A Note on
Uniform Power Connectivity
in the SINR Model

Chen Avin

Zvi Lotker

Francesco Pasquale

Yvonne-Anne Pignolet

Ben-Gurion University of the Negev

University of Rome Tor Vergata

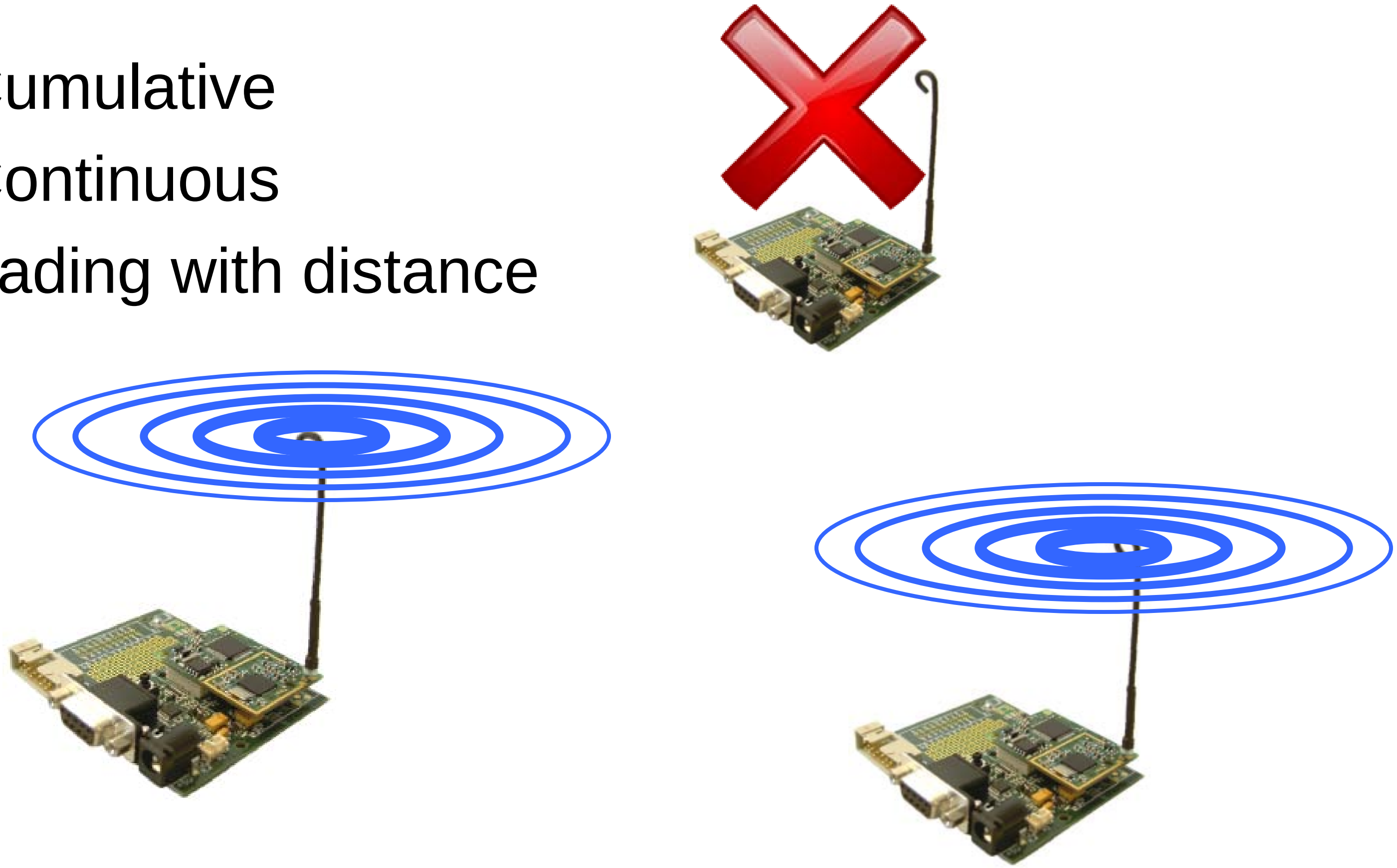
ETH Zurich Switzerland

Interference in Wireless Networks

Interference:

Concurrent transmissions disturb each other

- Cumulative
- Continuous
- Fading with distance



Interference Model

Interference:

Cumulative, continuous, fading with distance

Received signal power from sender

Path-loss exponent in [1.6,6]

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N+\sum_{w\in V\setminus\{u\}} \frac{P_w}{d(w,v)^\alpha}} \geq \beta$$

Hardware threshold

Noise

Interference

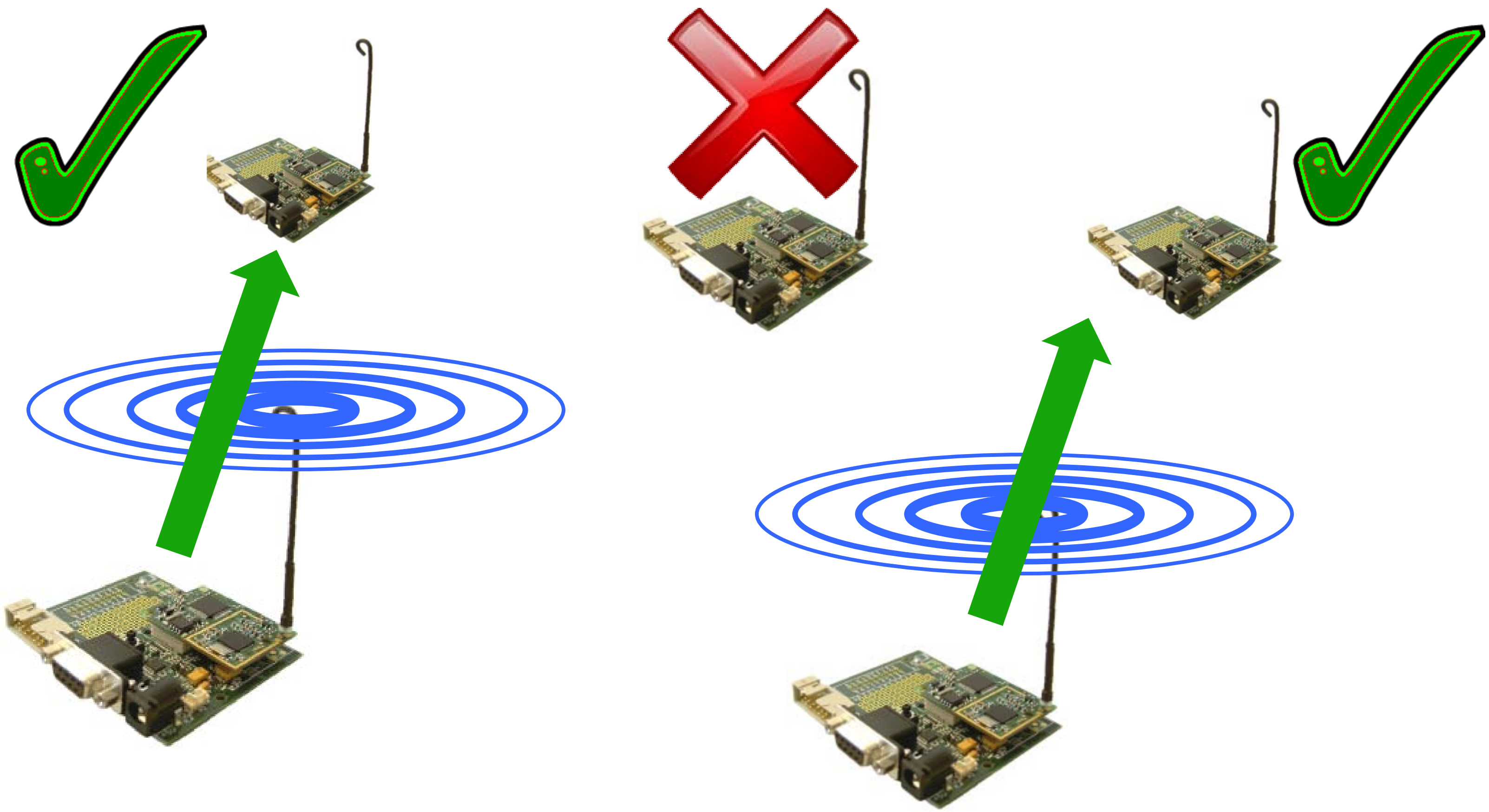
Distance between two nodes

SINR (Physical Model):

v receives from *u* if Signal-to-Noise+Interference Ratio $\geq \beta$

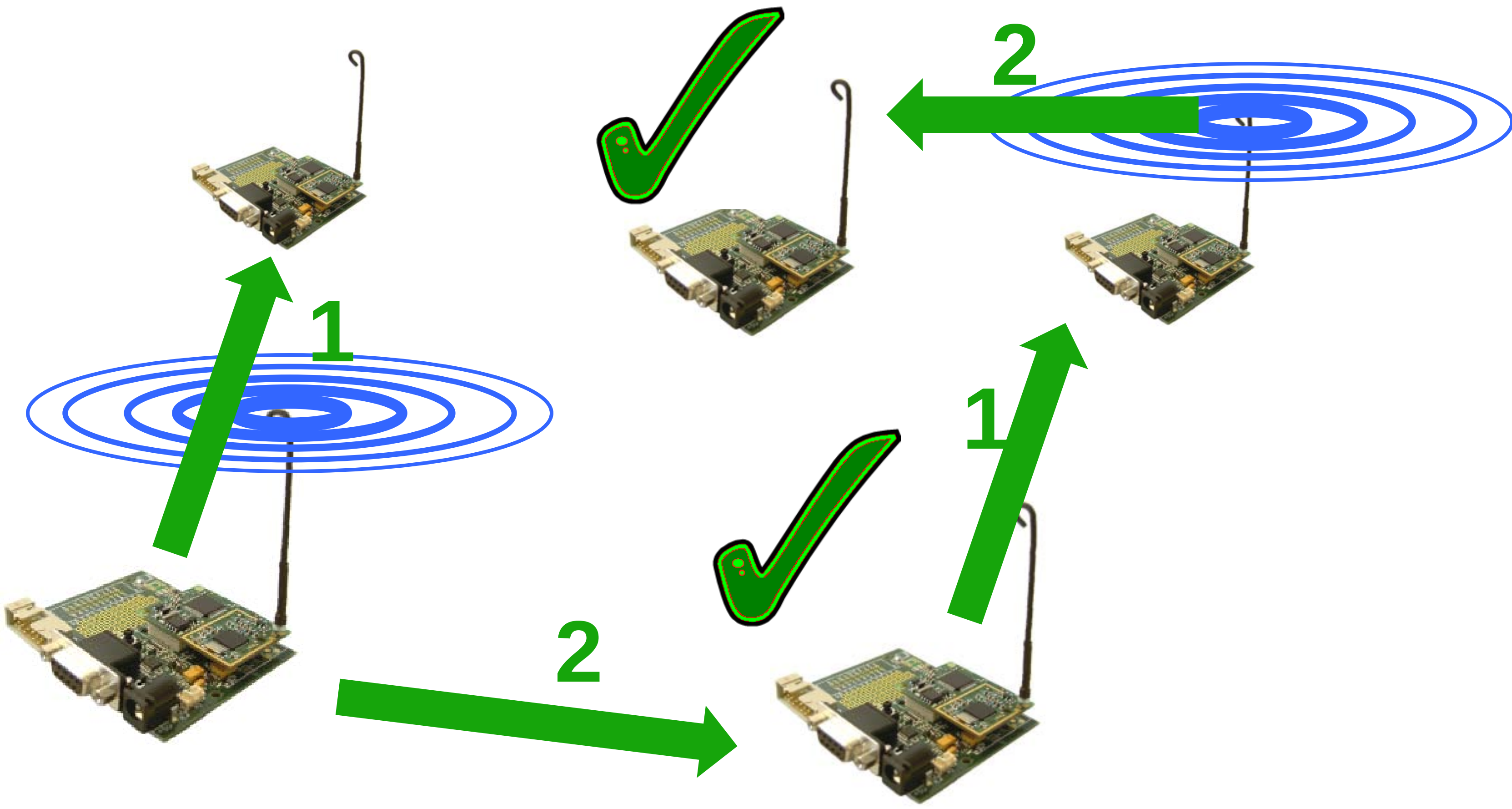
Interference:

Concurrent transmissions disturb each other



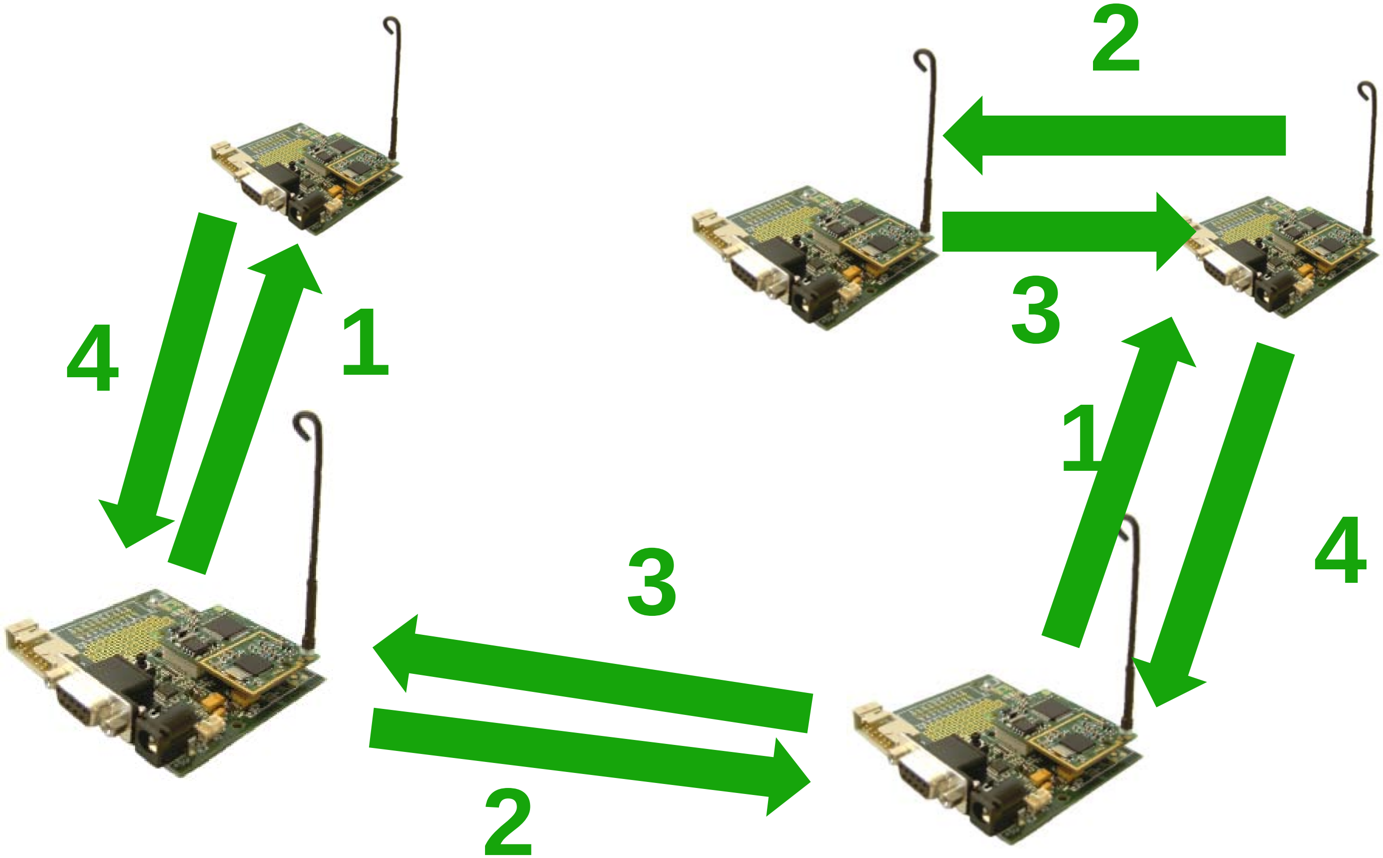
Complexity of Connectivity:

time slots until strongly connected communication graph



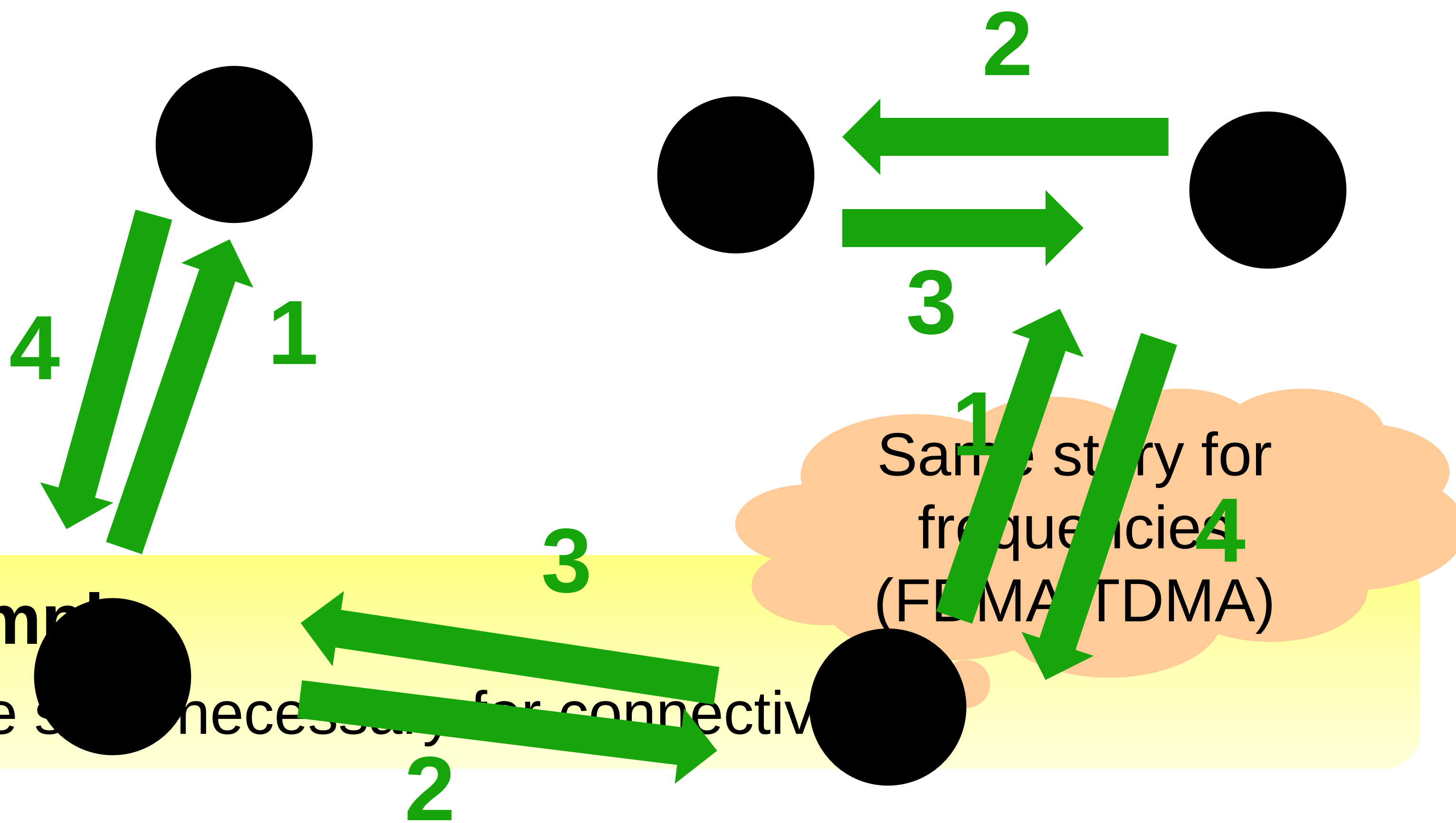
Complexity of Connectivity:

time slots until strongly connected communication graph



Complexity of Connectivity:

time slots until strongly connected communication graph



Exampl

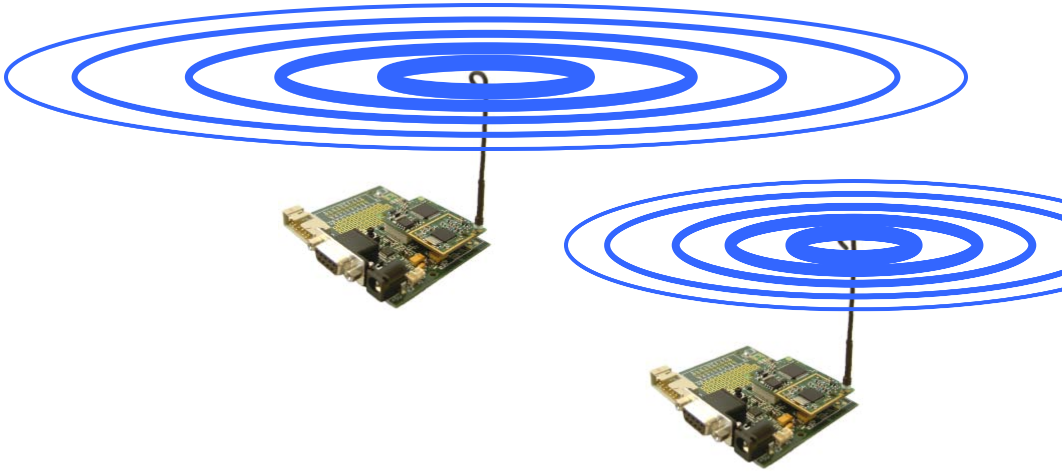
4 time s necessary for connectiv

Complexity of Connectivity:

colors until strongly connected communication graph

[Moscibroda and Wattenhofer, Infocom 06]

- Without power control:
  $\Omega(n)$ in worst case
- With power control:
  $O(\log^4 n)$ in worst case



Complexity with uniform density and power?

Interference Model

Received signal power from sender

Path-loss exponent in [1.6,6]

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{N+\sum_{w\in V\setminus\{u\}}\frac{P_w}{d(w,v)^\alpha}} \geq \beta$$

Noise

Interference

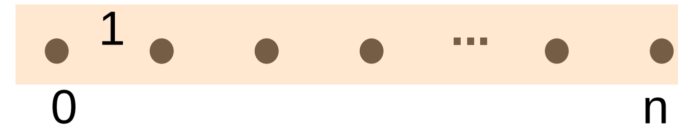
Distance between two nodes

SINR (Physical Model):

v receives from *u* if Signal-to-Noise+Interference Ratio $\geq \beta$

Connectivity with Uniform Power and Density

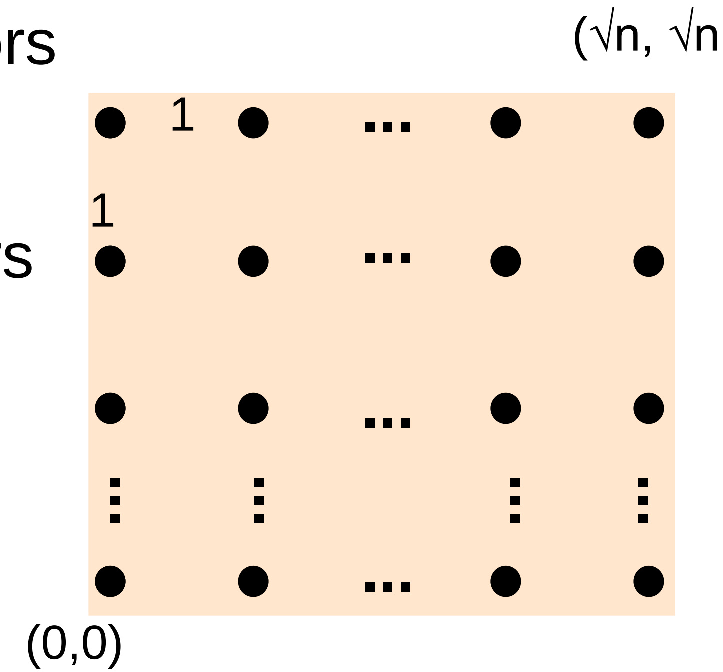
- **1D grid**



$\alpha > 0$: constant number of colors

- **2D grid**

$\alpha > 2$: constant number of colors



$\alpha = 2$: O (log n) colors
Ω(log n / log log n) colors

$\alpha < 2$: θ ($n^{2/\alpha - 1}$)

- **uniformly distributed 1D**

$\alpha = 2$: O(log n) colors
Ω(log log n) colors

2D Grids: Upper bounds

regular k^2– coloring:

- Partition into k^2 sets
- Shortest distance between same color nodes is k

Interference at (0,1):

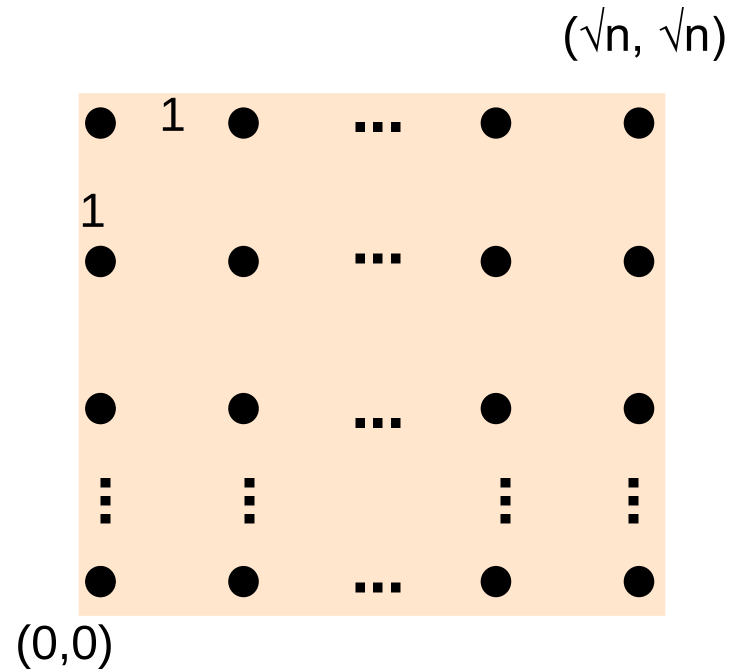
$$I(0,1) < \frac{3}{(k/2)^\alpha} \sum_{i=1}^{\sqrt{n}} \frac{1}{i^{\alpha-1}}$$

Riemann-Zeta Function:

Constant for $\alpha > 2$

Logarithmic for $\alpha = 2$

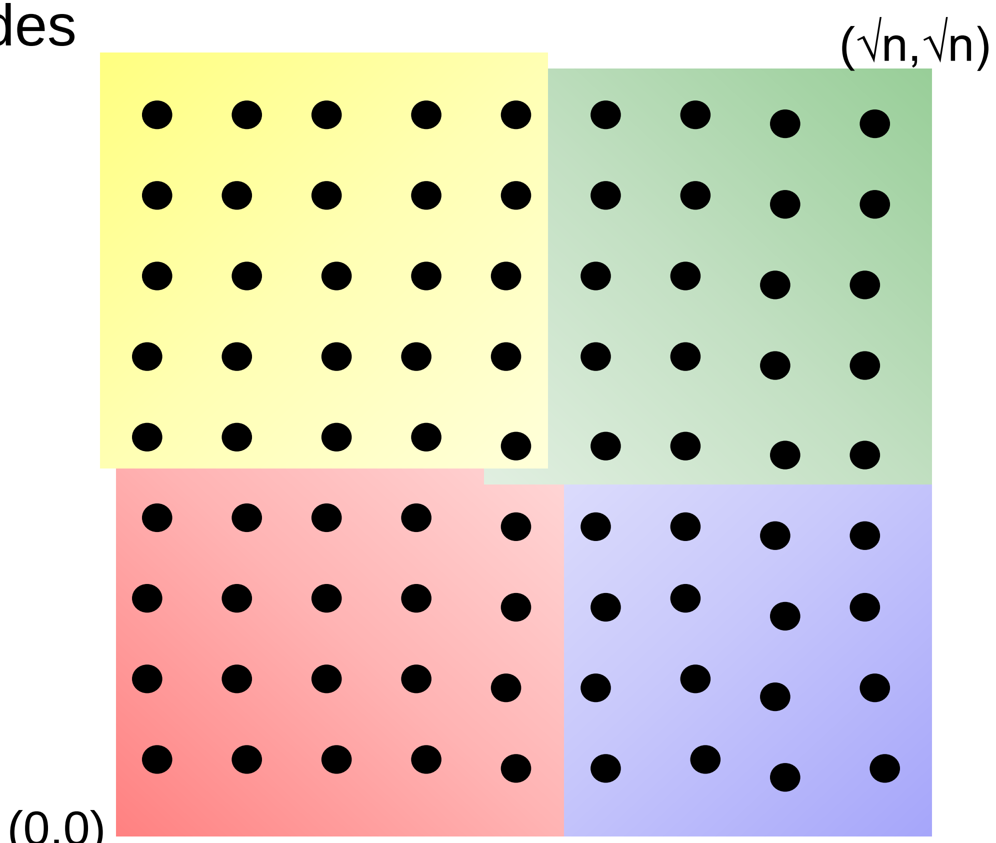
$O(n^{2/\alpha - 1})$ for $\alpha < 2$



2D Grids: Lower bound α = 2

Bound interference at (0,0) with 3 colors:

- 1 color with at least n/3 nodes
- Divide grid intro 4 parts
- Pick square with > n/12 nodes

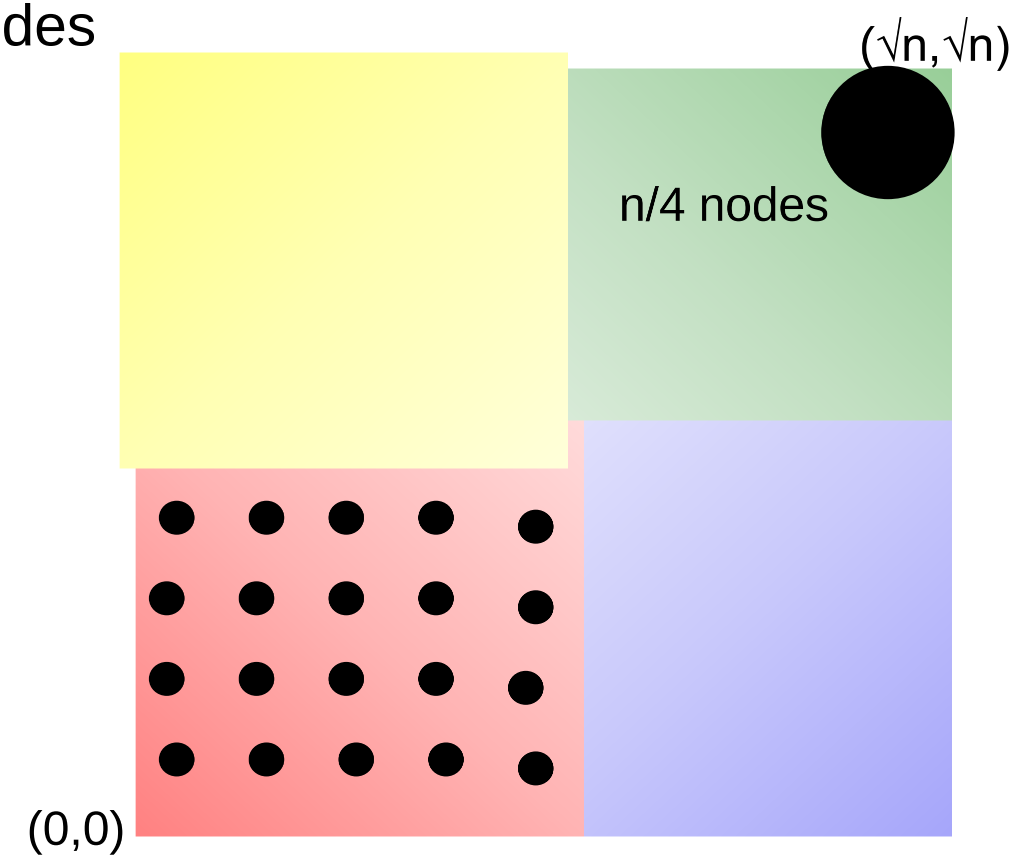


2D Grids: Lower bound $\alpha = 2$

Bound interference at (0,0) with 3 colors:

- 1 color with at least n/3 nodes
- Divide grid intro 4 parts
- Pick square with > n/12 nodes
- I(0,0) > 1/8

- O(log n) recursions ....
  I(0,0) > Ω(log n) 1/8



2D Grids: Lower bound $\alpha = 2$

Bound interference at (0,0) with 3 colors:

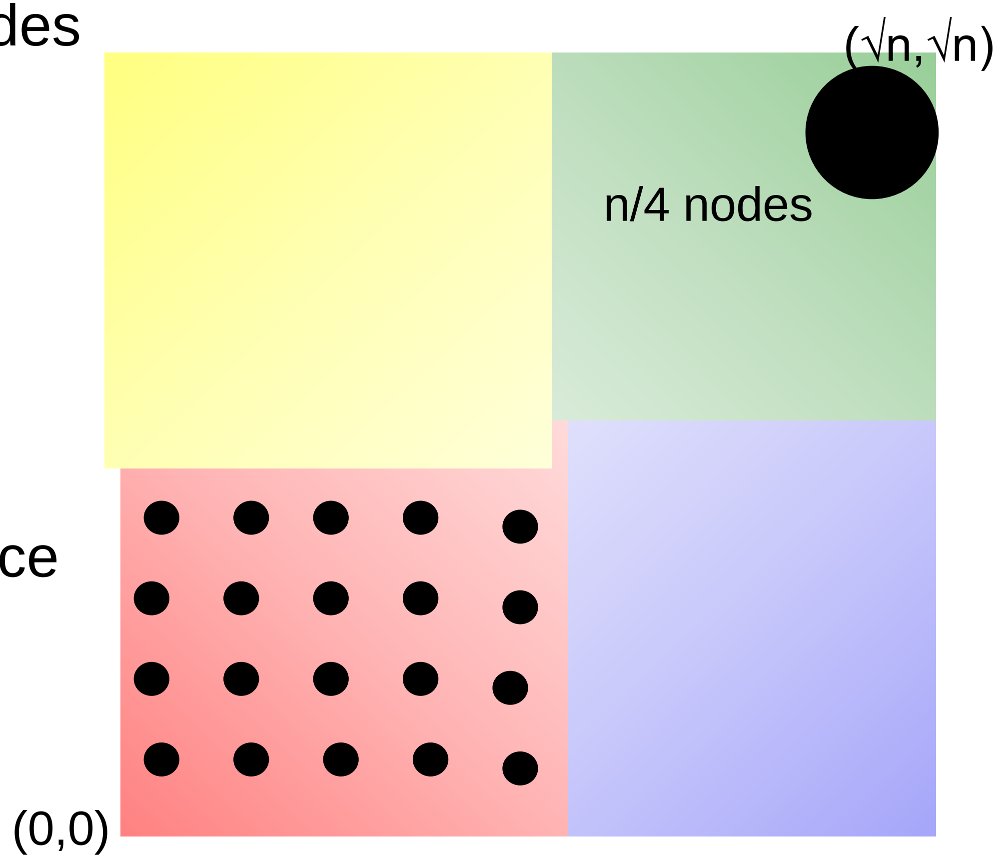
- 1 color with at least n/3 nodes
- Divide grid intro 4 parts
- Pick square with > n/12 nodes
- I(0,0) > 1/8

- O(log n) recursions ....
  I(0,0) > Ω(log n) 1/8

Generalize for k colors:

- Ω(log n/ (k log k)) interference

- Interference is constant
  for k in Ω(log n / log log n)



Conclusion

- Grids:

Complexity of connectivity is bounded in uniform power grids

Phase transition for $\alpha = 2$

- 1D uniform distribution, $\alpha = 2$:

Regular coloring needs O(log n) colors
General Ω(log log n) lower bound

- Many open questions

in 2D uniform distribution,communication graph needs to be determined as well...

That‘s it...

Thanks!

Questions?