: 100 nodes uniformly distributed, geometric link model

In Fig. 1, we show how C , R , and P vary with the node degree for $N = 100$ nodes. Observe that both the connectivity probability and reachability exhibit a phase transition, albeit at different values on the node degree axis. Whereas the connectivity probability goes from zero to one in the interval $d = 4..20$, reachability increases more sharply in $d = 0..12$.

The sizable region of network density values where connectivity lies between zero and one represents the area of

¹Note that an identical metric had been proposed earlier under the name “connectivity index” in [9]. Irrespective of its name, the metric serves as a less pessimistic measure of the communication capabilities provided by a sparsely-connected ad hoc network.

partially-connected networks, situated in between the (very) sparse networks commonly studied in the DTN community and the highly-connected networks studied in the MANET context. Note that the average length of the shortest path in the partially-connected region is greater than two; hence, the assumption that there are no multi-hop paths is too pessimistic even when $d = 1$. Below, we identify four areas of interest in the connectivity continuum and explain whether and how intermediate forwarding can be applied in each:

Area 1: (Connectivity $\rightarrow 1$, Reachability $\rightarrow 1$) This is the area (far right) commonly dealt with by MANET research, where end-to-end paths are assumed to exist most of the time. Yet, when the network is very dynamic (e.g., high node mobility), the initial path often breaks while the message is en route. In this case, rather than dropping the message after it has traversed a partial path, intermediate forwarding can re-route it from that node on through an alternate path that the routing protocol has cached or will discover after the break. This mechanism is traditionally called *salvaging* in the MANET literature, and is analyzed theoretically in Sec. V (Thm. 4).

Area 2: ($0 < \text{Connectivity} < 1$, Reachability > 0) Nodes have end-to-end connectivity only for some fraction of time. During this fraction, a tentative end-to-end routing path between the source and destination nodes can be established, but for the remainder of the time this path is disrupted with only disconnected segments (referred to as partial paths) of it being up each time. Using intermediate forwarding, nodes at the end points of these partial paths can store en route data for the duration of the disruption and wait until a new partial path comes up that allows to continue forwarding the data towards the destination. We compare the performance of such intermediate forwarding against source forwarding analytically in Sec. V (Thm. 5).

Area 3: (Connectivity $\rightarrow 0$, Reachability > 0) The network is always partitioned. Nevertheless, node density is high enough that each node can reach a non-negligible number of other nodes within the same connected component (cluster) using multi-hop partial paths. In this context, DTN algorithms are usually applied or hybrid DTN/MANET schemes (MANET routing when the destination is in the same cluster, and DTN routing if outside the cluster) [10], [11]. We believe that one could still take advantage of existing partial paths using an appropriately modified version of intermediate forwarding, and outperform existing proposals. However, we defer this study for future work.

Area 4: (Connectivity $\rightarrow 0$, Reachability $\rightarrow 0$) Nodes are essentially isolated. Forwarding opportunities arise only when two nodes come in contact (e.g., through mobility), but multi-hop paths are rare. Such sparse networks are properly treated by DTN schemes.

Summarizing, even for homogeneously distributed nodes, there is a significant region of network density where a substantial portion of nodes are connected by multi-hop paths even if the connectivity probability is negligible. In the remainder of the paper, we will show how this property allows one to perform significantly better using local recovery methods

to route around disruptions instead of relying on end-to-end mechanisms.

B. Partial Paths in Real-life Scenarios

Real-life networks are usually not uniform; either the network area structure, or the node distribution in it, or both may be non-uniform in many scenarios. Examples include campus scenarios where nodes accumulate in areas of interest (e.g., library, cafeteria, classrooms) with less connectivity available between these areas [12], and vehicular networks where nodes tend to gather at specific locations, e.g., due to decelerating or stopping at junctions or traffic lights [10]. This creates concentrations of nodes (clusters) in specific network locations, thus yielding even more opportunities for multi-hop forwarding than the uniform node distribution model predicts.

C. Discovering Partial Paths

Given that partial paths exist, actually discovering and using them requires a routing algorithm. For source forwarding, we assume the use of some established routing protocol; however, in order to be able to use partial paths, intermediate forwarding requires a different routing approach. We believe that neither DTN nor MANET proposals can directly be applied for this task. We are currently working on a suitable algorithm and studying analytically its properties in terms of correctness, convergence, and performance. The key idea and main difference to existing protocols in the MANET area is to *not* report route failures; instead, information about broken routes is still propagated. This allows nodes to keep forwarding data as far as possible towards the destination along those segments of broken routes that are still intact. Upon reaching the point of failure, packets are stored until a new—partial or complete—path becomes available, allowing further progress.

The selection among partial paths could be based on metrics such as those studied in the context of DTN routing, e.g., age of last encounter [10] or frequency of past encounters [13]. But in contrast to DTN routing, these metrics would not only be used to discriminate among neighbors but to select among *multi-hop* path segments.

III. MODELING SOURCE VS. INTERMEDIATE FORWARDING

We present our modeling assumptions for the network and the two forwarding mechanisms under investigation; they are common to all analytical arguments made in the remainder of this paper. As outlined in the previous section, we consider a scenario with a set of mobile nodes where partial paths exist and can be discovered by a routing algorithm.

A. Network Model

In our model, time is slotted and we trace packets being transmitted from a source to a destination node. The random variable (r.v.) $H(t)$ describes the hop count along the active or *primary* route from the node holding the packet to the destination in time slot t . To refer to nodes that are *according to the routing algorithm* on the primary route, we use the term *position* h , meaning that this node has hop distance h to

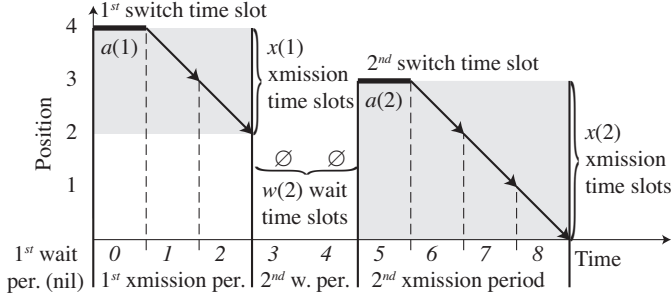


Fig. 2: A packet begins $a(1)$ hops from the destination, but the route breaks after $x(2)$ transmissions; after $w(2)$ time slots, an alternative route of length $a(2)$ hops is found

the destination; the destination being at position 0. Over time, the primary route may break and an *alternate route* needs to be determined, which will then become the new primary route. These “lifecycles” of the route between source and destination node are numbered consecutively by $l = 1, 2, \dots$. The time required for a path to become available and the routing algorithm to establish a route is called *waiting period* and represented by r.v. $W(l)$ taking values $w(l) = 0, 1, \dots$. The subsequent *transmission period* begins with a *switch time slot*, in which the packet switches to position $a(l) = 1, 2, \dots$ corresponding to the length of the route. The switch time slot is followed by a sequence of $x(l) = 0, 1, \dots, a(l)$ *transmission* (“*xmission*”) *time slots* during which the packet is transmitted towards the destination, as depicted in Fig. 2. Hence, in every transmission period l , the packet is transmitted from position $a(l)$ to positions $a(l) - 1, a(l) - 2, \dots, a(l) - x(l)$ and reaches the destination (position 0) in the earliest transmission period where $a(l) - x(l) = 0$. Every packet begins at initial hop distance $a(1)$; the initial wait time is $w(1) = 0$.

Regarding the distributions of these r.v.s, we assume that the link at the current position of a packet is up with probability p or down with probability $1 - p$, hence the distribution of r.v. X is geometric. For the waiting time W and length of the alternate route A , we will assume that they depend only on the length of the primary route, i.e., the position of the node requesting the alternate route. As will be described next, which node requests the alternate route is where the source and intermediate forwarding mechanisms differ.

B. Source and Intermediate Forwarding

Consider a packet that begins transmission period l at some position $a(l) > 0$ (the *source node*), and is then transmitted across $x(l) > 0$ hops until it gets stuck at an *intermediate node* at position $a(l) - x(l) > 0$. Now, under *source forwarding* (SrcFwd), the intermediate node *discards* the packet and the source node requests an alternative route to the destination; hence the distribution of $W(l + 1)$ and $A(l + 1)$ is conditioned on $a(l)$. In contrast, under *intermediate forwarding* (IntFwd), the intermediate node *stores* the packet and requests an alternative *partial route* from itself to the destination; thus the distribution of $W(l + 1)$ and $A(l + 1)$ is conditioned

on $a(l) - x(l)$. This is the only difference between the two forwarding mechanisms, hence the relationship between the length of the primary and the alternate route determines the advantage of intermediate forwarding.

IV. INTFWD *Not Necessarily* SUPERIOR

In our model, source and intermediate forwarding differ as to which node continues forwarding a packet when the primary route fails; in particular, the length of the alternate route (hop count) depends on the length of the primary route from the requesting node to the destination. In this section, we use r.v. H to describe the length of the primary route, A for the alternate route, and A_h to describe the length of the alternate route conditioned on the primary route having length h . We are interested in a sufficient condition on this relationship to guarantee that intermediate is faster than source forwarding. Such a condition must guarantee that the time to forward a packet to the destination along a route of given length h , denoted by T_h , is monotonically increasing in h , i.e., $T_k \geq T_j$ holds for every $k > j$.

Contrary to intuition, even if the length of the alternate route is tightly related to that of the primary route, monotonicity does not necessarily hold. In particular, we show that neither the positive correlation between alternate and primary route length (Thm. 1) nor the monotonic increase of the *expected* alternate route length in the primary route length (Thm. 2), are sufficient conditions.

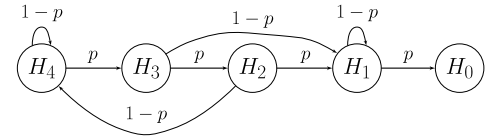


Fig. 3: Counter example for positive correlation between alternate and primary route length

Theorem 1. *Assume that the length of the alternate route is positively correlated with the length of the primary route, i.e., the correlation coefficient of r.v.s H and A , $\rho_{HA} > 0$. Then, $T_k \geq T_j$ does not necessarily hold for every $k > j$.*

Proof: By counter example. Let the primary route length take values 1 to 4 with equal probability. Assume that the corresponding alternate route lengths A_h are related to the primary route lengths through the following conditional distribution: $A_1 = 1, A_2 = 4, A_3 = 1$, and $A_4 = 4$, all with probability one (cf. Fig. 3).

Observe that alternate and primary route length are positively correlated with correlation coefficient $\rho_{HA} = 1/\sqrt{5} > 0$. To derive this, we used $\rho_{HA} = \sigma_{HA}/(\sigma_H \sigma_A)$, where $\sigma_{HA} = \sum_k \sum_l ((h_k - \mu_H)(a_l - \mu_A) \Pr[H = h_k, A = a_l])$.

Despite the positive correlation, the alternate route of H_2 is four hops, whereas the one of H_3 is only one hop. The expected values of the time to destination T_k coincide with the mean times to absorption starting from position k , when viewing the node chain as an absorbing Markov chain with

a single absorbing state (destination node); cf. [14] for the complete derivation. Using this, we can relate T_3 and T_2 with the following equation:

$$T_2 - T_3 = \frac{1 + 2p}{p^2} - \frac{2p^2 + 2}{p^3 - p^2 + p}.$$

This implies that $T_3 < T_2$ for all values of $p \in (0, (\sqrt{5}-1)/2)$; in other words, T_h is not monotonically increasing in h . ■

Next we show that even the *expected* alternate route length being monotonically increasing in the primary route length does not guarantee that T_h is monotonically increasing in h .

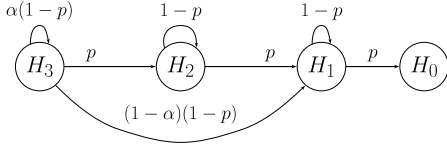


Fig. 4: Counter example for expected value of alternate route length monotonically increasing in primary route length

Theorem 2. Assume $E[A_k] \geq E[A_j]$ for every $k > j$. Then $T_k \geq T_j$ does not necessarily hold for every $k > j$.

Proof: By counter example. Let the primary route length take values 1, 2, or 3; the alternate route lengths are $A_1 = 1, A_2 = 2$, each with probability one, whereas $A_3 = 3$ with probability α and $A_3 = 1$ with probability $1 - \alpha$ (cf. Fig. 4).

Observe that the expected length of the alternate route is monotonically increasing in the length of the primary route as long as $1/2 < \alpha < 1$: $E[A_k] = 1, 2, 2\alpha + 1$, for $k = 1, 2, 3$, respectively. Yet a packet at H_3 may cut through to H_1 with non-negligible probability $(1 - \alpha)(1 - p)$. Invoking again the absorbing Markov chain argument, as in Thm. 2, it can be shown (cf. [14]) that:

$$T_2 - T_3 = \frac{2}{p} - \frac{3}{1 - \alpha + \alpha p}.$$

implying that $T_3 < T_2$ for all values of $p \in (0, (2-2\alpha)/(3-2\alpha))$. Consider, e.g., $\alpha = 5/8$ and $p = 1/3$, then $T_k = 5, 10, 6$ for $k = 1, 2, 3$, which is not monotonically increasing in k . ■

V. INTFWD SUPERIOR GIVEN STOCHASTIC MONOTONICITY OF ROUTE LENGTH

We now turn to a condition, *stochastic monotonicity*, which as we will show yields that the time to deliver a packet to the destination under source forwarding *stochastically dominates* that under intermediate forwarding. We first introduce the concept of stochastic monotonicity and derive a fundamental result on its transitivity; we then use this result to prove the above claim, first under the assumption of immediate availability of alternate routes, then under a relaxed assumption.

Definition 1 (*Stochastic Dominance — Stochastic Monotonicity*). An r.v. X stochastically dominates r.v. Y , written as $X \succ Y$, if $\Pr[X > t] \geq \Pr[Y > t] \forall t$, or equivalently

if $F_X(t) \leq F_Y(t) \forall t$.² Drawing on the stochastic dominance concept, we define *stochastic monotonicity*. An r.v. X is stochastically monotonic in r.v. A , written as $X \succ A$, if $\Pr[X \geq x|A = a_1] \geq \Pr[X \geq x|A = a_0] \forall a_1 > a_0$; or if $F_{X|A}(x|A = a_1) \leq F_{X|A}(x|A = a_0) \forall a_1 > a_0$.

Stochastic monotonicity is *transitive* under the conditions outlined in Lem. 1.

Lemma 1. Let X, Y, Z be r.v.s with strictly monotonic and continuously differentiable cumulative distribution functions (CDF). Then $Y \succ X$ implies $Z \succ X$ if the following holds:

$$F_{Z|Y,X}(z|Y = y_1, X = x_1) \leq F_{Z|Y,X}(z|Y = y_0, X = x_0) \forall y_1 > y_0, x_1 > x_0.$$

The proof is based on the law of total probability and provided in [14]. In the following, we are mainly interested in the corollary below, emerging from Lem. 1 as the special case where r.v. Z only depends on r.v. X via Y .

Corollary 1. Let X, Y, Z be r.v.s with strictly monotonic and continuously differentiable CDFs. If $Y \succ X$, and $Z \succ Y$, then $Z \succ X$ if $F_{Z|Y,X} = F_{Z|Y}$.

We now use these concepts to compare source and intermediate forwarding, based on the network model and notation from Sec. III and under the following assumption.

Assumption 1 (Alternate route length). The distribution of the alternate route length A is time-invariant and is stochastically monotonic in the primary route length H , i.e., $A \succ H$ or equivalently if F_A denotes the CDF of A , then $F_{A|H}(a|H = k) \leq F_{A|H}(a|H = j)$ for every $k > j$.

Evidence that this assumption holds in a sizable subset of networks is provided in Sec. VI.

A. Immediate Route Discovery

In this part, we assume that alternate routes can be determined immediately, corresponding to Area 1 (Sec. II); this assumption is relaxed in Sec. V-B.

Assumption 2 (Immediate route discovery). The alternate route is available immediately, allowing the packet to continue along the alternate route in the subsequent time slot.

We first show that the position of a packet forwarded under *SrcFwd* stochastically dominates that of a packet under *IntFwd* (Thm. 3) if they start from the same initial position; this will imply that the same relationship holds for the time to deliver a packet to the destination (Thm. 4). To this end, we will compare the positions of packet S forwarded by *SrcFwd* against packet I forwarded by *IntFwd* over the course of time; when referring to one of them, we will use corresponding superscripts S and I as appropriate.

For the purposes of the proof, we will track the two packets given the condition that the links at their positions are always

²This is also referred to as *First-order Stochastic Dominance*, e.g., [15].

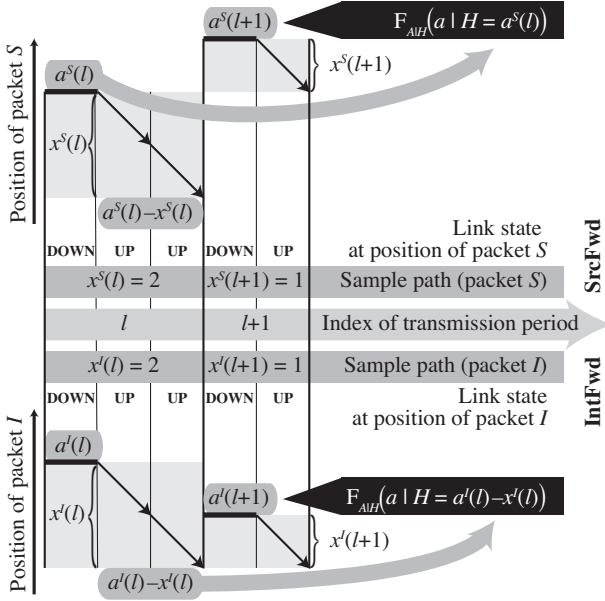


Fig. 5: Comparing the positions of packets S and I during exemplary transmission periods l and $l+1$ given they trace identical link state sample paths

in the same state, i.e., up or down. This condition means that r.v.s $X^S(l), X^I(l)$ yield *identical outcomes* (“samples”) $x^S(l) = x^I(l)$ for every transmission period l ; we refer to such a sequence of outcomes as a *sample path* \mathbf{L} . As illustrated in Fig. 5, in light of Assm. 2 the waiting periods shown in Fig. 2 are nil and transmission periods back-to-back. A sample path describing the link states of τ time slots comprises outcomes $x(1), x(2), \dots, x(\alpha)$, where α is the number of switch time slots (i.e., link down); similarly $\xi := \tau - \alpha$ denotes the number of transmission time slots (link up).

For a finite number of time slots τ , the number of possible link state sample paths is given by 2^τ and can be enumerated; we use subscript k to refer to individual sample paths $\mathbf{L}_k, k = 1, 2, \dots, 2^\tau$ and corresponding quantities α_k, ξ_k . We say that a packet *traces* a certain sample path \mathbf{L}_k if the link at the packet’s position is always in the state described by the sample path. The probability of this event is given by $\Pr[\mathbf{L}] = p^\xi \cdot (1-p)^\alpha$, as all links are independent and up with probability p (cf. Sec. III-A). In the following theorem, we will *match corresponding sample paths* $\mathbf{L}_k^S = \mathbf{L}_k^I = \mathbf{L}_k$ and show that the claim holds under this condition; at the end we will uncondition this result using the law of total probability [16].

Theorem 3. *Let packet S be forwarded by SrcFwd and packet I by IntFwd. If the positions of packets S and I at time 0 are given by $a(1) = 1, 2, \dots$, then their future positions obey*

$$H^S(\tau) \succ H^I(\tau) \quad \forall \tau \geq 1. \quad (1)$$

Proof: Let both packets S and I trace identical sample paths, i.e., $\mathbf{L}_k^S = \mathbf{L}_k^I = \mathbf{L}_k$; given this condition, denote their position at time t by $H_k^S(t), H_k^I(t)$ and their position

in the l^{th} switch time slot by $A_k^S(l), A_k^I(l)$, respectively. We use $\mathbf{L}_k[u]$ to denote a sample path prefix comprising only the first $u \leq \alpha_k$ elements $x_k(1), x_k(2), \dots, x_k(u)$ of \mathbf{L}_k . We next use induction on the length u of identical sample path prefixes $\mathbf{L}_k^S[u] = \mathbf{L}_k^I[u] = \mathbf{L}_k[u]$ to show that the following holds for every length $u = 1, 2, \dots, \alpha_k$:

$$A_k^S(u) \succ A_k^I(u). \quad (2)$$

Inductive basis. For $u = 1$, we have $A_k^S(1) = A_k^I(1) = a(1)$. *Inductive step.* Assume (2) holds for u and prove for $u+1$. By Assm. 1, the alternative route length $A(u+1)$ is stochastically monotonic in some primary route length. For packet S , this means that $A_k^S(u+1) \succ A_k^S(u)$, $A_k^S(u)$ being the position the packet had in the previous switch time slot. For packet I , it means that $A_k^I(u+1) \succ (A_k^I(u) - X_k^I(u))$, $A_k^I(u) - X_k^I(u)$ being the position the packet reached at the end of the previous transmission period. Hence, the inductive assumption $A_k^S(u) \succ A_k^I(u)$ implies immediately $A_k^S(u) \succ (A_k^I(u) - X_k^I(u))$, i.e., stochastic dominance between the primary route lengths in which the alternate route lengths are stochastically monotonic; hence application of Cor. 1 yields that (2) also holds for $u+1$.

Still given that both packets trace identical sample paths, (2) valid for every $u = 1, 2, \dots, \alpha_k$ yields that (1) holds *for every switch time slot* of these sample paths. In addition, their sample paths being identical also means that both packets are transmitted simultaneously over the same number of hops $x_k^S(u) = x_k^I(u)$ after every switch time slot, hence (1) valid for switch time slots also yields that it holds *for every transmission time slot*.

Summarizing, given that the packets trace identical sample paths $\mathbf{L}_k^S = \mathbf{L}_k^I = \mathbf{L}_k$, their positions obey $H_k^S(t) \succ H_k^I(t)$, which can also be written as

$$F_{H_k^S}(h, t) \leq F_{H_k^I}(h, t). \quad (3)$$

Summation over all possible sample paths $\mathbf{L}_k, k = 1, 2, \dots, 2^\tau$ then yields:

$$F_{H^S}(h, t) = \sum_{k=1}^{2^\tau} \Pr[\mathbf{L}_k^S] F_{H_k^S}(h, t),$$

$$F_{H^I}(h, t) = \sum_{k=1}^{2^\tau} \Pr[\mathbf{L}_k^I] F_{H_k^I}(h, t).$$

Since the sample paths are identical, they are also equiprobable ($\Pr[\mathbf{L}_k^S] = \Pr[\mathbf{L}_k^I]$); this together with (3) implies $F_{H^S}(h, t) \leq F_{H^I}(h, t)$, which is equivalent to the claim. ■

In the following theorem, we show that the time to the destination of a packet forwarded by SrcFwd stochastically dominates that of a packet forwarded by IntFwd if they both start from the same position.

Theorem 4. *Let packet S be forwarded by SrcFwd and packet I by IntFwd. If their positions at time 0 are given by $a(1)$, then their times to the destination obey*

$$T^S(\tau) \succ T^I(\tau) \quad \forall \tau \geq 1.$$

Proof: Observe, first, that since 0 is the minimal outcome of r.v. $H(t)$, $\Pr[H(t) = 0] = \Pr[H(t) \leq 0]$. Second, the probability that a packet reaches the destination by some time t is the same as the probability that, at time t , the packet is at position $H(t) = 0$, i.e., $\Pr[T \leq t] = \Pr[H(t) = 0]$.

From Thm. 3, $H^S(\tau) \succ H^I(\tau)$ implies $\Pr[H^S(\tau) \leq 0] \leq \Pr[H^I(\tau) \leq 0]$, which by the first observation is equivalent to $\Pr[H^S(\tau) = 0] \leq \Pr[H^I(\tau) = 0]$. By the second observation, this implies $\Pr[T^S(\tau) \leq \tau] \leq \Pr[T^I(\tau) \leq \tau] \forall \tau \geq 1$, concluding the proof. ■

B. Accounting for Stochastic Route Discovery Latency

Lastly, we relax the assumption that the alternate route is always available immediately. The analysis carried out so far assumes that an alternate route is available immediately upon failure of the primary route. However, in many scenarios (e.g., Area 2 in Sec. II) the intermediate (or source node) will have to wait for some time until an alternate route becomes available. This time mainly depends on the mobility patterns of the nodes and the network density.

In the following, we generalize our analysis by assuming that the time to discover an alternate route is an r.v. W . We will make the following assumptions on W :

Assumption 3 (Waiting time). *The distribution of W is time-invariant and may depend only on the length of the primary route H , similarly to how the length of the alternate route A depends on H by Assm. 1. Specifically, W is stochastically monotonic in the length of the primary route, i.e., if F_W denotes the CDF of W , then $F_{W|H}(w|H = k) \leq F_{W|H}(w|H = j)$ for every $k > j$.*

The assumption of time-invariance holds when the mobility model is stationary and network density is time-invariant. Regarding stochastic monotonicity, we argue as follows: in the worst case, the intermediate hop will have to wait for a time longer than the *mixing time* [16] for the given mobility model and network. In this case the waiting time to find an alternate route for the intermediate and source nodes follows the same distribution; such equality is allowed under our assumption of stochastic monotonicity. Nevertheless, it can be shown that, for many mobility models with high mixing time (e.g., two-dimensional random walk) and sufficiently high node density, the waiting time till a new route is found from an intermediate node is actually smaller than for the source node.

Making this assumption instead of Assm. 2, we next claim that Thm. 4 still holds.

Theorem 5. *Under Assm. 3, Thm. 4 holds, namely*

$$T^S(\tau) \succ T^I(\tau) \quad \forall \tau \geq 1.$$

Proof: Due to space constraints, we only give a sketch of the proof, which is provided in [14]. We again let packets S and I trace pairs of corresponding sample paths and compare their times to the destination under this condition. The difference to the previous derivation is that the waiting time, previously zero, is now a random variable. By Assm. 3,

the waiting time following transmission period l , represented by r.v. $W(l+1)$, depends on previous packet positions in the same way as the alternate route length r.v. $A(l+1)$; namely for *SrcFwd* it depends on the position of the source, $A^S(l)$, and for *IntFwd* it depends on the position of the intermediate node, $A^I(l) - X^I(l)$. This means that if we know the exact sequence of positions a packet visits till it is delivered to the destination, we also know the outcomes $a(l)$ and $x(l)$ in every transmission sequence l , on which the subsequent waiting period's duration, $W(l+1)$, depends. Therefore, for this theorem we use sample paths that include outcomes $a(l)$ and $x(l)$ of every transmission period $l = 1, 2, \dots, \alpha$.

Let us ignore waiting periods for a moment. Using standard coupling techniques [17], one can match sample paths of packets S and I such that (i) they are equiprobable, (ii) $x^S(l) = x^I(l)$ and (iii) $a^S(l) \geq a^I(l)$ is valid in every transmission period. This can be achieved by defining all sample paths of packet I such that they end with the delivery of the packet, then match to each of them a corresponding sample path of packet S comprising the same number of transmission periods such that properties (i)–(iii) hold.

Now, again considering waiting periods, observe that (ii) together with (iii) implies directly $a^S(l) \geq a^I(l) - x^I(l)$. Since by Assm. 3, $W^S(l+1) \succ A^S(l)$ and $W^I(l+1) \succ A^I(l) - X^I(l)$, this inequality implies $W^S(l+1) \succ W^I(l+1)$. Stochastic dominance is preserved under summation, hence the total waiting time of every sample path pair also obeys this relationship. This means that in addition to the (equal) number of switch and transmission time slots, the time to the destination is increased by the sum of all waiting periods; therefore, the total sum of time slots (switch, transmission, and wait time slots) still obeys the stochastic dominance relationship. Lastly, every sample path of packet I ending with the delivery of the packet and none of the sample paths of packet S delivering the packet earlier (if at all), un-conditioning from the sample paths yields the claim. ■

VI. STOCHASTIC MONOTONICITY OF ROUTE LENGTH

Through the analysis in Sec. IV-V, we have demonstrated that the stochastic monotonicity of alternate path length (hop count) in the primary path length is sufficient for intermediate forwarding to outperform source forwarding. We now turn to the question of whether this stochastic monotonicity indeed holds in realistic wireless multi-hop networks. Whereas a rigorous proof along the lines of the analysis in the previous sections appears hard to devise, we draw on available analytical results in literature to argue that such monotonicity exists.

In [18], Ta et al. derive a recursive expression for the probability $\Phi_k(d)$ that the length of the shortest path between two nodes is k hops, given their Euclidean distance, d . They assume uniform node distribution and the geometric link model (cf. Sec. II). From the set $\Phi_k(d)$, $k = 1, 2, \dots$ we may derive the discrete conditional probability distribution function of the shortest path length between two nodes given d as

$$f_{H|D}(h|D = d) = \sum_k \Phi_k(d) \cdot \delta(h - k), h = 1, 2, \dots,$$

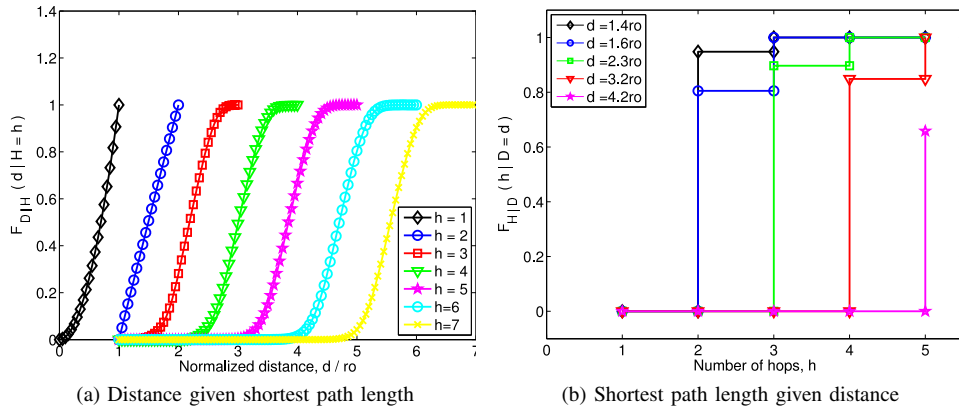


Fig. 6: Relationship between length of shortest path and Euclidean distance between end points; parameters: $r_0 = 2$, $\varrho = 1.25$, area: circle($R = 5 \cdot r_0$)

where $\delta(\cdot)$ is Dirac’s delta function. Applying Bayes’ theorem for probability densities (cf. [16]), the conditional probability density function $f_{D|H}(d|H = h)$ of the Euclidean distance between two points given the number of hops of the shortest connecting path can be written as

$$f_{D|H}(d|H = h) = \frac{\Phi_k(d) \cdot f_D(d)}{\int \Phi_k(d) \cdot f_D(d)}$$

where $f_D(d)$ corresponds to the unconditional probability distribution function of the distance between two random nodes over a given surface. This distribution is available in literature for multiple surfaces, including circles of radius R (“disk line picking” problem; e.g., [19]) and rectangles of sizes a and b (“rectangle line picking” problem; e.g., [20]).

In [14], we plot the respective probability and cumulative distribution functions for multiple combinations of the transmission range r_0 and network density ϱ parameters. We pick two samples of those plots, cf. Fig. 6a and Fig. 6b, to support our arguments.

One may directly observe that for all pairs of curves in the two plots, $F_{D|H}(d|H = h_1) \leq F_{D|H}(d|H = h_0) \forall h_1 > h_0$ and $F_{H|D}(h|D = d_1) \leq F_{H|D}(h|D = d_0) \forall d_1 > d_0$. In other words, the stochastic monotonicity property, as defined in Def. 1 pertains to both distributions, namely the distance is stochastically monotonic in the path length, $D \succcurlyeq H$, and vice-versa. The same observation holds for all plots in [14].

The distribution of the length of the alternate route, i.e., the route obtained by the intermediate node after the primary route to the destination breaks, will have a distribution $f_{A|D}(a|D = d)$, which, in the general case, is different from the one of the primary route. We do however conjecture the following:

Conjecture 1. *The conditional distribution of the alternate route length, A , given the Euclidean distance, D , preserves the stochastic monotonicity property, i.e., $A \succcurlyeq D$.*

Given this conjecture, application of Cor. 1 yields that the length of the alternate route A is stochastically monotonic in the length of the primary route H , i.e., $A \succcurlyeq H$.

VII. RELATED WORK

Route salvaging proposed in the context of MANETs exploits partial paths; failed routes are repaired locally at the point of failure if an alternate route is known (cached). Salvaging might be performed for few packets, as is the case with the Dynamic Source Routing protocol (DSR) [21], or for the whole data stream [4], and has been shown, mainly with simulation studies, to improve performance. A first step towards analytical comparison of end-to-end against local route recovery protocols was presented in [22], yet under a less generic set of assumptions. Finally, hop-by-hop transport for wireless networks has also been recently explored [2], [3]. Nevertheless, none of these studies *analytically* addresses the existence of partial paths or under which conditions they might help to improve performance over traditional end-to-end approaches. In fact, the implicit assumption in most of these studies has been that the network is almost surely connected.

It was only more recently that this assumption was relaxed in the context of DTNs [23]. However, the majority of DTN routing protocols rely on single-hop transfer opportunities (“contacts”) and node mobility to eventually deliver a message. In a few cases, hybrid approaches that combine elements from both DTN and MANET routing have been proposed. In [10] the authors look into a vehicular network where connected clusters are formed at various locations, with little or no connectivity between them. They propose regular routing to be used within these clusters, while DTN schemes can be used to move messages between clusters when the destination is not locally available. Similarly, in [11] AODV is used when end-to-end connectivity is available, while a node may fall back to DTN forwarding when an end-to-end path cannot be discovered. Finally, in [24] a Kalman filtering approach is taken to predict the delivery probability (“utility”) of each node in a DTN context, but regular routing is used to query and identify the highest utility node within the cluster. These approaches more formally target Area 3 of the connectivity continuum discussed in Sec. II. Further, we believe the analysis presented

in this paper could be applied with appropriate modifications to such hybrid solutions.

Along a slightly different line of research, there have been several theoretical studies revisiting the network connectivity dynamics [6], [9], [25], [26]. Whereas the phase transition phenomena for asymptotically large networks had been already studied in [27], [28], more recent studies have focused on more realistic network scenarios and propose metrics that can capture the smoother evolution of connectivity with network density for a finite number of nodes. The connectivity index (reliability) in [9] and the giant component in [26], where a lognormal radio model is superimposed on the geometric link model, are two examples of attempts to capture better the partially connected nature of finite-size networks and assess the extent to which forwarding opportunities are provided by them. The remarks in these studies, which are practically in line with our discussion in Sec. II, have been the main motivation for looking closer at the regime of partial connectivity in this paper. Finally, an interesting study and classification of the connectivity continuum with the goal of understanding what is most suitable for each network class is provided in [29].

VIII. CONCLUSION

In this paper, we have shown that a considerable regime between fully connected and fully disconnected networks can be defined, namely *partially-connected networks*, where partial paths of multiple hops often exist and could be taken advantage of to improve performance. DTN schemes usually ignore multi-hop paths altogether and rely on single-hop paths (“contacts”) and speculative forwarding. MANET schemes, on the other hand, require complete end-to-end paths to operate.

We have argued here that local recovery mechanisms (“intermediate forwarding”), which store data at the point of path breakage and locally try to discover an alternate partial route from that point to the destination, can outperform established end-to-end mechanisms (“source forwarding”) over a range of connectivity regimes. Specifically, we show analytically that *stochastic monotonicity* of the length of alternate paths given the length of the current path, is a sufficient condition for intermediate forwarding to *stochastically dominate* end-to-end forwarding. At the same time, we show that, contrary to intuition, weaker conditions relating alternate to the current path lengths are not sufficient to prove the desired result of superiority.

In future work, we are planning to explore the applicability of our analytical methodology in connectivity regimes where connectivity is close to zero (i.e. end-to-end paths never exist), but reachability is still considerable (Area 3 in Sec. II). Further, we are planning to look in more detail into the protocol-related issues of intermediate forwarding for partially-connected networks, building on the insight acquired by this analytical study. In particular, we are currently studying analytically a routing algorithm suitable to discover and maintain partial paths.

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