Deterministic Multi-Channel Information Exchange

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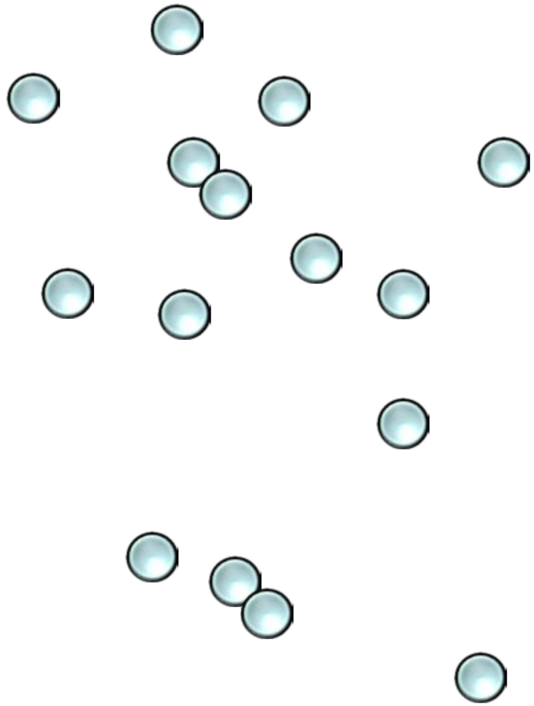
Roger Wattenhofer - ETH Zürich

Deterministic Multi-Channel Information Exchange

n:= # nodes

Problem:



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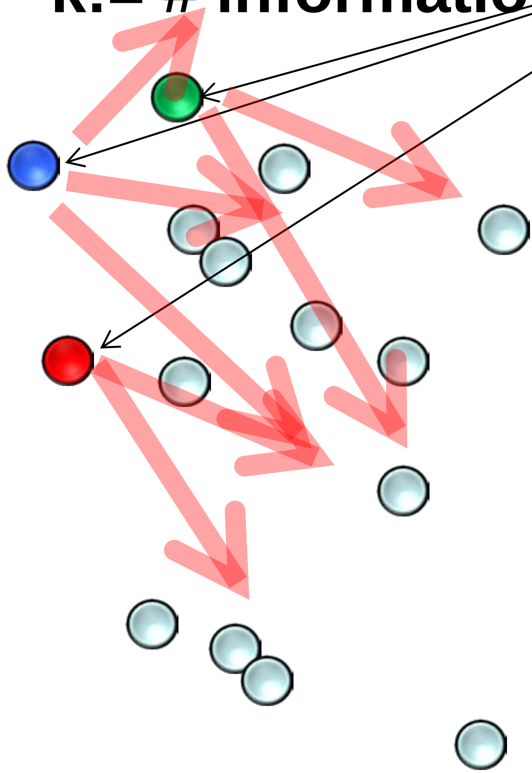
n:= # nodes

k:= # information

Have information



Disseminate to all!

?

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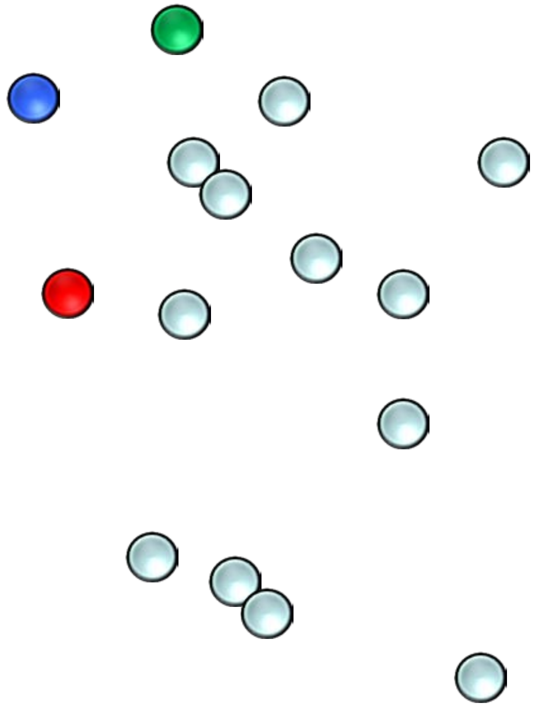

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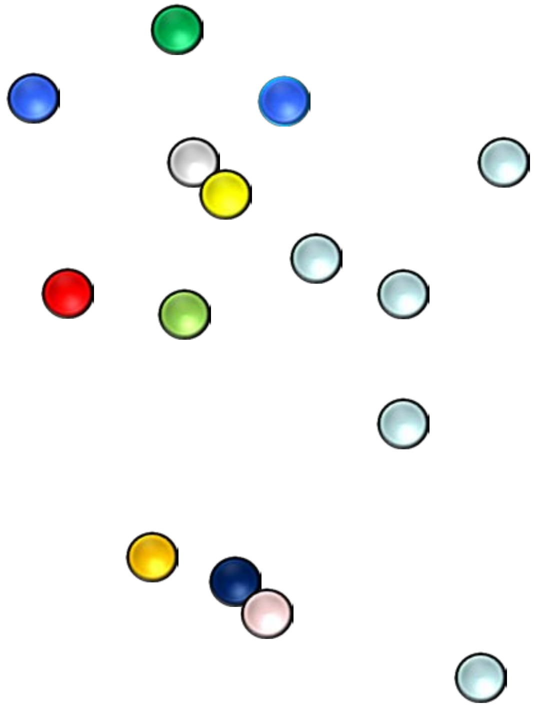

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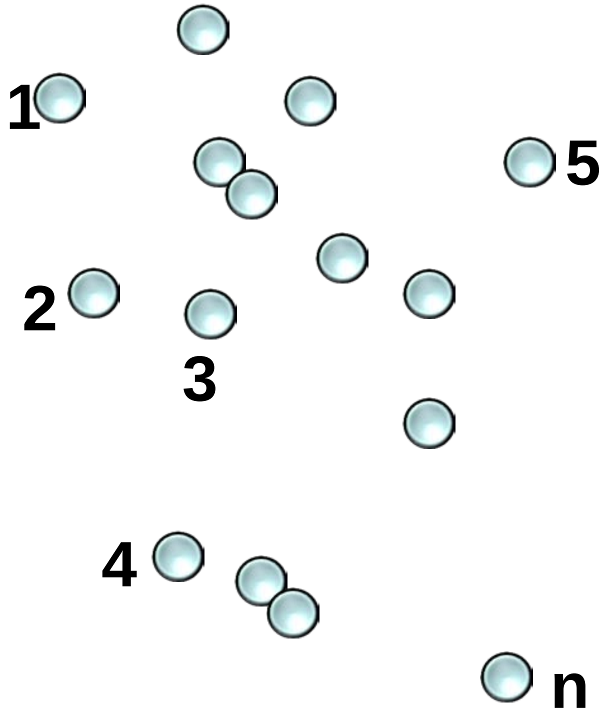

Unique IDs 1…n

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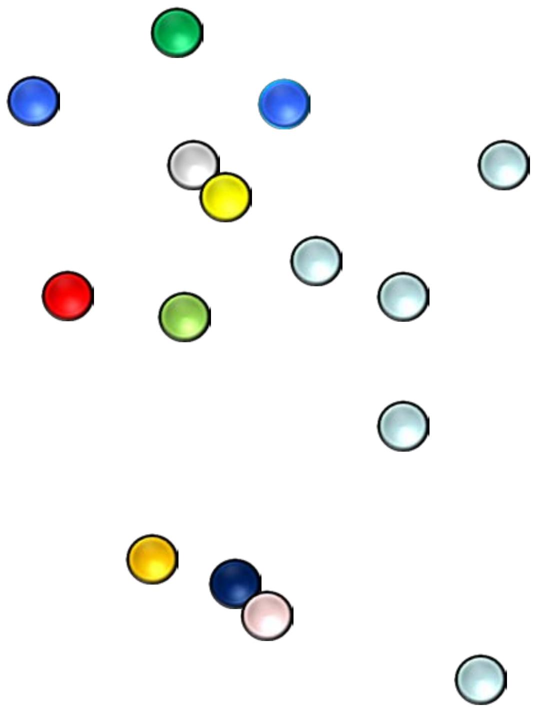

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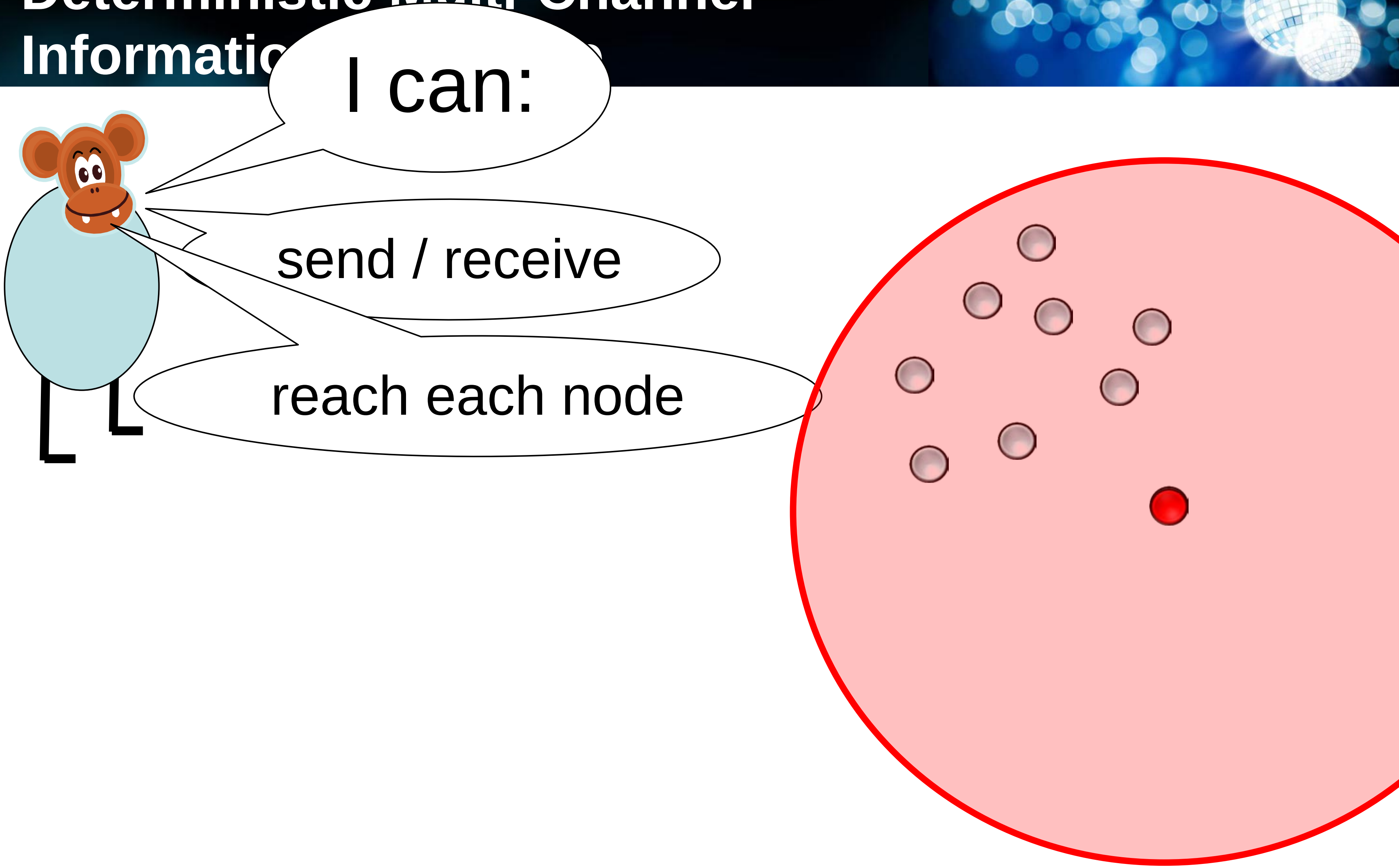
Easy: O(n)

Faster?

?

Deterministic Multi-Channel Informatio



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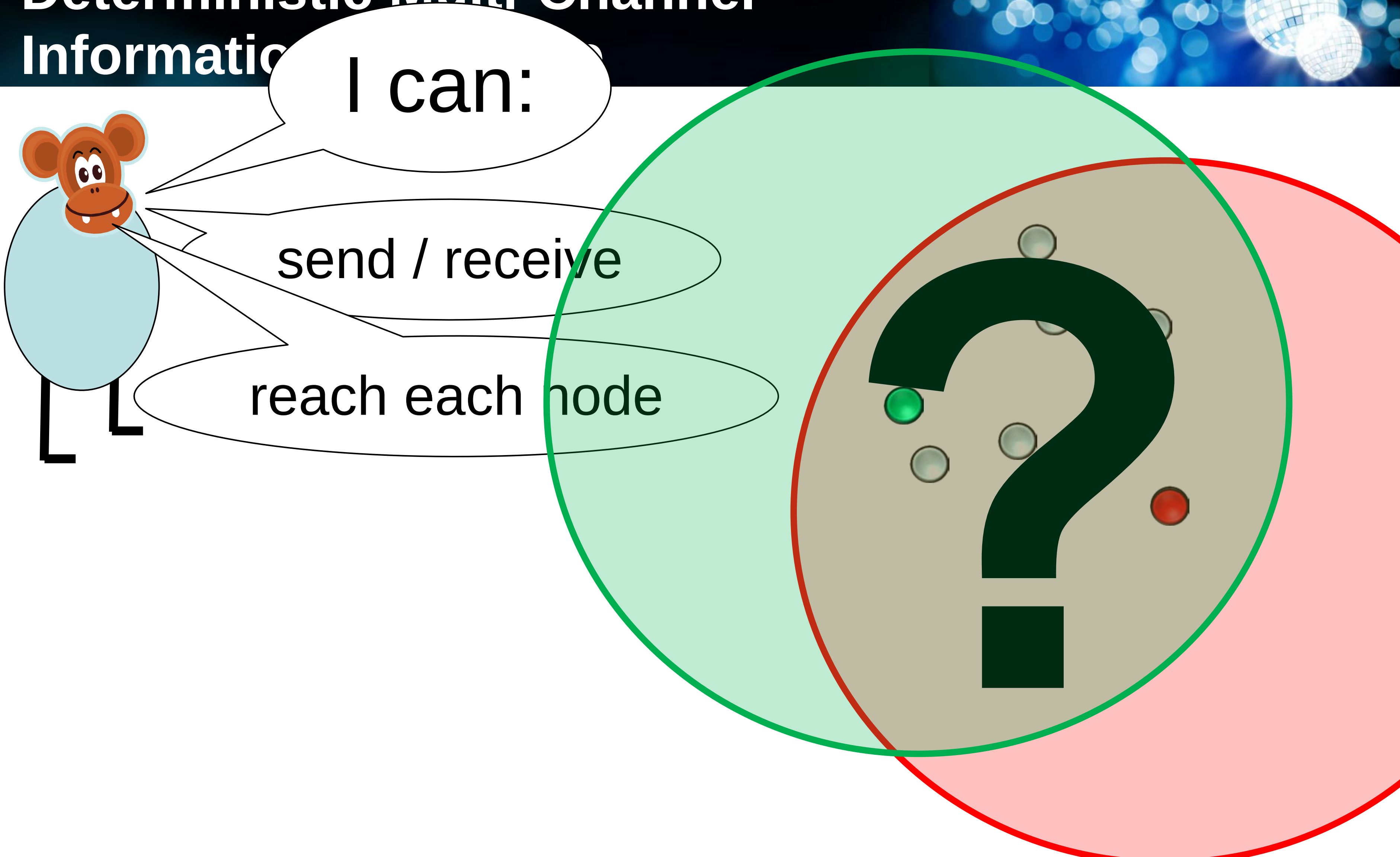

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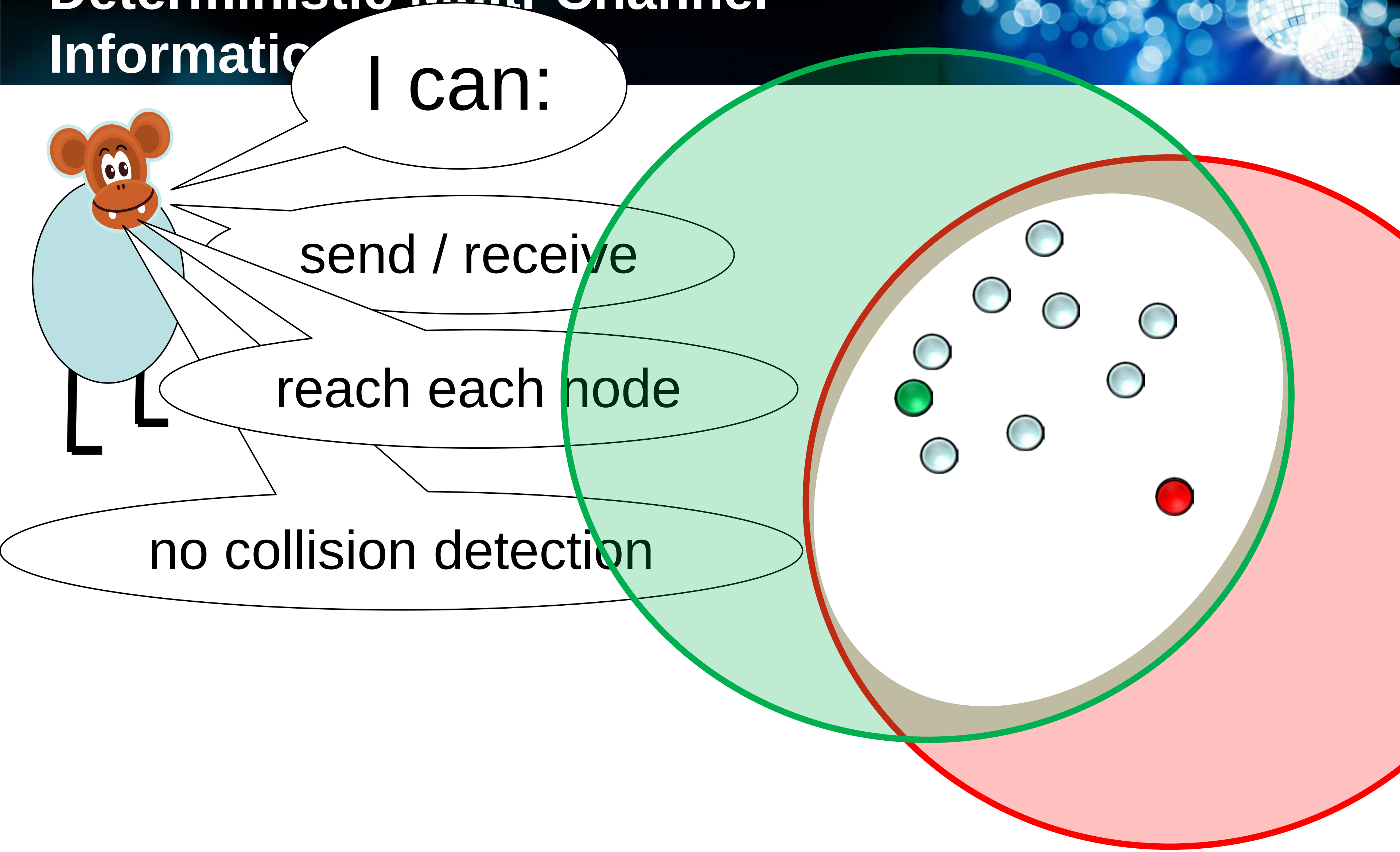
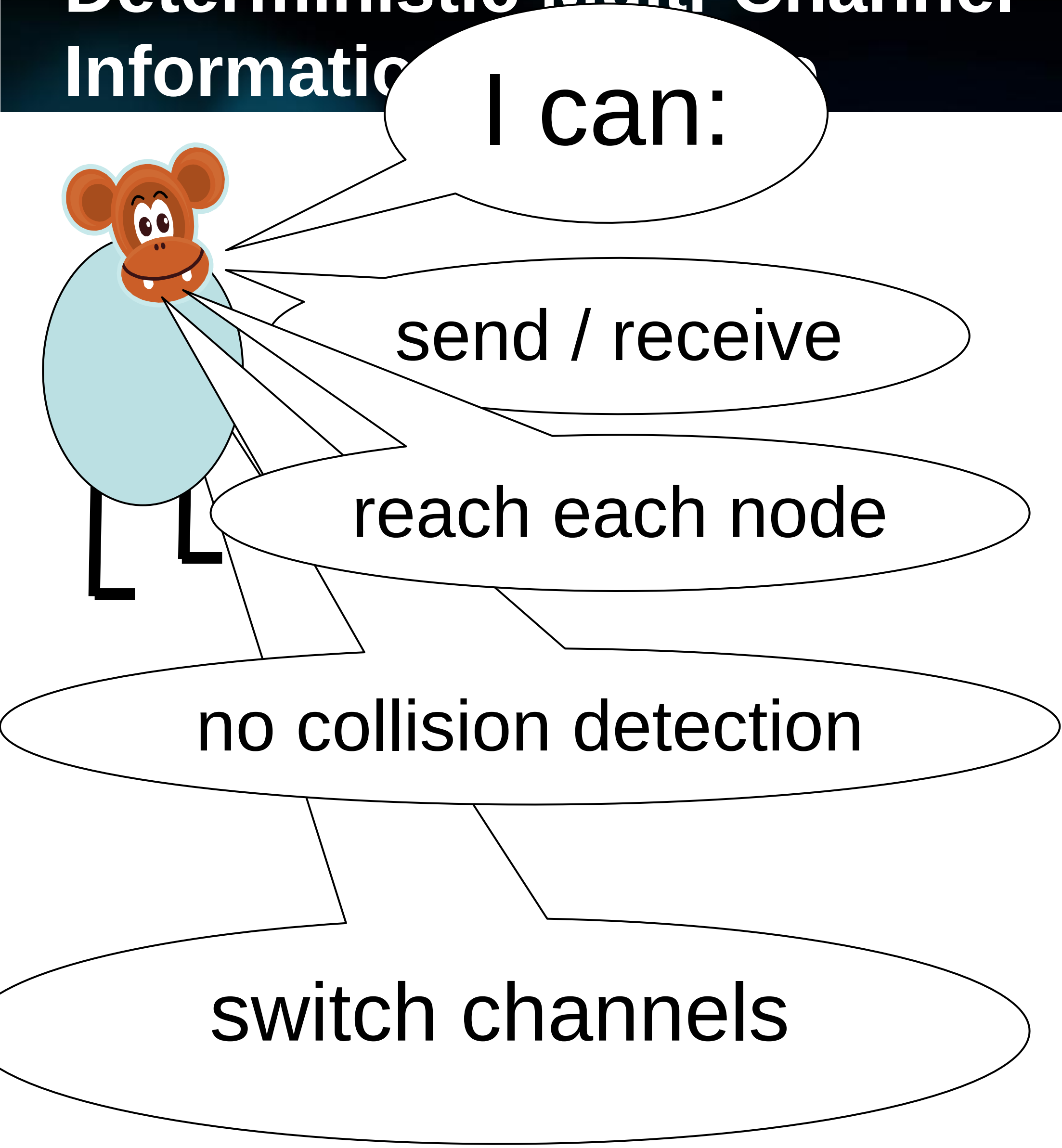
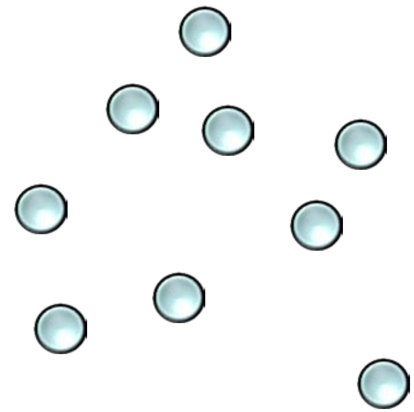


Deterministic Multi-Channel
Informatic

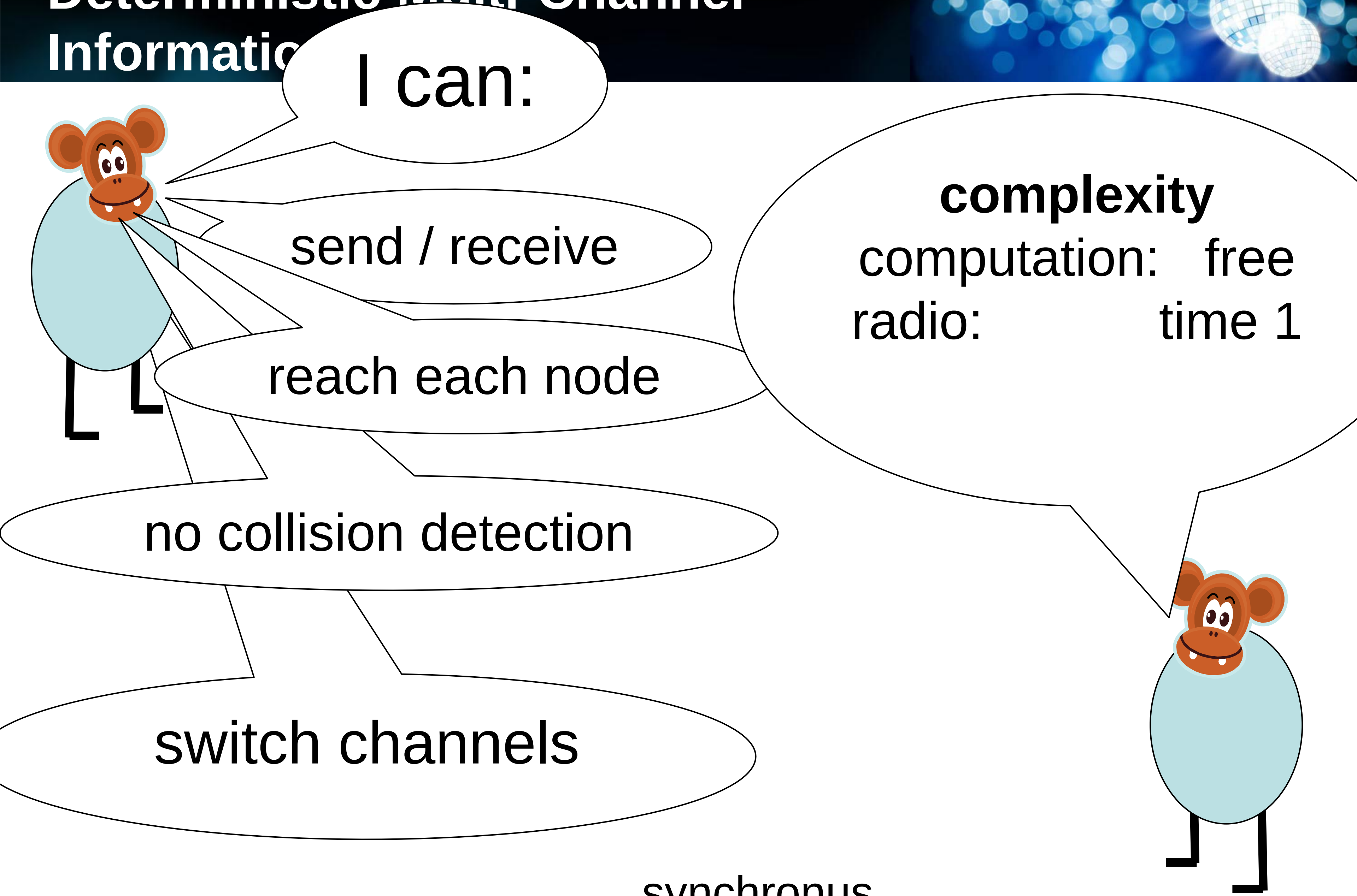




101 Mhz
117 Mhz
132 Mhz
...

synchronus

Deterministic Multi-Channel Informati…



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	Time	**Channels**
[GW85]:	$\Omega(k + \log_k n)$	1

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Optimal

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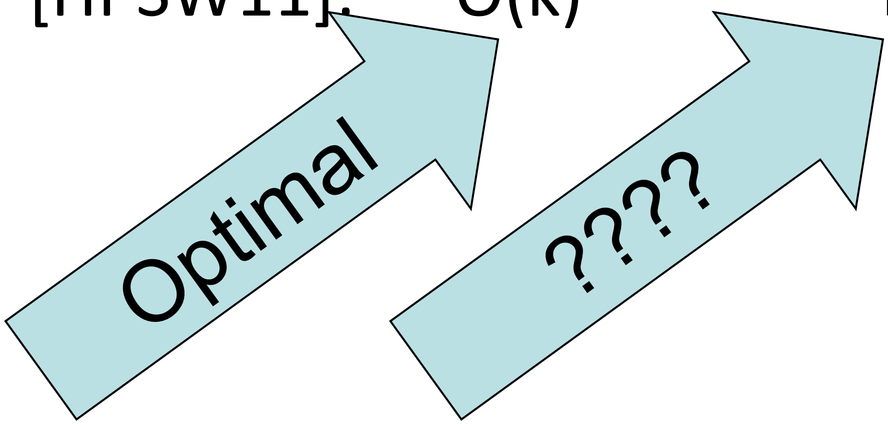

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Range of k	**[1, $\sqrt{\log n}$]**	**($\sqrt{\log n}$, log n)**	**[log n , n]**
Upper bound On channels	$O(n^{\frac{\log(k)}{k}})$	$O(2^k)$	1

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Lower bound On channels	$\Omega\left(n^{\frac{1}{k}}\right)$	$\Omega\left(\frac{\log n}{\log\log n}\right)$	$\Omega\left(\log_k(n)\right)$	1

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Main ingredient:

Specially taylored graphs.

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Topology: Still single hop.

Graphs used to select channel.

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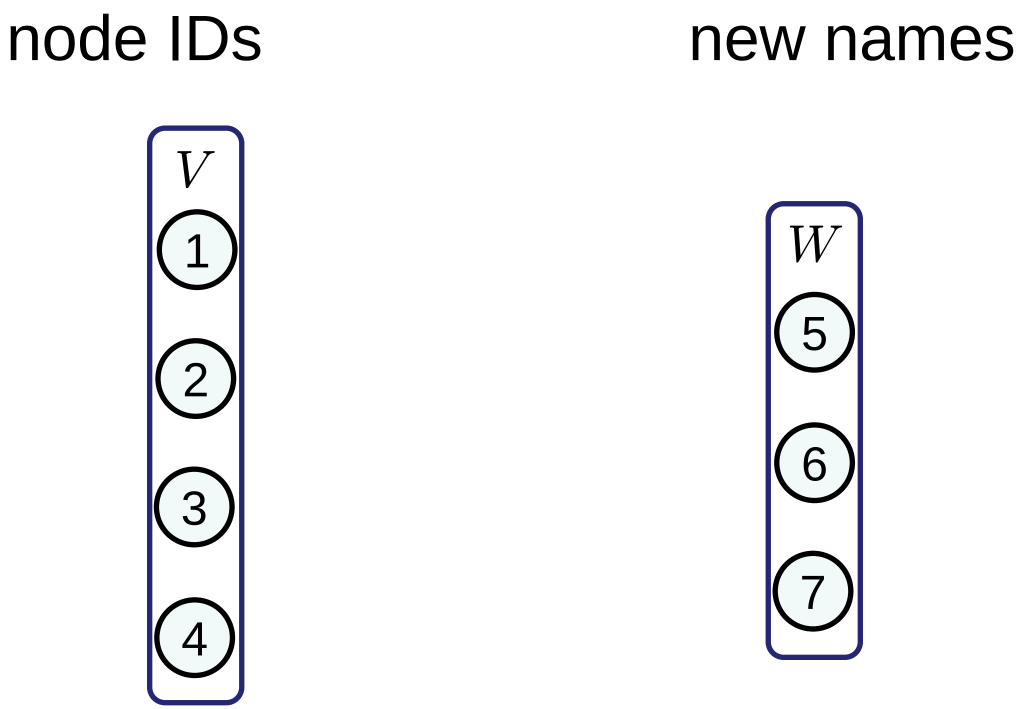
Bipartite :



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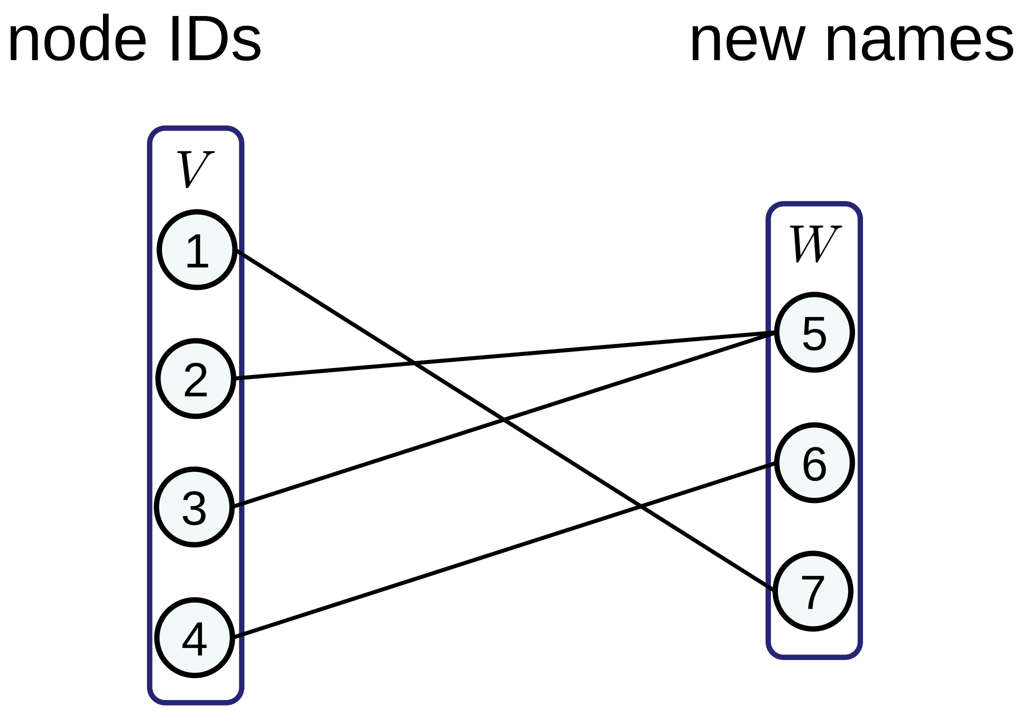
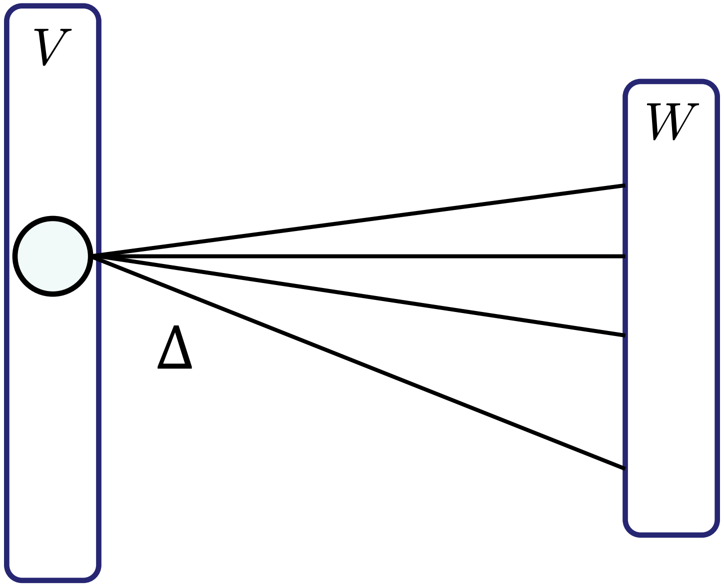


Deterministic Multi-Channel Information Exchange

Matching Graphs:

- Nodes in V have degee Δ



Deterministic Multi-Channel Information Exchange

Matching Graphs:

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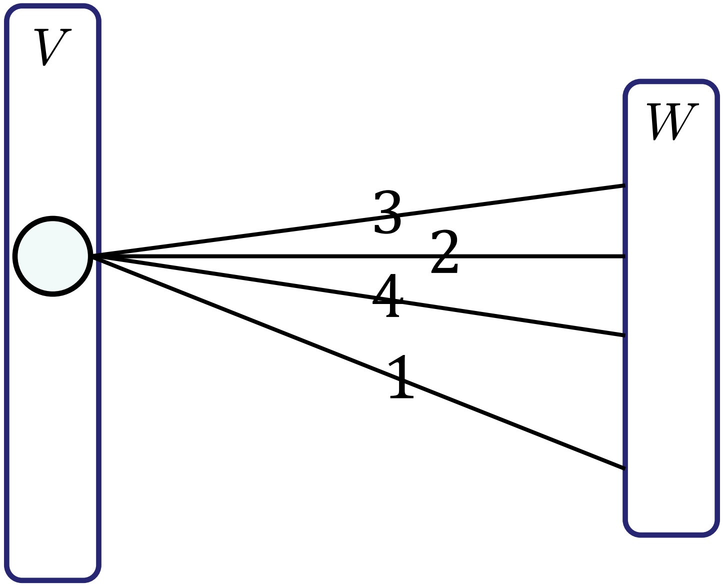

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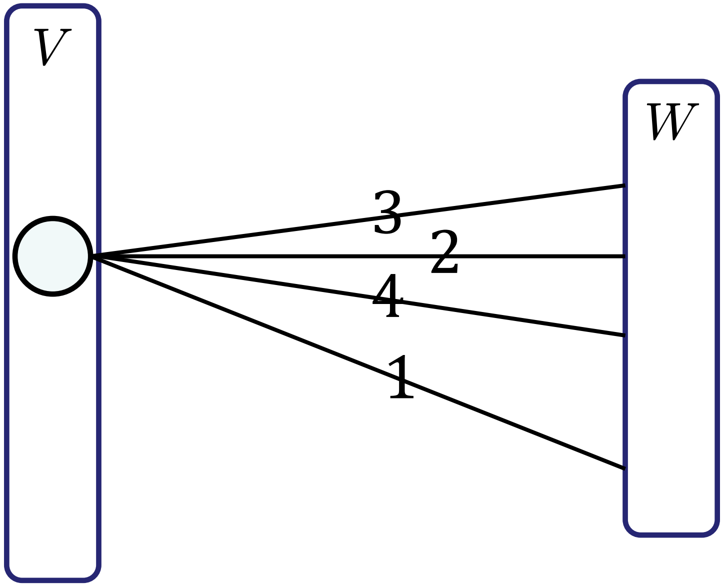

- For any $X \subseteq V$ of size at most k

  there is $i \in [1, \Delta]$

  at least $\varepsilon|X|$ nodes in X have a unique i-neighbor.

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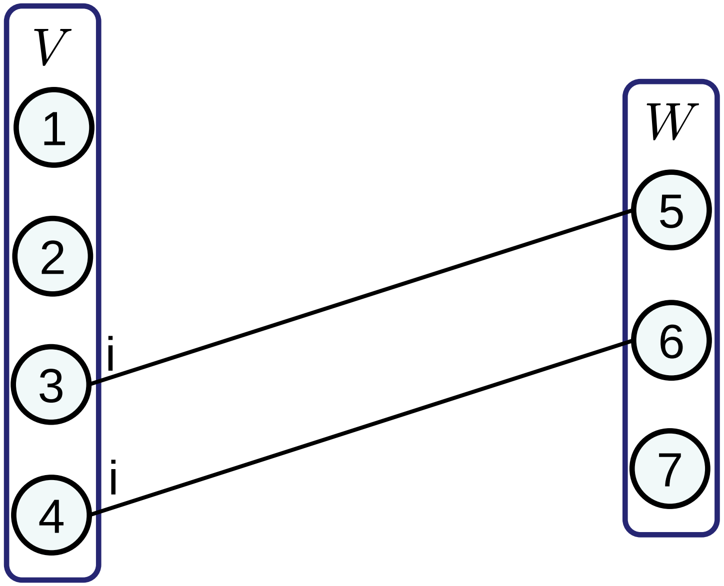

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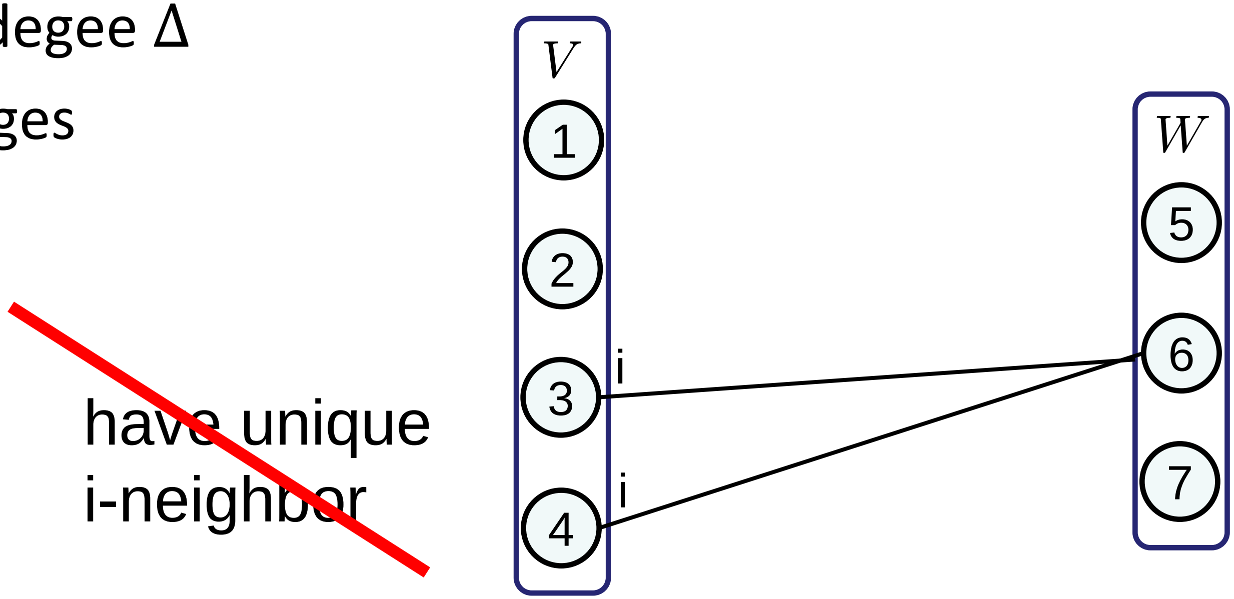

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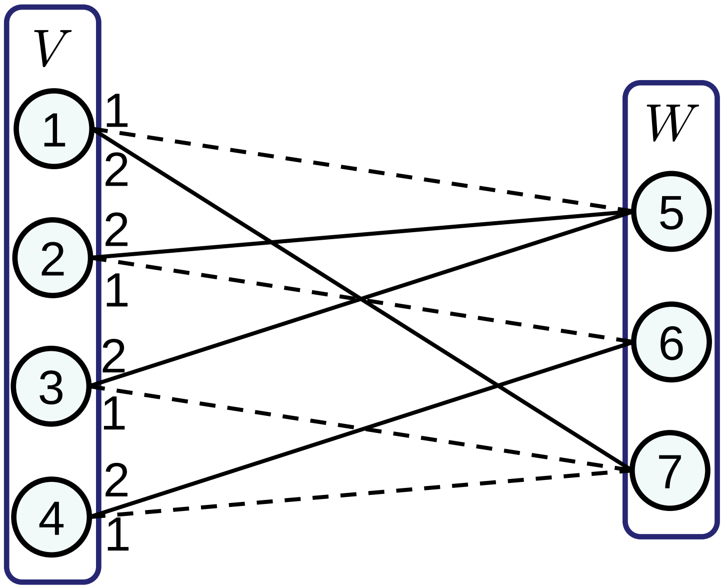

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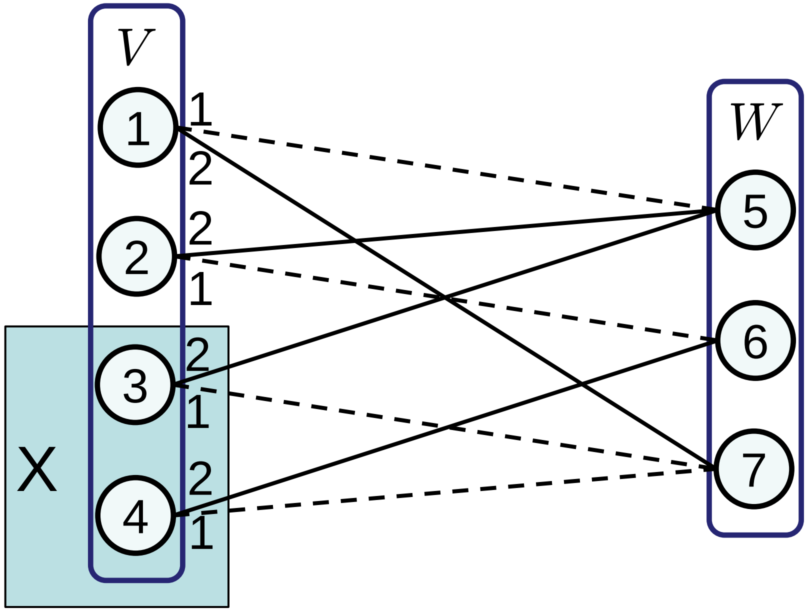

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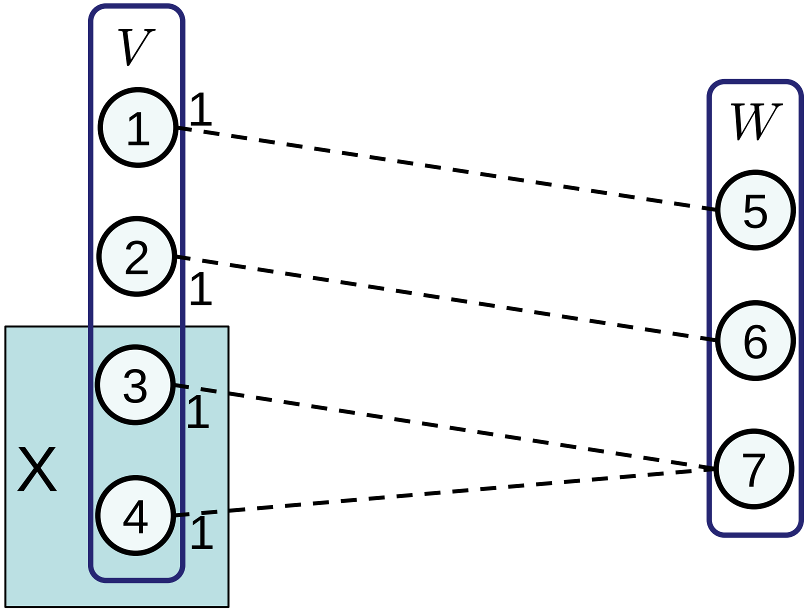

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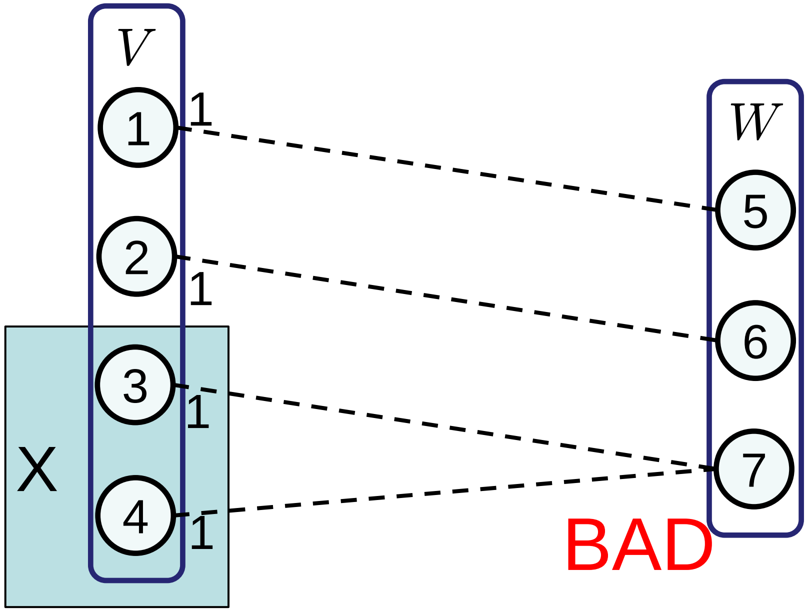

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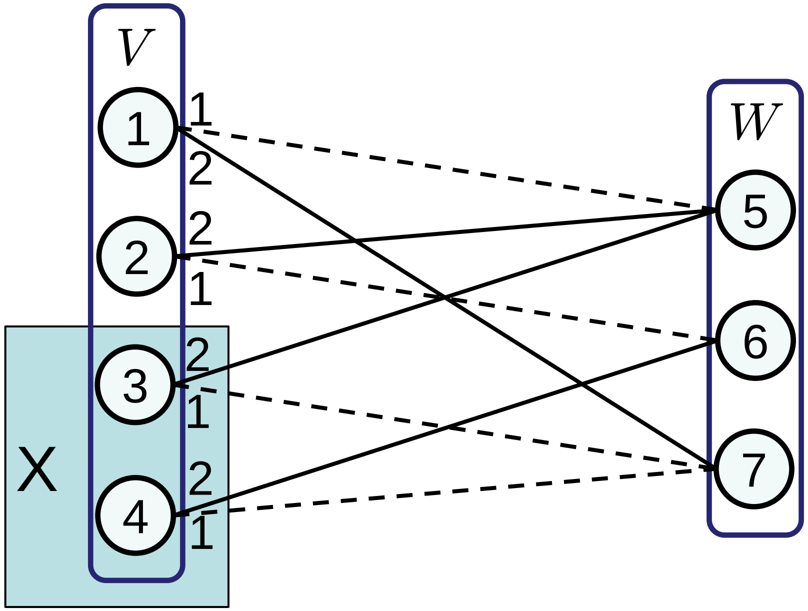

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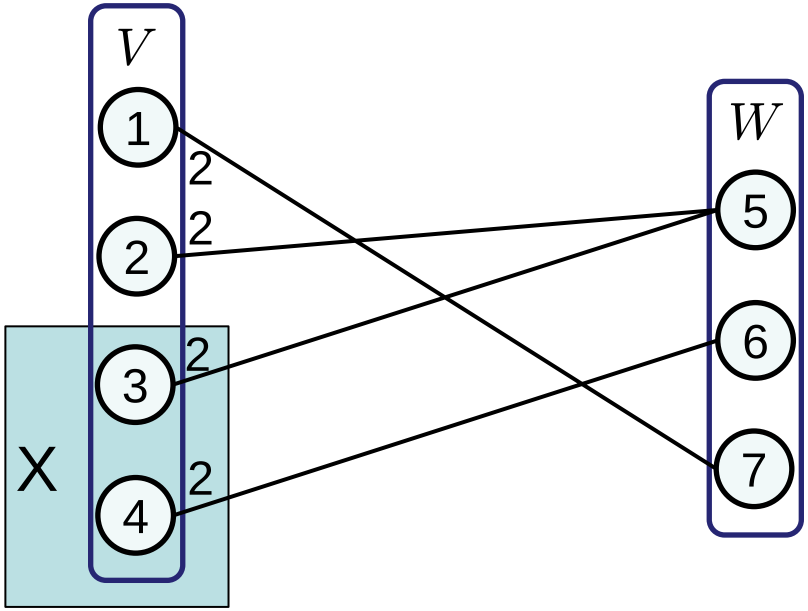

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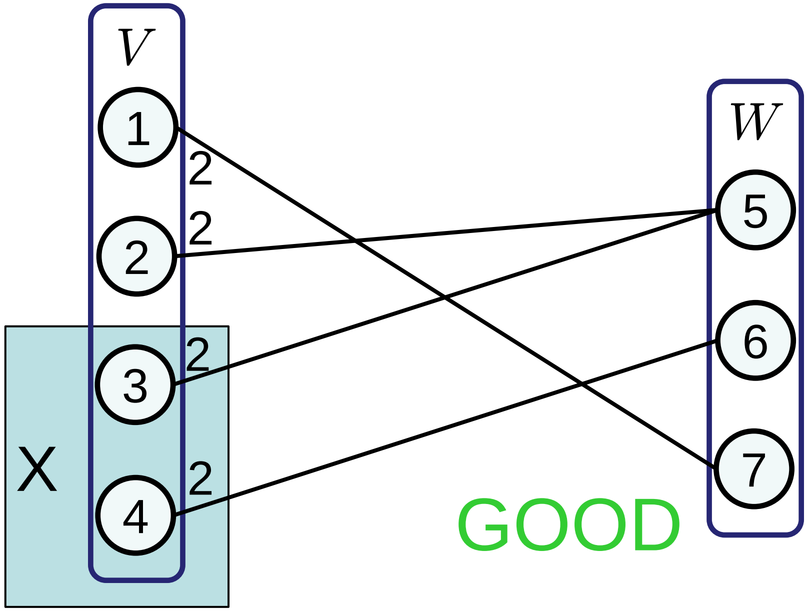

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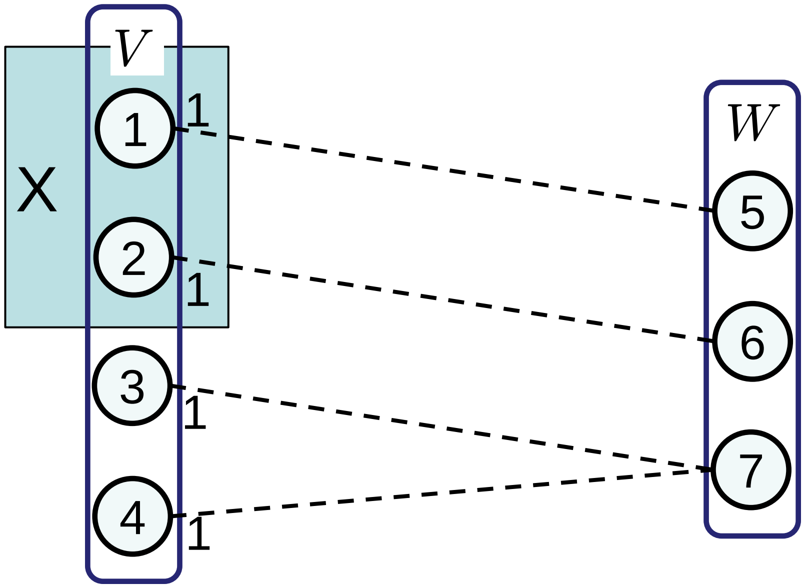

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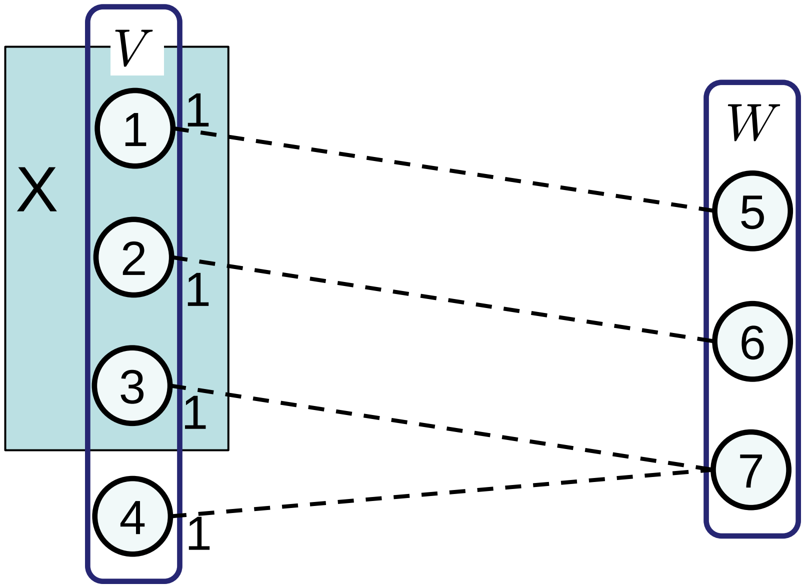

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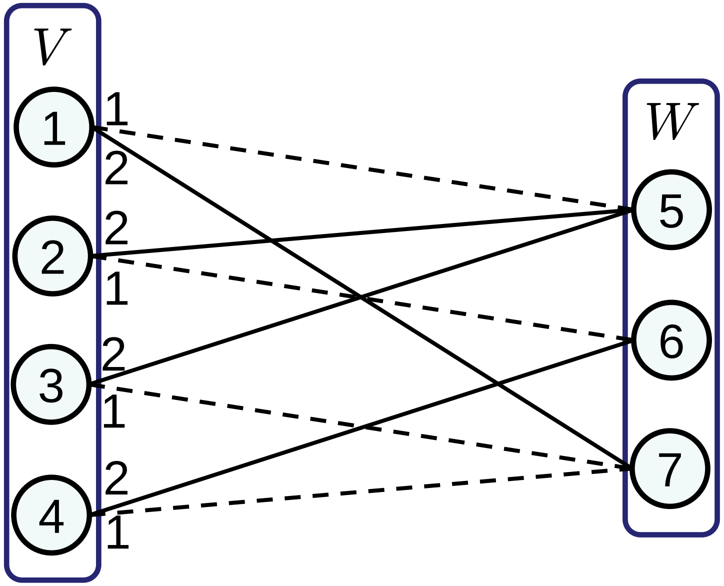

exist if

$$|W| \geq |V|^{\frac{1}{\Delta}} + K^{f(\varepsilon)}$$

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Renaming ☺

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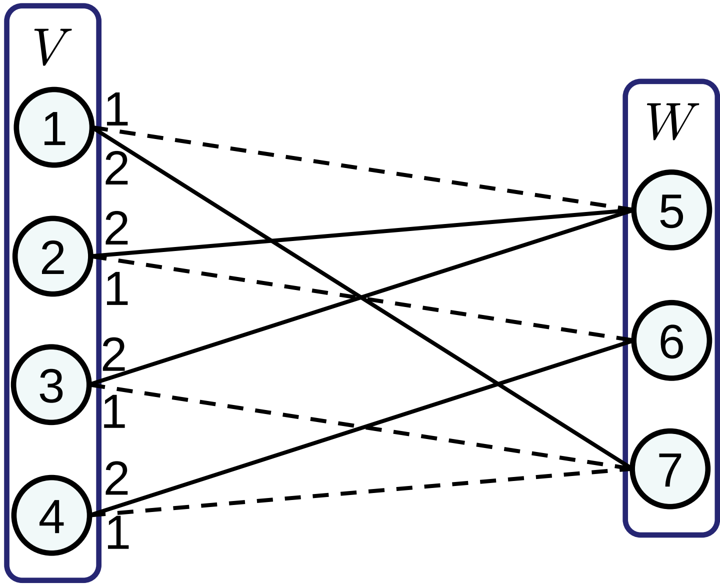

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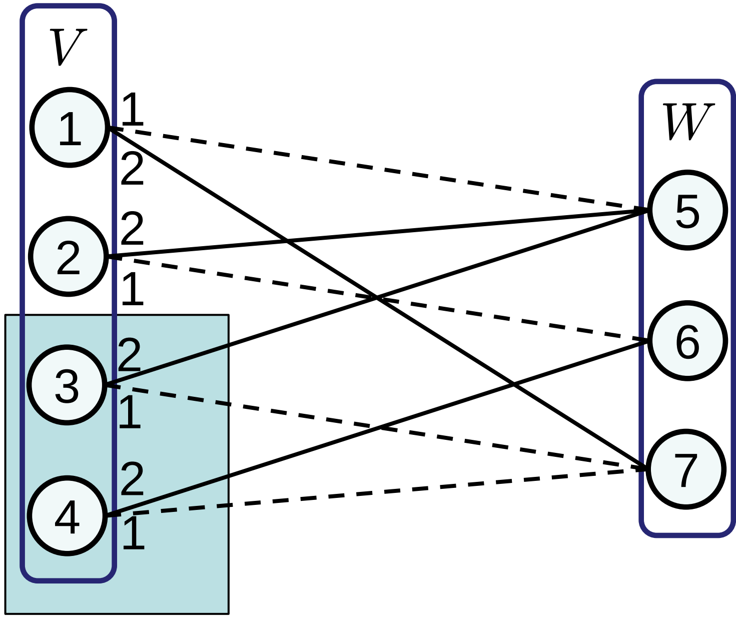

- To each of the k «reporters»

  we can assing a new unique name in |W|

  in time O($\Delta \log k + k$)

  using |W| channels.

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Deterministic Multi-Channel Information Exchange

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Assignment of reporters to channels!

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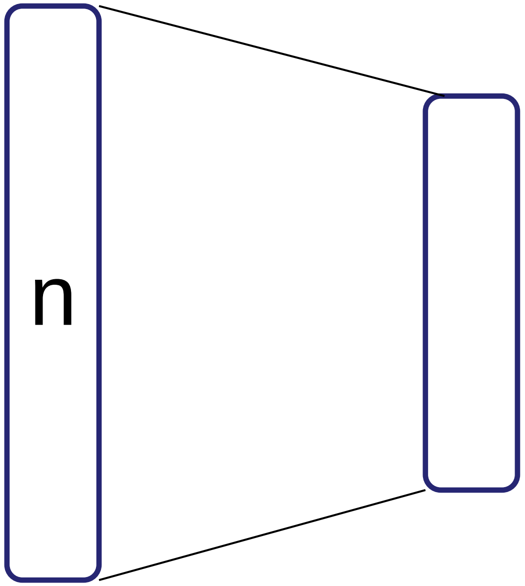
Original names



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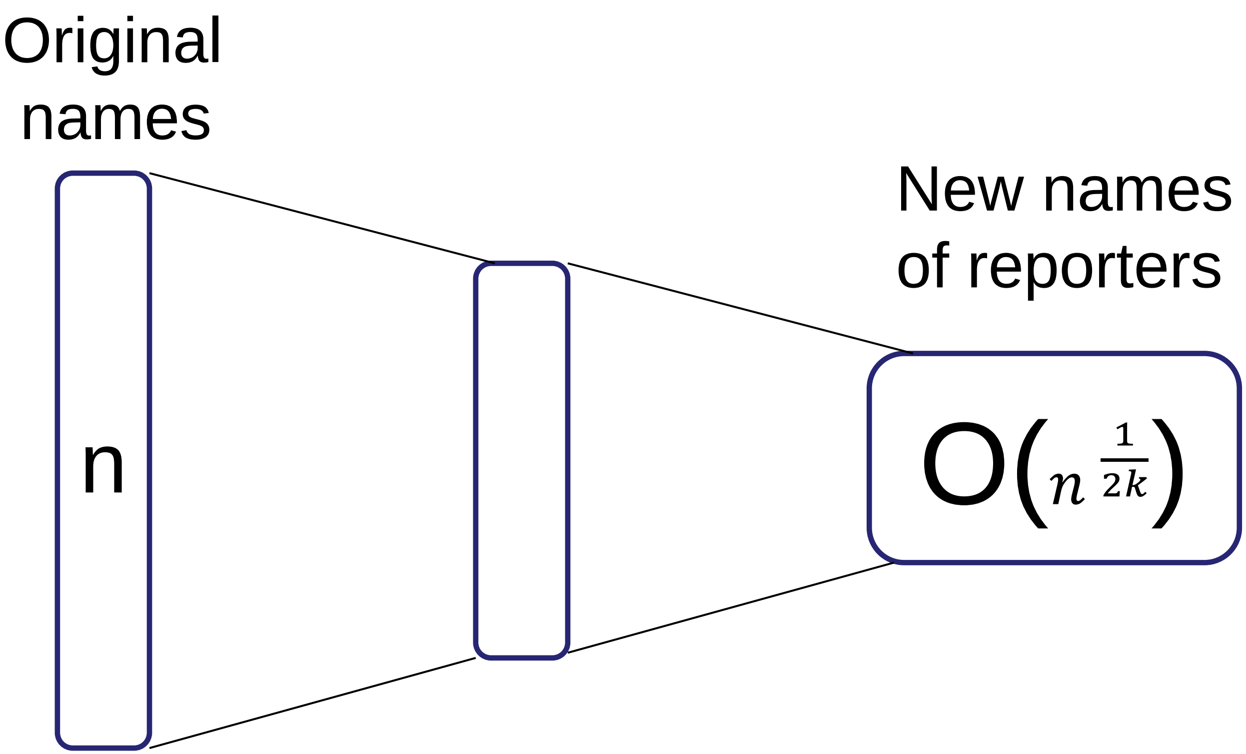

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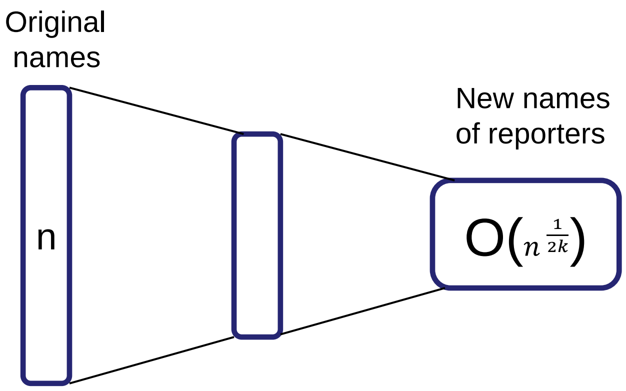

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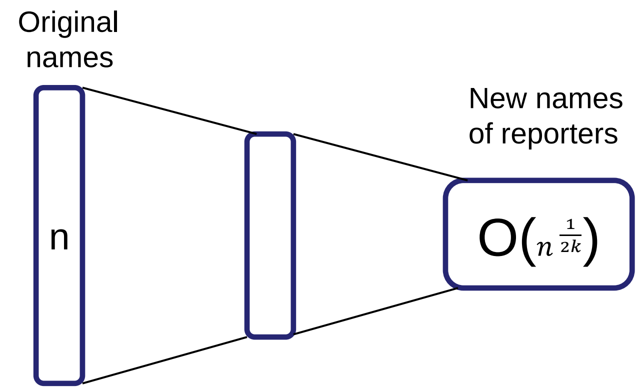

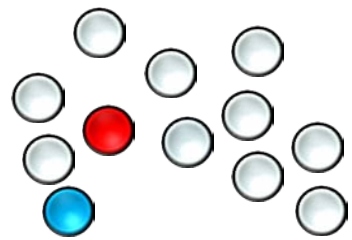

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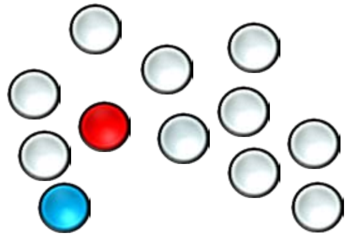
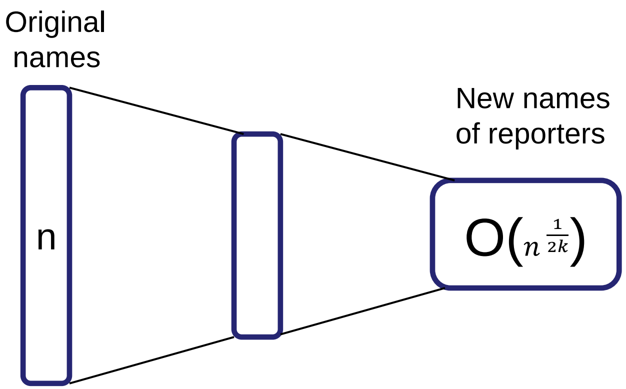
$Size: n/2$





Time: O(k)

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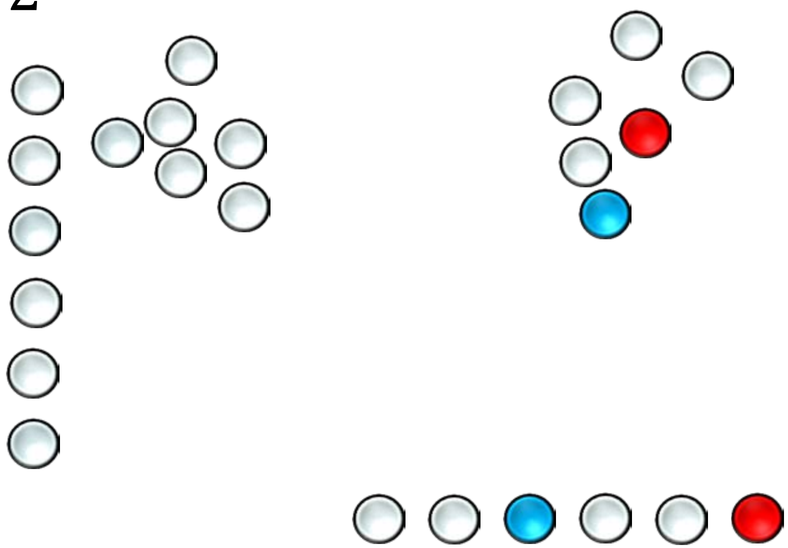

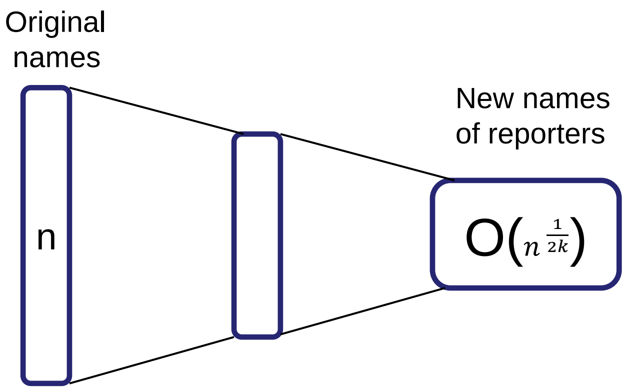

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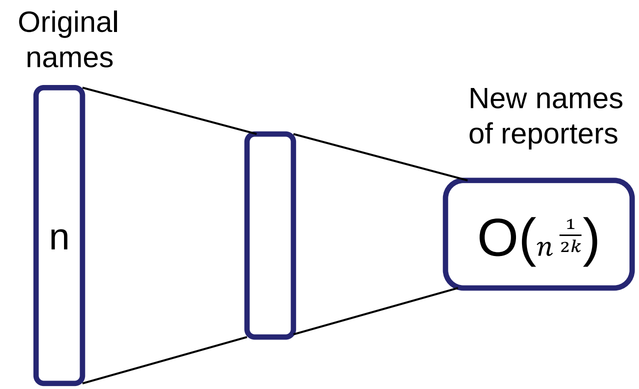

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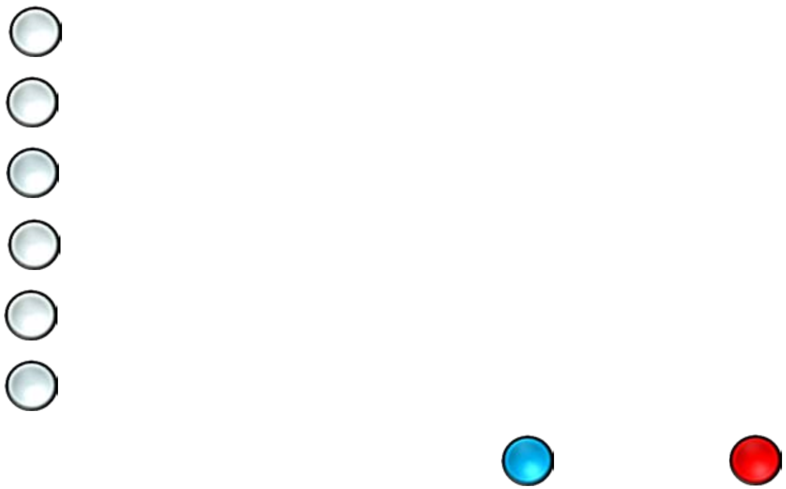

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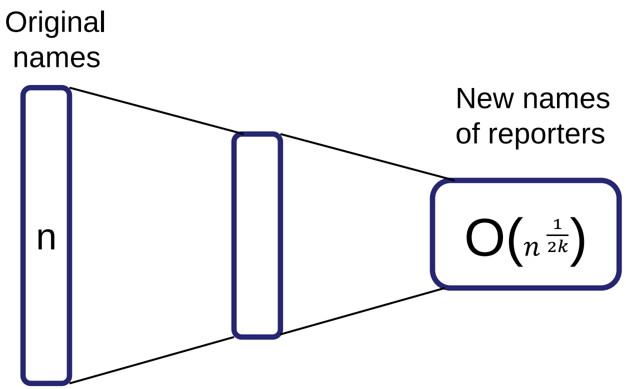
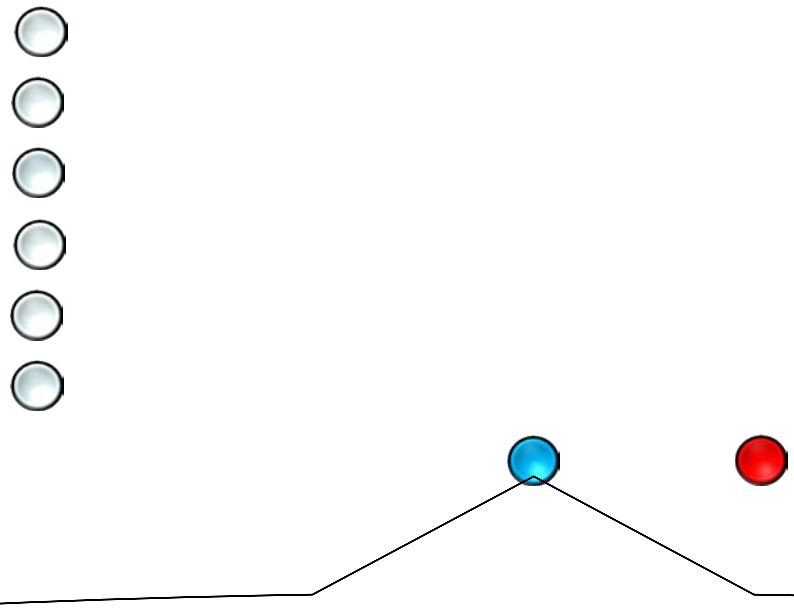


$Size: n/2$

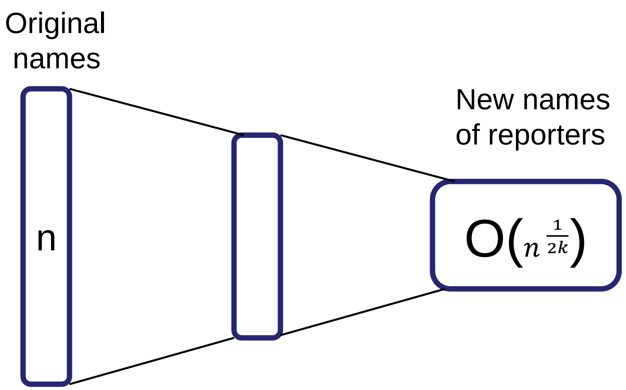


Send on channel "new name" $\in \{1, \ldots, n^{\frac{1}{2k}}\}$.

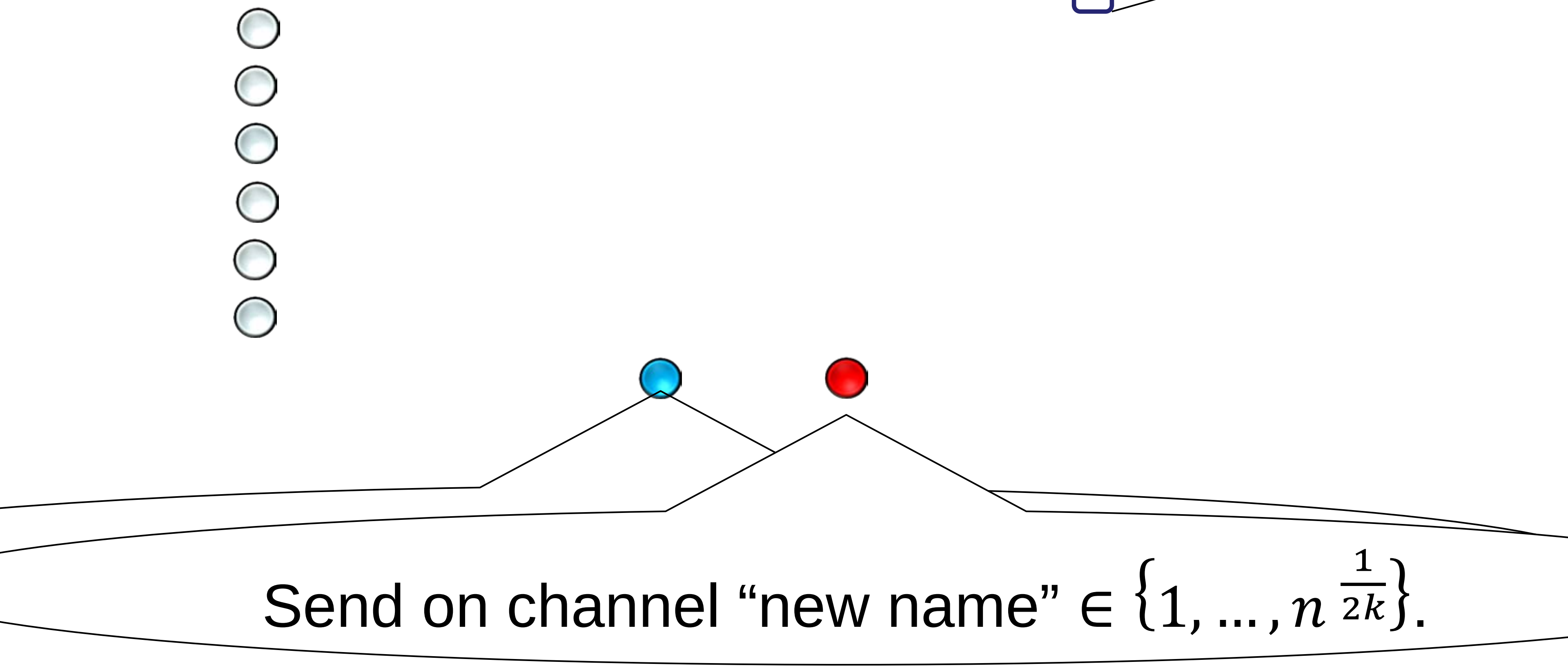
Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information



$Size: n/2$



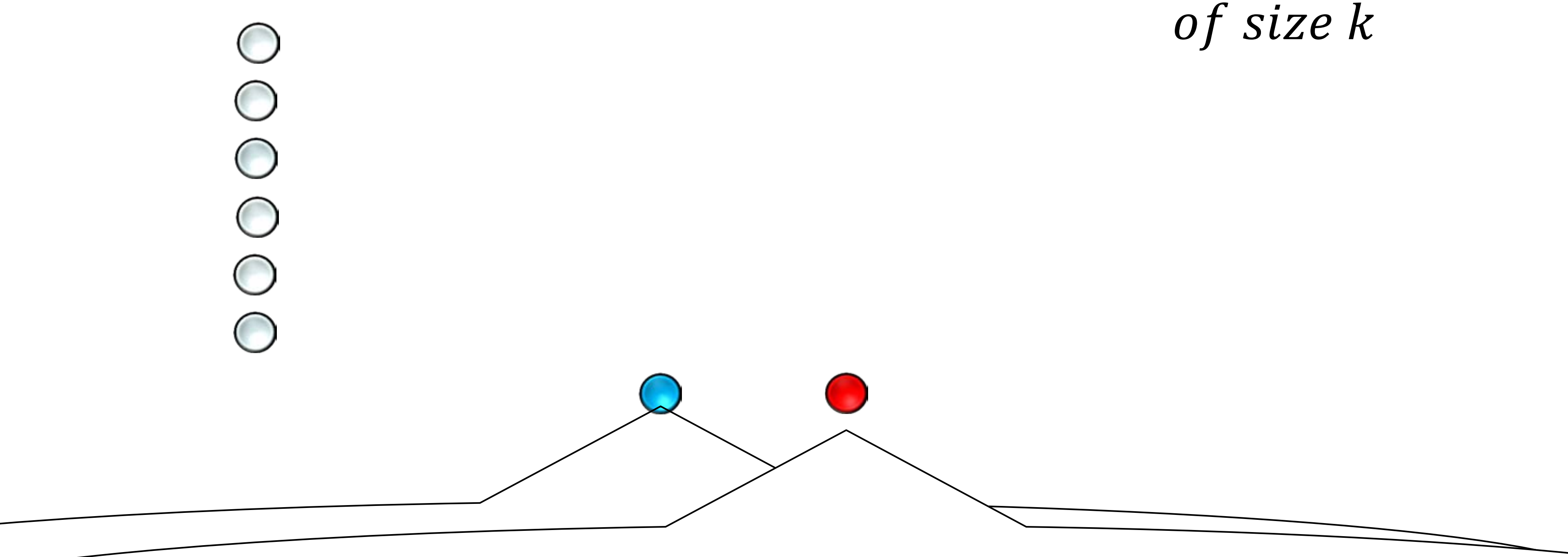
Send on channel “new name” $\in \{1, \ldots, n^{\frac{1}{2k}}\}$.

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

$Size: n/2$ $\quad$ $map: \{1, \ldots, n/2\} \longrightarrow subsets\ of\ \{1, \ldots, n^{\frac{1}{2k}}\}$ $of\ size\ k$



Send on channel "new name" $\in \{1, \ldots, n^{\frac{1}{2k}}\}$.

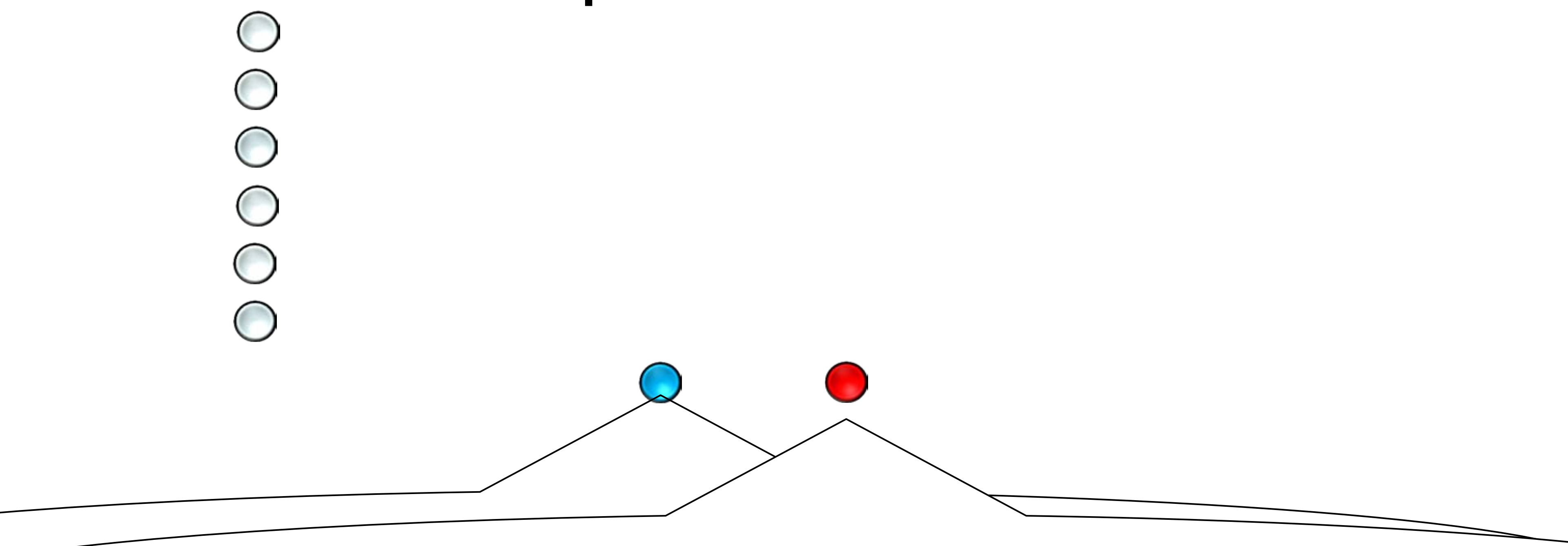
Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

$Size: n/2$

Example: 3 channels



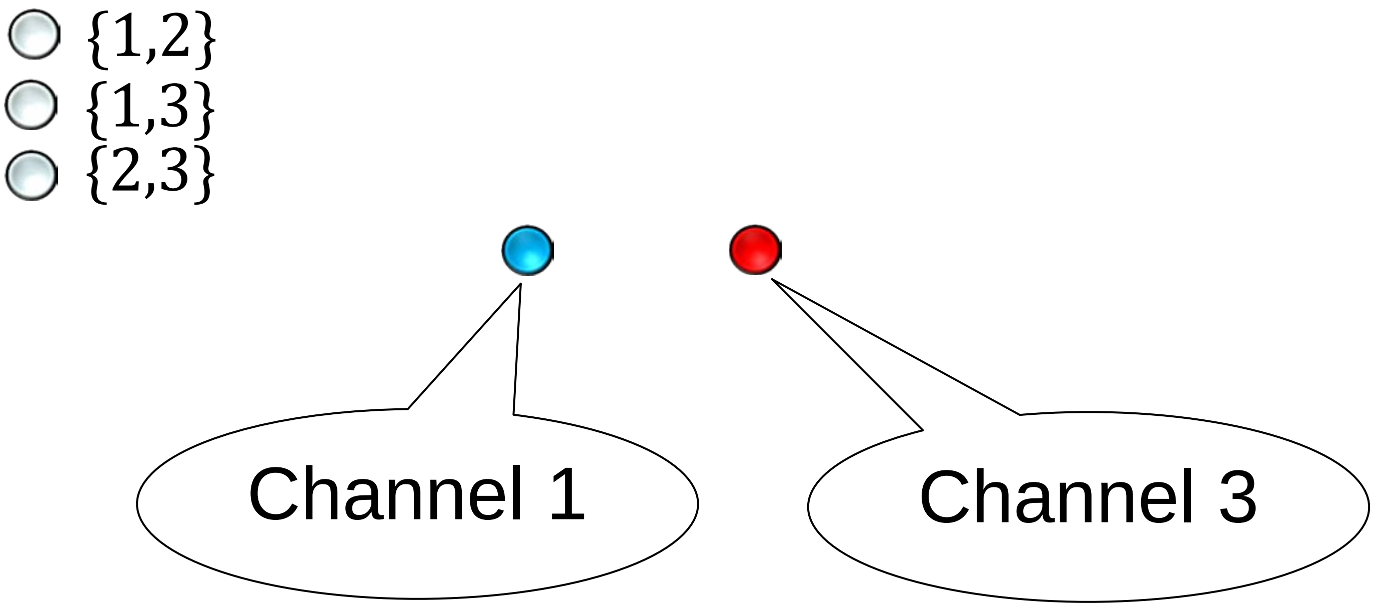
Send on channel “new name” $\in \left\{1, \ldots, n^{\frac{1}{2k}}\right\}$.

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels

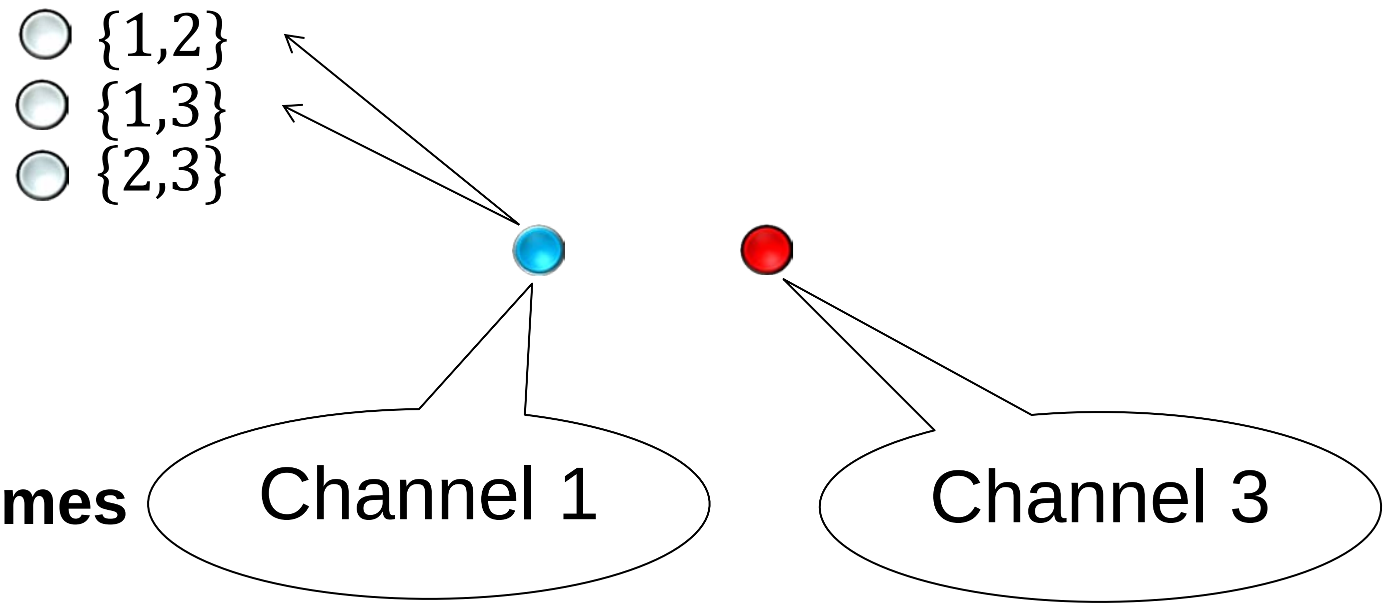


Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels



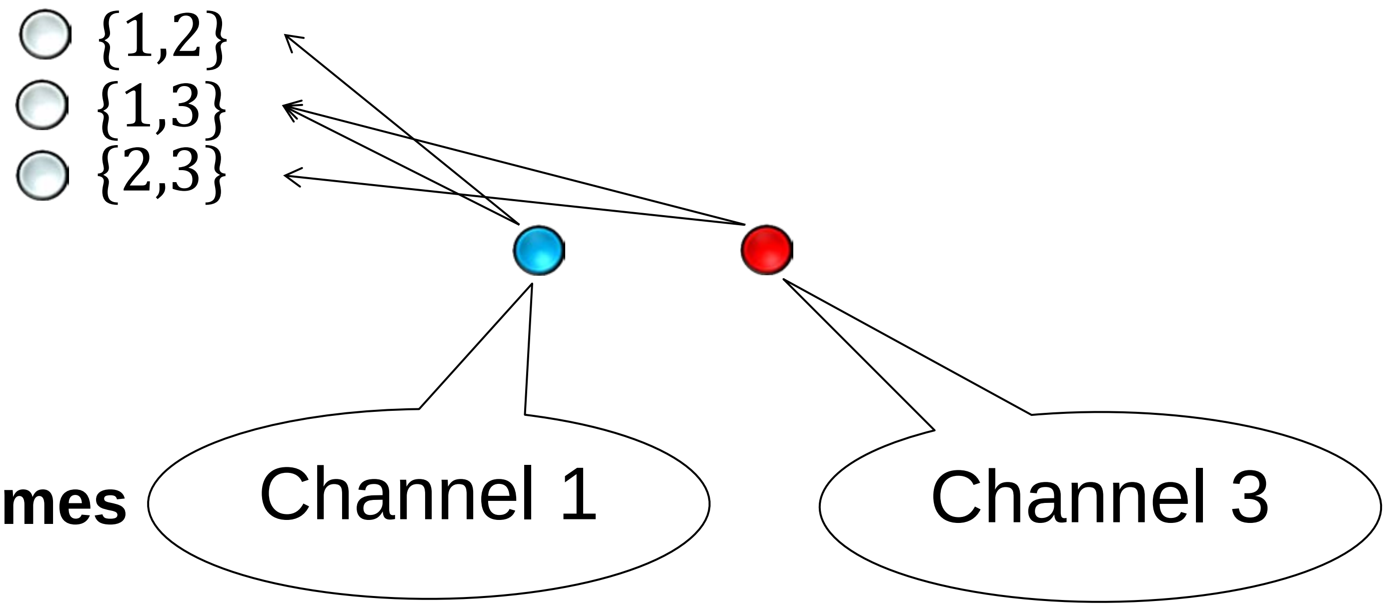
Send 2 times

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels



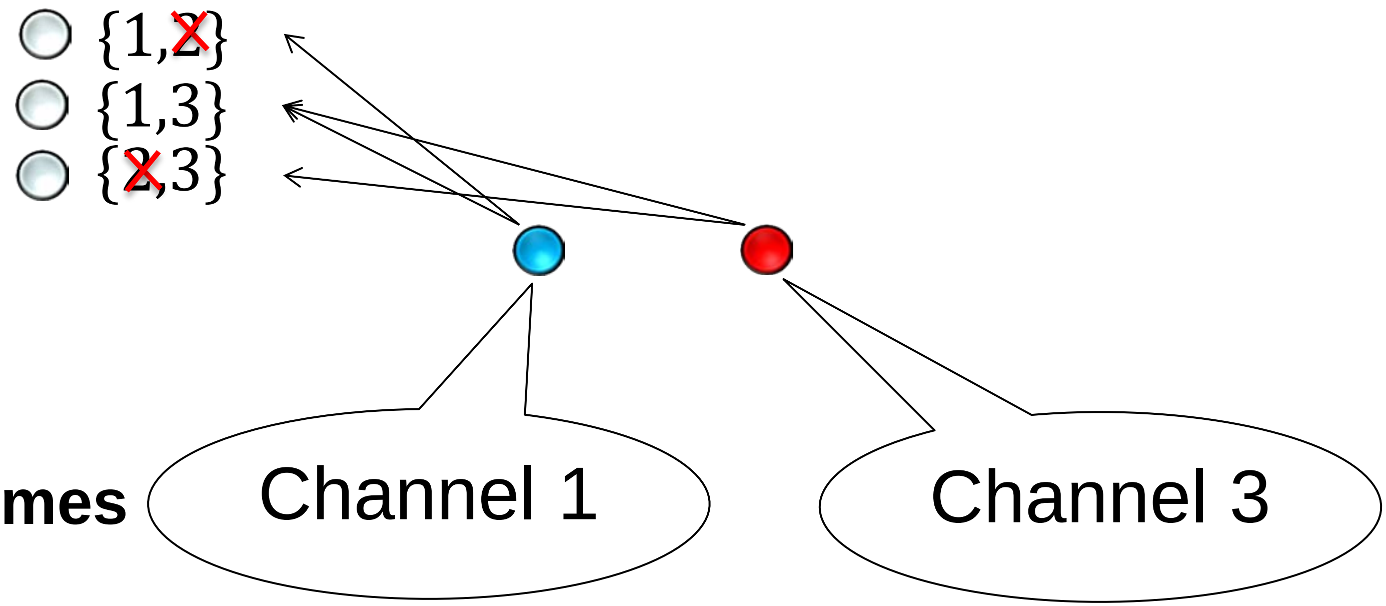
Send 2 times

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels



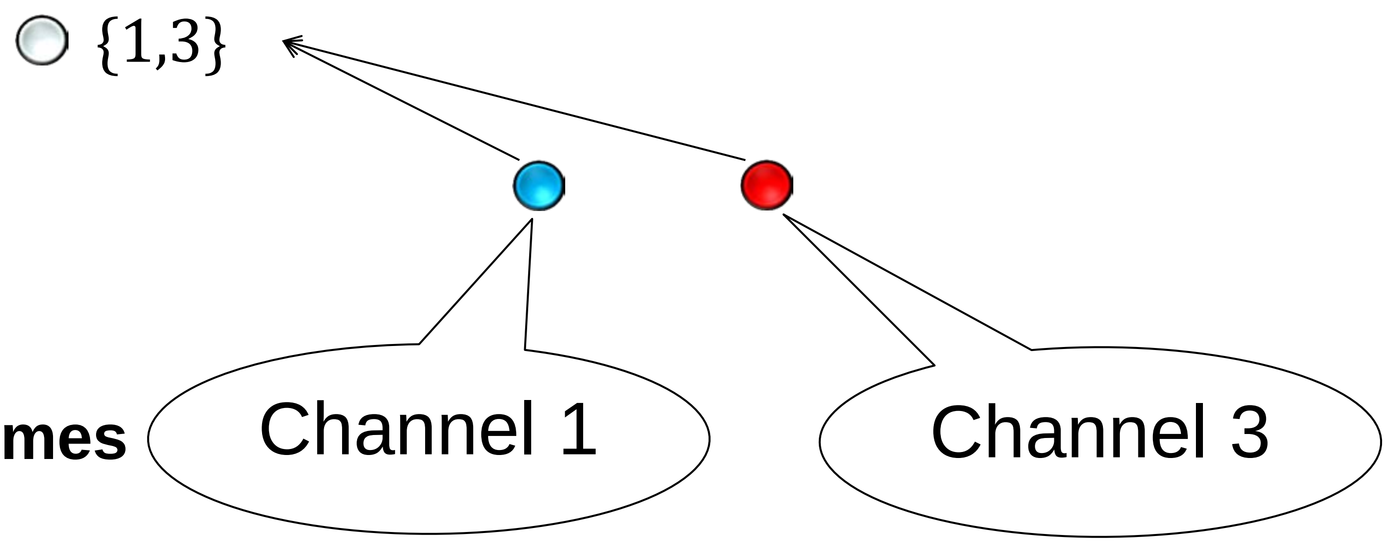
Send 2 times

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels



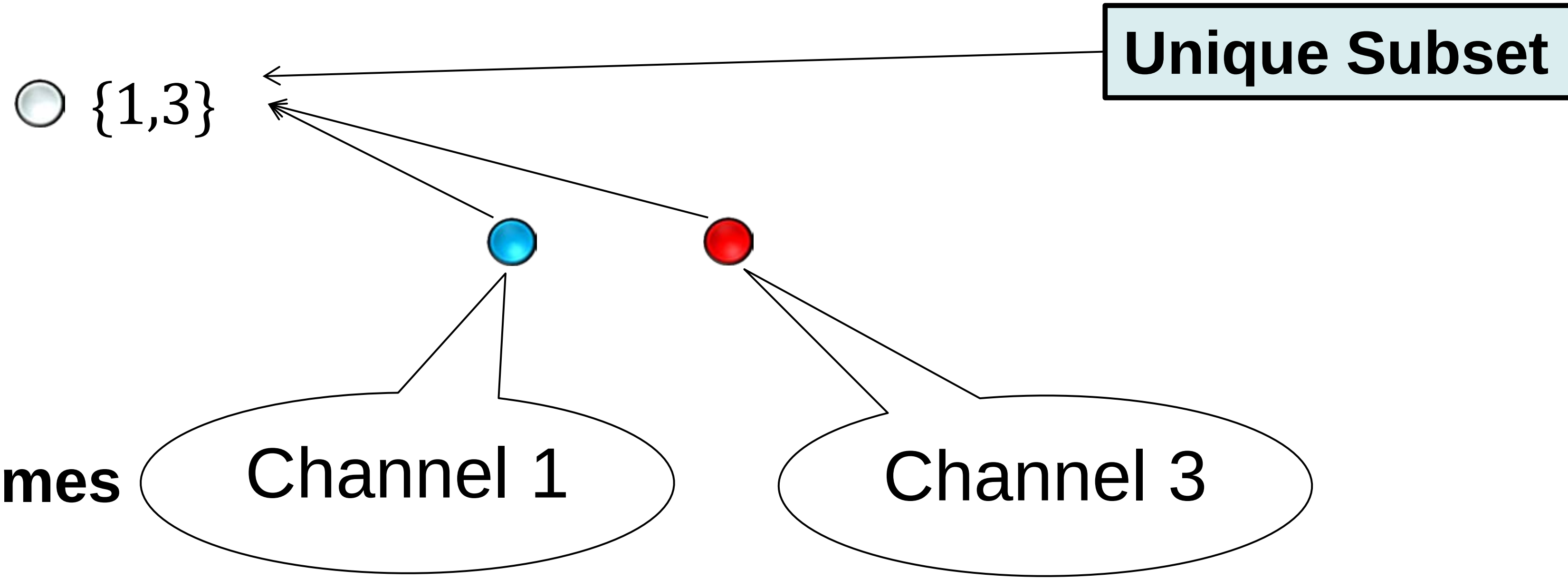
Send 2 times

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels



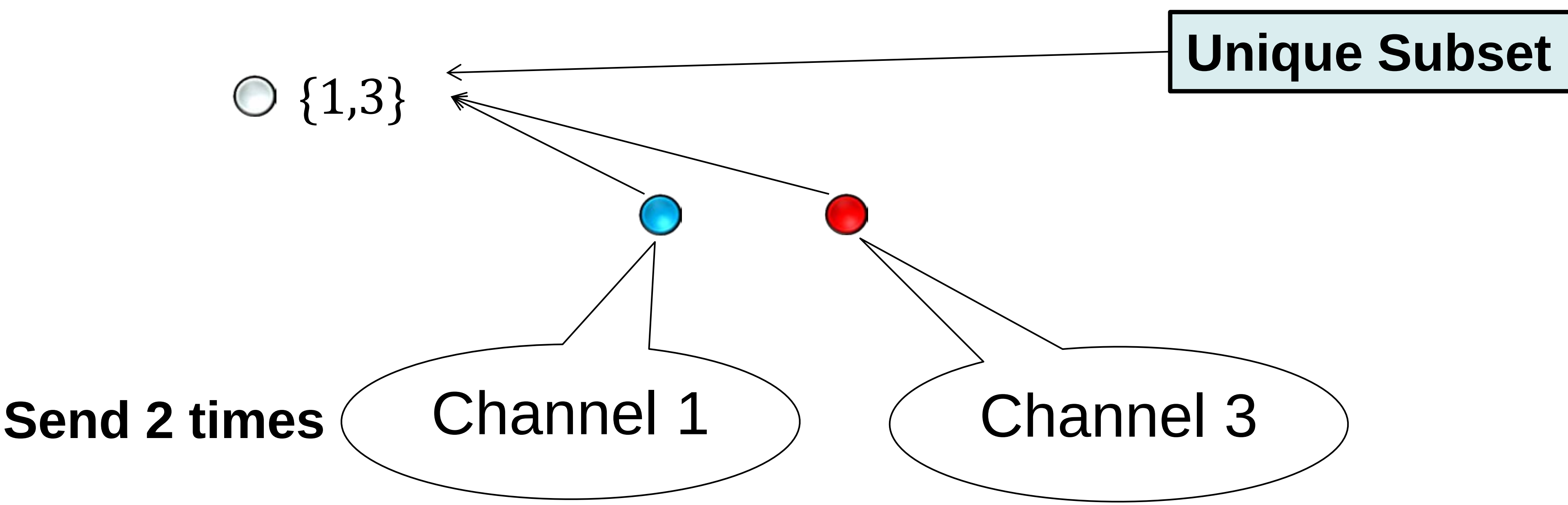
Send 2 times

Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels

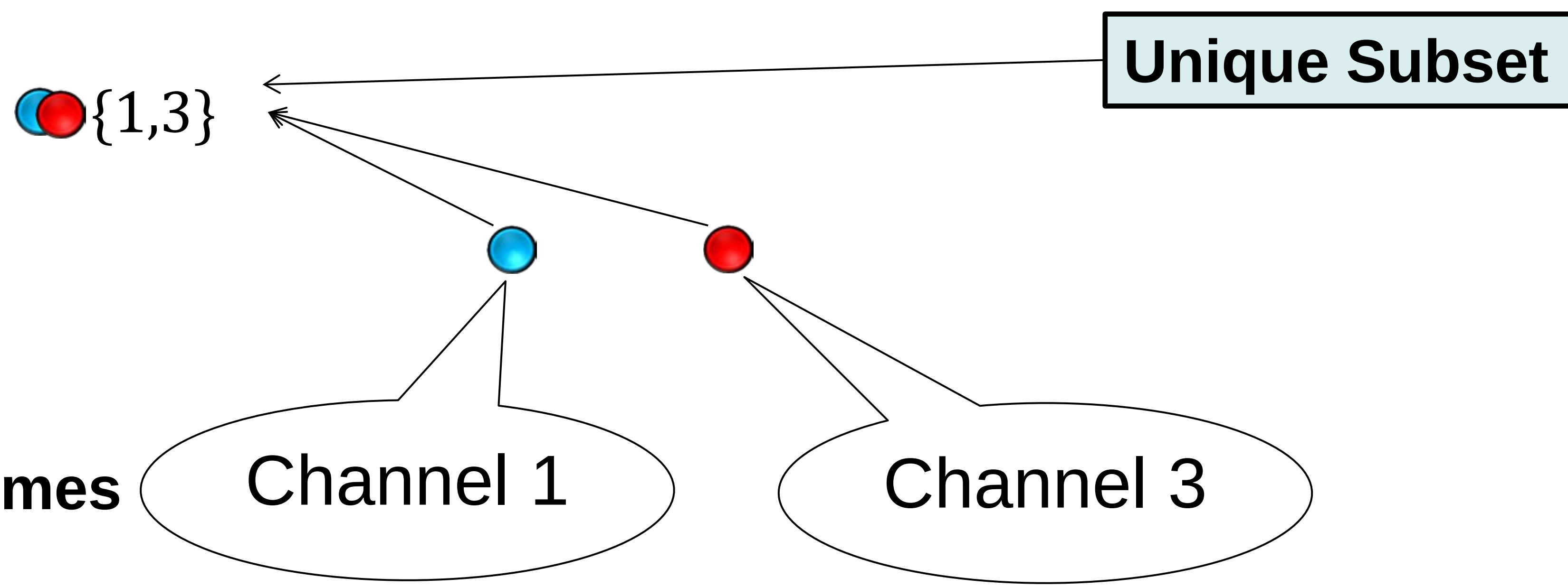


Deterministic Multi-Channel Information Exchange

n:= # nodes

k:= # information

Example: 3 channels



Send 2 times

Deterministic Multi-Channel Information Exchange

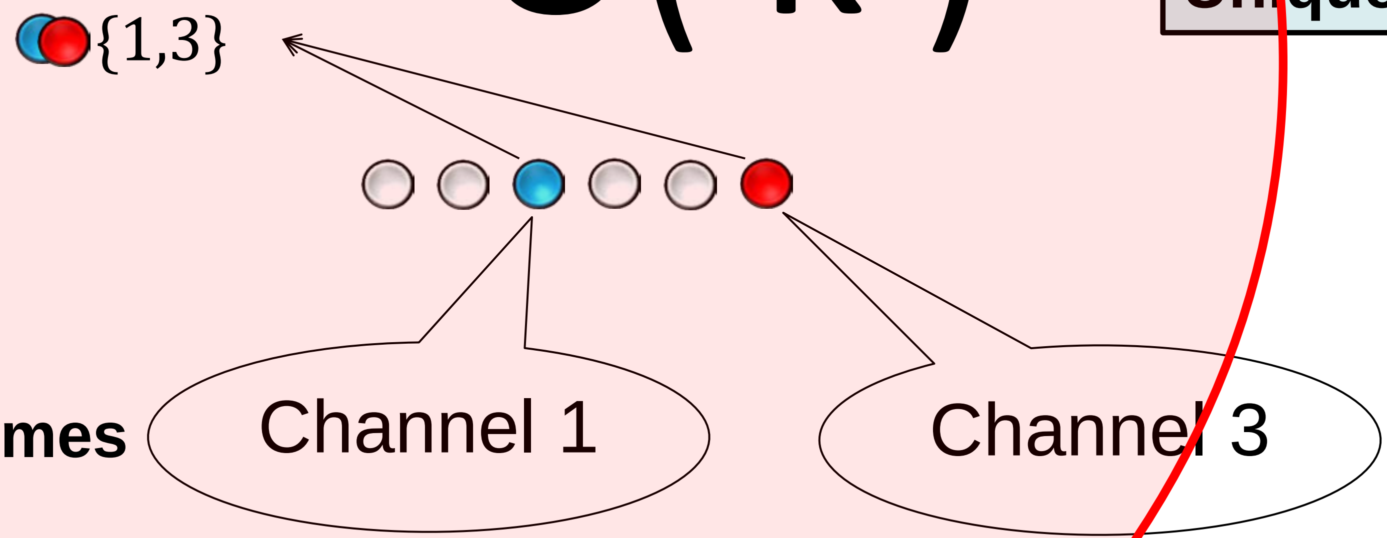
n:= # nodes

k:= # information

Example: 3 channels

$$O(\,k\,)$$

Unique Subset

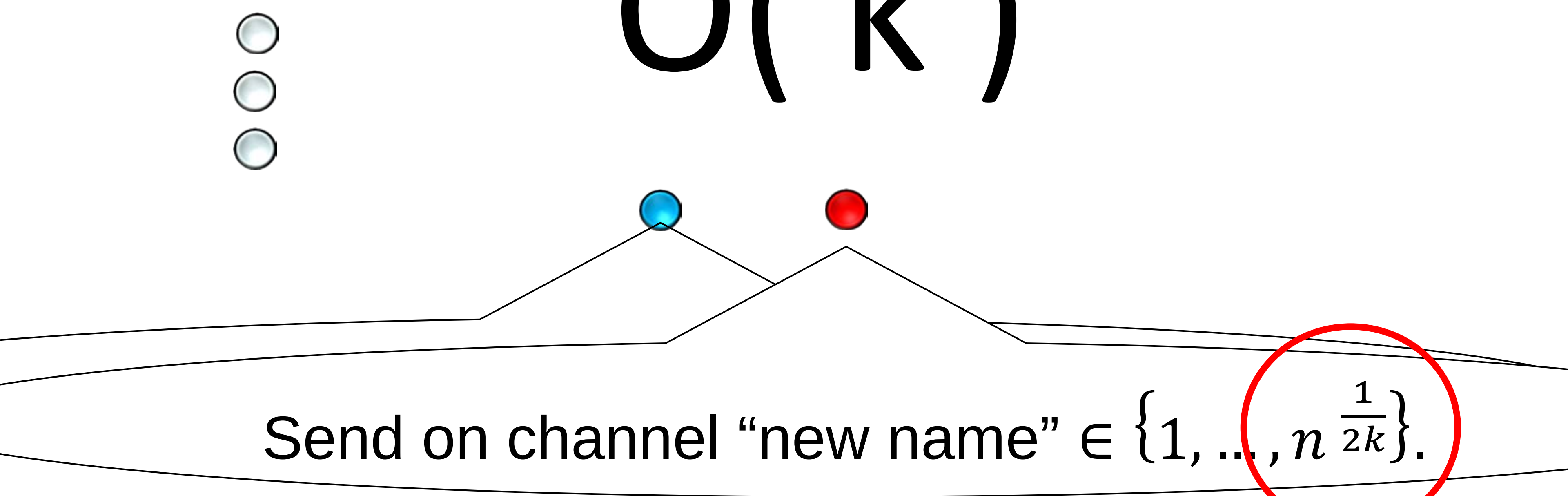


Send k times

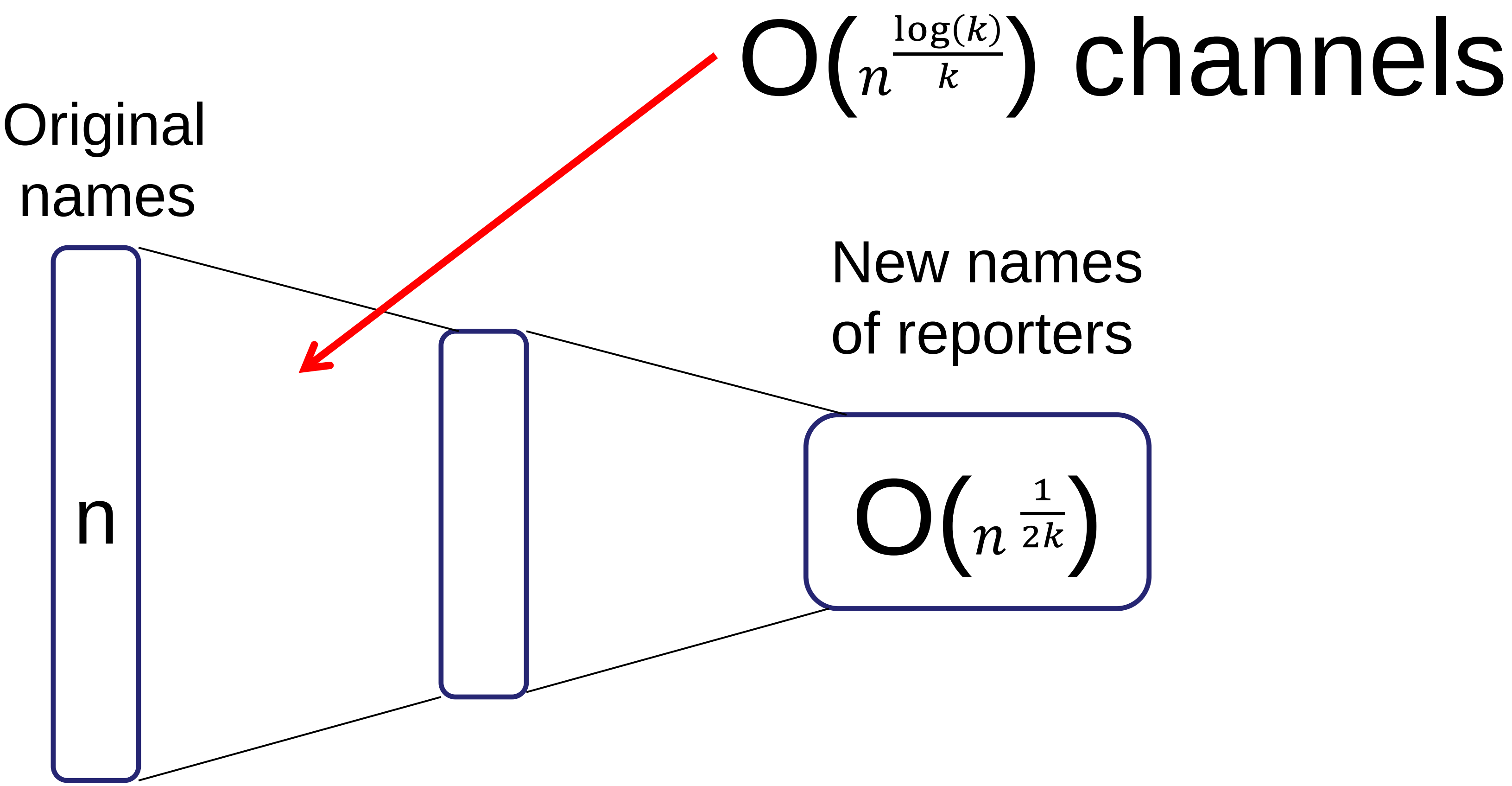
Deterministic Multi-Channel

Range of k	**[1, log n]**	**(log n , log n loglog n)**	**[log n loglog n , n- log n)**	**[n – log n, n]**
Upper bound On channels	$O\left(n^{\frac{\log(k)}{k}}\right)$	$O\left(log^{1+p}(n)\right)$	$O\left(\log(n/k)\right)$	1
Lower bound On channels	$\Omega\left(n^{\frac{1}{k}}\right)$	$\Omega\left(\frac{\log n}{\log\log n}\right)$	$\Omega\left(\log(n)\right)$	1

O(k)



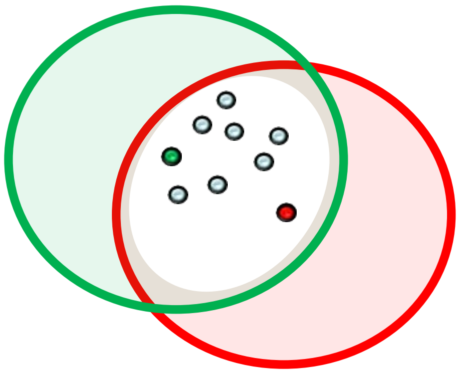
Deterministic Multi-Channel Information Exchange



Deterministic Multi-Channel Information Exchange

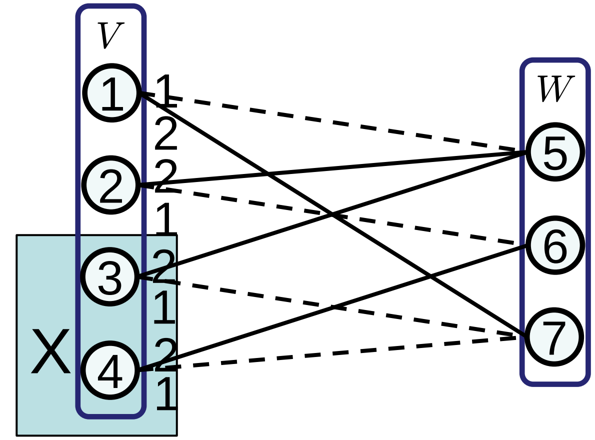
in Summary …

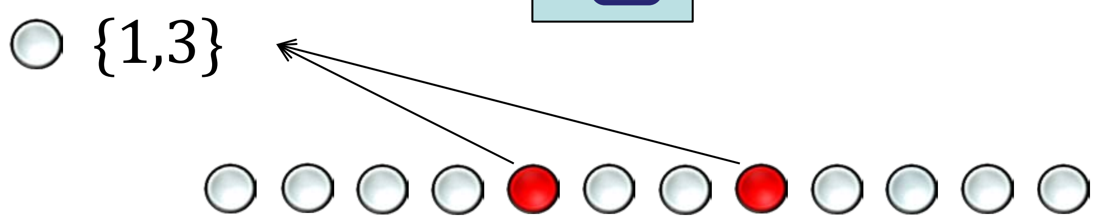
Detect / Disseminate Information!



101 Mhz
117 Mhz
132 Mhz

…





$$\boldsymbol{\Theta(k)}$$

Range of k	[1, log n]	(log n , log n loglog n)	[log n loglog n , n- log n)	[n – log n, n]
Upper bound On channels	$O\left(n^{\frac{\log(k)}{k}}\right)$	$O\left(log^{1+p}(n)\right)$	$O\left(\log(n/k)\right)$	1
Lower bound On channels	$\Omega\left(n^{\frac{1}{k}}\right)$	$\Omega\left(\frac{\log n}{\log\log n}\right)$	$\Omega\left(\log(n)\right)$	1

Thank You!

Questions & Comments?

Stephan Holzer - ETH Zürich
Thomas Locher - ABB Switzerland
Yvonne Anne Pignolet - ABB Switzerland
Roger Wattenhofer - ETH Zürich

Thank You!

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