# Energy-Efficient Scheduling on Homogeneous Multiprocessor Platforms*

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## ABSTRACT

Low-power and energy-efficient system implementations have become very important design issues to extend operation duration or cut power bills. To balance the energy consumption resulting from the dynamic power consumption and the static power consumption, the concept of *critical speed* has been adopted widely in the literature. Most scheduling algorithms for such systems assume that the critical speed is the lowest speed for scheduling and then perform job/task procrastination to turn the processor(s) to the dormant mode when there is no job for execution. This paper shows that the critical speed might be too optimistic and turning the processor(s) to the dormant mode might be energy-inefficient. By allowing tasks to run at lower speeds than the critical speed, in this paper, a new approximation algorithm is developed for homogeneous multiprocessor systems with a $1.21$-approximation factor, which significantly improves the state-of-the-art approximation algorithm with a $1.667$-approximation factor. Performance evaluation shows the effectiveness of the proposed algorithm with comparison to the state-of-the-art approximation algorithm. Our algorithm can reduce the energy consumption by at most 15% in our simulations.

## Categories and Subject Descriptors

D.4.1 [**Operating Systems**]: Process Management–Scheduling

## General Terms

Algorithms, Performance

## Keywords

static power consumption, dynamic voltage scaling, task partitioning, real-time scheduling.

## 1. INTRODUCTION

Power management has become an important system design issue for embedded systems and server systems to prolong the operation duration or reduce power bills. Dynamic power consumption (mainly) due to switching activities and static power consumption (mainly) due to leakage current are two major sources of energy consumption of a CMOS processor [8]. The dynamic voltage scaling (DVS) technique was introduced to balance the dynamic energy consumption and the performance of a system, in which different supply voltages lead to different execution speeds/frequencies. The dynamic power consumption is usually a convex and increasing function of speed/frequency, which motivates to execute as slowly as possible. However, executing at a lower speed stretches the execution time with more leakage/static energy consumption.

For systems with real-time demands, energy-efficient scheduling in DVS systems has been studied extensively. For real-time systems, energy-efficient scheduling is to minimize the energy consumption under timing constraints. As chip makers are releasing multi-core chips and multiprocessor system-on-a-chip (MPSoC), multiprocessor platforms have become even more popular to improve the system performance and accommodate the growing demand of application functionality.

In nano-meter manufacturing, static power consumption in CMOS circuits has significant contribution of the total power consumption, in which the static power consumption is comparable to the dynamic power dissipation [8]. To reduce the static energy consumption resulting from the leakage current, a processor might enter a dormant mode (or be turned off), in which the power consumption of the processor in the dormant mode is much smaller than the static power consumption. However, turning the processor to the active mode requires time and energy overheads, due to the wakeup/shutdown of the processor and data fetch in the register/cache. For example, the Transmeta processor in 70nm technology has $483\mu J$ energy overhead and less than 2 msec (ms) timing overhead [8].

Energy-efficient scheduling has been recently explored on DVS platforms with non-negligible static power consumption, such as [2, 4, 8, 9], in which the static power can be reduced by turning the processor to the dormant mode. In particular, for scheduling periodic tasks in uniprocessor systems, Chen and Kuo [2], Jejurikar et al. [7, 8], and Lee et al. [9] develop DVS and procrastination scheduling strategies to decide the execution speeds and when to turn the processor to the dormant mode. For uniprocessor scheduling of aperiodic real-time tasks, Irani et al. [5] propose a 3-approximation algorithm for the minimization of energy consumption, where a $\rho$-approximation algorithm guarantees to derive solutions no more than $\rho$ times of the optimal energy consumption. Niu and Quan [11] apply similar procrastination strategies for periodic real-time tasks by considering the hyper-period of the given tasks. For homogeneous multiprocessor systems, Xu et al. [13] propose algorithms to determine the number of activated processors by considering workload requests instead of real-time tasks, and de Langen and Juurlink [4] provide heuristic algorithms for systems with discrete speeds. Chen et al. [1] develop a polynomial-time approximation algorithm for energy consumption minimization.

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Most of the above approaches [1,2,5,7,8,11,13] apply the *critical speed* as the lower-bounded speed for execution and then perform job/task procrastination without violating the timing constraints, in which the critical speed is the available speed with the minimum energy consumption for execution. By definition, if $P(s)$ is the power consumption at speed $s$, the critical speed $s^*$ is the speed $s$ with the minimum energy consumption $P(s)/s$. However, as the critical speed is defined only for the minimization of energy consumption for job execution, it, in fact, optimistically assumes to turn the processor(s) to the dormant mode after the execution. However, it might not be worthy to turn the processor to the dormant mode because the energy overhead might be larger than the reduced energy. In such cases, executing at the critical speed might lead to a solution that consumes more energy. Furthermore, the critical speed is used in [1] as the maximum speed for task assignment and processor allocation. As executing at the critical speed might consume more energy, task assignment based on the critical speed with load balancing might also consume more energy.

To clarify the non-optimality of critical speeds, we will first explore energy-efficient scheduling for frame-based real-time tasks, in which every task has the same deadline and period. For tasks with precedence constraints, the proposed algorithm and analysis here can be applied by pipelining the tasks. (For detail, please refer to [10].) We show that it is not good to stick with the critical speed by taking it as the lower-bounded speed for execution [2, 8] or by treating it as the upper-bounded speed for task remapping in multiprocessor systems [1]. Optimal solutions, regardless of task mapping, might execute tasks at lower speeds than the critical speed when the energy overhead to turn on/off the processor is relatively large. Moreover, we will also show that load balancing might not be good since assigning tasks on a light-loaded processor might waste energy to turn on/off the processor. Based on the above observations, we propose a polynomial-time algorithm to minimize the energy consumption for homogeneous multiprocessor systems. When multiple processors are necessary for energy minimization, with theoretical analysis, we prove that the derived solutions of our algorithm consume energy no more than $1.21$ times of the energy consumption of the corresponding optimal solutions when the minimum available speed is 0. The algorithm significantly improves the state-of-the-art approximation algorithm with a $1.667$-approximation factor developed in [1]. For systems with non-zero minimum available speed, extending the analysis of this paper can yield to a $1.43$-approximation factor. This also greatly improves the algorithm with a 2-approximation factor in [1]. Performance evaluation reveals that the proposed algorithm is more effective not only in worst cases but also in average cases.

The rest of this paper is organized as follows: Section 2 provides system models. Section 3 shows the non-optimality of critical speeds. Section 4 presents the improved approximation algorithm for homogeneous multiprocessor systems. Experimental results are presented in Section 5. Section 6 concludes this paper.

# 2. SYSTEM MODELS

## 2.1 Processor models

The (system) power consumption function $P(s)$ of speed $s$ on a processor has two parts: $P_\delta(s)$ and $P_{ind}$, where $P_\delta(s)$ ($P_{ind}$, respectively) is dependent (independent, respectively) on the speed [2, 14]. The speed-dependent power consumption $P_\delta(s)$ comes from the short-circuit power consumption and the dynamic power consumption due to the charging and discharging of gates on a CMOS DVS processor, while the leakage power consumption mainly contributes to $P_{ind}$. For example, the dynamic power consumption $P_{switch}(s)$ due to gate switching at speed $s$ is $P_{switch}(s) = C_{ef} V_{dd}^2 s$, where $s = k\frac{(V_{dd}-V_t)^2}{V_{dd}}$, and $C_{ef}, V_t, V_{dd}$, and $k$ denote the effective switch capacitance, the threshold voltage, the supply voltage, and a design-specific constant, respectively ($V_{dd} \geq V_t \geq 0$, $k > 0$, and $C_{ef} > 0$). The speed-dependent power consumption function $P_\delta(s)$ can be modeled as a strictly convex and increasing function of the adopted speed, while the speed-independent power consumption is a constant. We explore the scheduling on processors with a continuous spectrum of the available speeds between the upper-bounded speed $s_{\max}$ and the lower-bounded speed $s_{\min}$. It is easy to apply the algorithms in [6] to revise the derived solutions to systems with discrete speeds only.

We assume that $P(s)$ is a strictly convex and increasing function, while $P(s)/s$ is merely a convex function [5]. For example, the most adopted speed-dependent power consumption function $P_\delta(s) = \alpha s^\gamma + \lambda s$ satisfies the assumption for any $\gamma > 1$ and any positive constants $\alpha$ and $\lambda$. The *critical speed* $s^*$, defined as the available speed that minimizes the energy consumption for execution [2, 5, 8], is the speed $s$ with the minimum $\frac{P(s)}{s}$. For example, when $P_\delta(s) = \alpha s^\gamma + \lambda s$, the critical speed $s^*$ is $\max\left\{\min\{\sqrt[\gamma]{\frac{P_{ind}}{\alpha(\gamma-1)}}, s_{\max}\}, s_{\min}\right\}$.

A processor has two modes: *dormant mode* and *active mode*. When the processor is in the dormant mode, the power consumption of the processor is assumed to be 0 by scaling the static power consumption [2, 5], but the results in this paper still hold when the power consumption in the dormant mode is not 0. The processor has to be in the active mode for task execution. However, switching between the two modes takes time and consumes energy. Since periodic tasks are considered, the procedure to turn the processor to the dormant mode can be assumed instantaneously with negligible energy overhead by treating the overhead as a part of the overhead to turn on the processor. We denote $E_{sw}$ ($t_{sw}$, respectively) as the energy (time, respectively) of the *switching overhead* from the dormant mode to the active mode.

When the processor is idle in the active mode, the processor executes NOP instructions at speed $s_{\min}$ for energy minimization. When the processor is idle and the idle interval is longer than the *break-even time* $\frac{E_{sw}}{P(s_{\min})}$, turning it to the dormant mode is worthwhile. Let $t_\theta$ be the break-even time, i.e., $t_\theta = \frac{E_{sw}}{P(s_{\min})}$. Without loss of generality, we assume that $t_{sw}$ is no more than $t_\theta$.

We explore energy-efficient scheduling on $m$ homogeneous DVS multiprocessors, where the power consumption function of each task is the same for every processor. For brevity, we denote these $m$ processors by $M_1, M_2, \ldots, M_m$.

For the simplicity of presentation, we assume $P_\delta(s)$ as $\alpha s^3 + \lambda s$ for constants $\alpha$ and $\lambda$, where $P_{ind}$ is abbreviated by $\beta$. The proposed algorithm can be extended to any other convex and increasing functions of $P_\delta()$ easily. Moreover, for simplicity, we implicitly take $s_{\min} = 0$ and $s_{\max} = \infty$ if there is no specific statement. How to handle the case that $s_{\min} \neq 0$ and $s_{\max} \neq \infty$ will also be presented at the end of Section 4. Moreover, we do not intend to deal with the schedulability of real-time tasks under the speed constraint. As shown in [1], it is $\mathcal{NP}$-complete to determine whether there exists a feasible solution when $s_{\max} \neq \infty$. Generally, the maximum available speed is quite higher than the critical speed.

## 2.2 Task models

We explore the scheduling of a set of periodic tasks in a multiprocessor DVS system. A periodic task is an infinite sequence of task instances, referred to as *jobs*, where each job of a task comes in a regular period. The period of task $\tau_j$ is denoted by $p_j$. The relative deadline $d_j$ of task $\tau_j$ is equal to its period $p_j$. The amount of the worst-case execution cycles of task $\tau_j$ is profiled and denoted by $c_j$. The value of $c_j$ could also be obtained by profiling the worst-case execution time at a certain profiling speed.

Executing at speed $s$ for $t$ time units is assumed to complete $s \times t$ cycles with energy consumption $P(s)t$. Without loss of generality, we only deal with the case that $\frac{c_j}{s_{\max}} \leq p_j$, in which the tasks can complete before their deadline at speed $s_{\max}$.

Note that, for a task $\tau_j$, no matter which speeds the scheduler uses for executing a task instance of $\tau_j$, the (partial) energy consumption comes from the power consumption term $\lambda s$ is $\lambda c_j$, which does not related to the schedule. As a result, for algorithmic design and analysis, *we will simply assume that* $\lambda$ *is* 0, which does not affect the applicability of the designed approach and analysis for the general case $\lambda > 0$.

When all the tasks are with the same period $D$ and arrival time 0, these tasks are called frame-based real-time tasks with frame size $D$. As shown in the literature [10], we can transform frame-based real-time tasks with precedence constraints to independent frame-based real-time tasks by pipelining the execution of the tasks. The proposed algorithm in this paper can then be adopted. Therefore, throughout this paper, we only focus on tasks without precedence constraints.

## 2.3 Problem definition

The problem explored in this paper is defined as follows:

Consider a set $\mathbf{T}$ of independent tasks over $m$ identical processors with a common power consumption function $P(s) = \alpha s^3 + \beta$, where $\alpha, \beta \geq 0$. Each of the processors can operate at any speed in the range of $s_{\min}$ and $s_{\max}$. Each periodic task $\tau_j \in \mathbf{T}$ is associated with a computation requirement in $c_j$ CPU-cycles and a period $p_j$, where the relative deadline of $\tau_j$ is $p_j$. The energy of the switching overheads from the dormant mode to the active mode of a processor is $E_{sw}$, while the timing overhead is $t_{sw} \leq \frac{E_{sw}}{P(s_{\min})}$. The objective is to partition tasks in $\mathbf{T}$ to $m$ processors so that the energy consumption is minimized and tasks can be done before their timing constraints.[1] $\square$

For brevity, the studied problem is referred to as the *leakage-aware multiprocessor energy-efficient scheduling* (LAMS) problem, which has been shown to be $\mathcal{NP}$-hard in a strong sense in [1]. This paper pursues polynomial-time approximation algorithms with worst-case guarantees on the quality of the derived solutions for the LAMS problem. A $\rho$-approximation algorithm for the LAMS problem (or, an algorithm with a $\rho$-approximation factor) derives solutions with at most $\rho$ times of the corresponding optimal solutions.

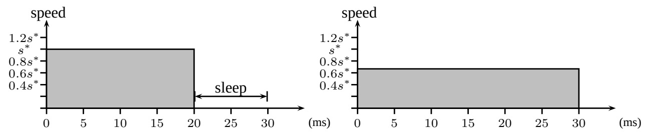
# 3. CRITICALITY OF CRITICAL SPEEDS

This section presents the reasons why the critical speed should not be used as the only lower bound for execution or the only upper bound for task assignment. We then present the weakness of the approximation algorithm in [1] by providing a tight example.

## 3.1 Non-optimality of critical speeds

Throughout this subsection, we will demonstrate the weakness of the critical speed by using the power consumption function for Intel XScale, in which the power consumption can be approximately modeled as $P(s) = (1.52(\frac{s}{1GHz})^3 + 0.08)$ Watt [2, 12]. For such a case, the critical speed $s^*$ is around 297MHz and the power consumption at the critical speed is 0.12Watt where $s_{\min}$ is assumed 0 in this example. We assume that the switching overhead $E_{sw}$ is 0.8 mJoule. Therefore, the break-even time $t_\theta$ is 10 msec, provided that the idle power consumption of the processor is 0.08 Watt. We will only use frame-based real-time tasks with a common

[1] The energy consumption is defined by executing these tasks in a certain interval $L$. If the hyper-period of the tasks exists, $L$ should be defined as the hyper-period. Otherwise, $L$ should be a number that is large enough to show the representative.



(a) At the critical speed. (b) At a lower speed.

**Figure 1: Different schedules on one processor.**

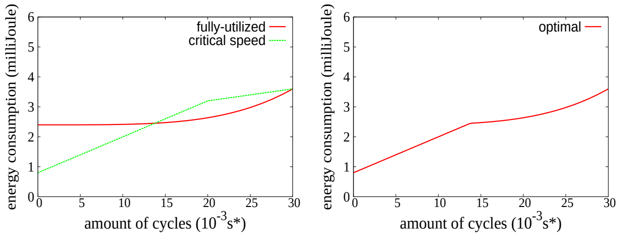
deadline $D$ equal to 30 msec in this subsection. For simplicity, we assume there is no task/job procrastination [2, 8], but the results in this subsection can be easily extended to deal with task/job procrastination for frame-based real-time tasks.

As the critical speed is defined only for the minimization of energy consumption for job execution, it, in fact, optimistically assumes to turn the processor(s) to the dormant mode after the execution. However, it might not be worthwhile to turn the processor to the dormant mode because the energy overhead might be larger than the reduced energy. In such cases, executing at the critical speed might lead to a solution that consumes more energy.

For example, consider the case that we are going to schedule a task with computation cycle equal to $0.02s^*$. Executing at the critical speed completes this task at time 20 msec. As there will be no job for execution in time interval $(20, 30]$ msec, the processor can then be turned to the dormant mode and then is awaken at time 30 msec. As it is equal to the break-even time $t_\theta$, turning to the dormant mode or being idle at the active mode consumes the same amount of energy. As a result, the energy consumption in time interval $(0, 30]$ msec in the above schedule is $0.12 \times 20 + 0.8 = 3.2$ mJoule. However, executing at speed $\frac{2s^*}{3}$ in time interval $(0, 30]$ msec leads to a schedule with energy consumption $30(0.08 + 0.04\frac{2}{3}^3) = 2.756$ mJoule. Figure 1 presents the above two schedules. It is then clear that executing at the critical speed does help if the resulting schedule has no room for turning the processor to the dormant mode. However, even when there is still some room to turn the processor to the dormant mode, it might not be energy-saving, neither. Consider the case that the amount of computation cycles is $0.019s^*$. As $0.12 \times 19 + 0.8 = 3.08 > 30(0.08 + 0.04\frac{19}{30}^3) \approx 2.705$, executing at the critical speed is again not worthwhile.

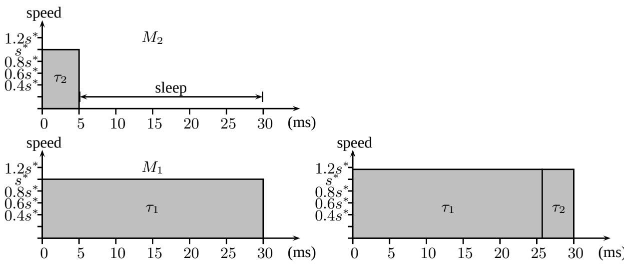
Figure 2 depicts the energy consumption in time interval $(0, 30]$ msec when the amount of computation of the task is between 0 and $0.03s^*$. Figure 2(a) shows the energy consumption for two different scheduling strategies: (1) the *fully-utilized* strategy is to execute at the speed to fully utilize the time interval $(0, 30]$ to minimize the energy consumption resulting from the dynamic power circuits, and (2) the *critical-speed* strategy is to execute at the critical speed and then either turn the processor to the dormant mode or be idle depending on the completion time. Figure 2(b) is the minimum energy consumption between the above strategies. As shown in Figure 2, in this example, executing at the critical speed is better only when the amount of the computation requirement is no more than $0.0125s^*$ in this example.

It seems to have benefit when the amount of the computation requirement is more than $0.03s^*$ by executing at the critical speed on two processors. But, it is not always true, again. Consider the input instance with two tasks $\tau_1$ and $\tau_2$ when $m = 2$. Figure 3 shows the schedules for the above tasks when $c_1 = 0.03s^*$ and $c_2 = 0.005s^*$ by using two processors (in Figure 3(a)) and by using one processor only (in Figure 3(b)). The energy consumption for the schedule in Figure 3(a) (Figure 3(b), respectively) is 5 (4.31, respectively) mJoule. To see when to use two processors, suppose that the total amount $C = c_1 + c_2$ of computation cycles required by these two tasks is between 0 and $0.05s^*$. If $C > 0.03s^*$, then $c_1$ is $0.03s^*$;




(a) Energy curves (b) Optimal energy curves

**Figure 2: An example of the energy consumption curves when the total amount of computation cycles ($\sum_{\tau_j \in \mathbf{T}} c_j$) is between $0$ and $0.03s^*$ on one processor.**




(a) On two processors. (b) On one processor.

**Figure 3: Different schedules for two processors.**

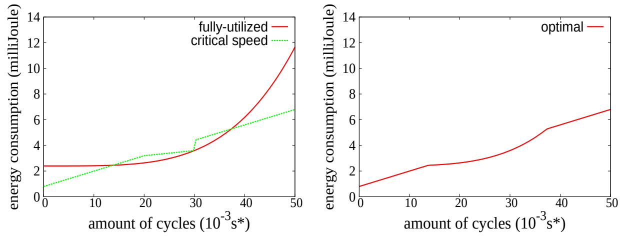
otherwise, $c_1 = c_2$. Figure 4(a) shows the energy consumption for two different scheduling strategies: (1) the *fully-utilized* strategy is to execute *on one processor* at the speed to fully utilize the time interval $(0, 30]$ to minimize the energy consumption resulting from the dynamic power circuits, and (2) the *critical-speed* strategy is to execute at the critical speed (*on one processor if* $C \leq 0.03s^*$ *or on two processors if* $C > 0.03s^*$) and then either turn the processors to the dormant mode or be idle depending on the completion time. Figure 4 reveals that violating the critical speed might help.

The critical speed is used in [1, 13] as the maximum speed for task assignment and processor allocation. The convexity is shown [1] to prove the optimality of load balancing under the critical speed when systems are with negligible overhead to turn the processor to/from the dormant mode. As executing at the critical speed might consume more energy, task assignment based on the critical speed with load balancing might also consume more energy.

## 3.2 Weakness of existing algorithms

The algorithms developed in [1] have been shown to have a 2-approximation factor when $s_{\min} > 0$ and a 1.667-approximation factor for $s_{\min} = 0$. The algorithms in [1] first perform task partitioning by assuming that there is no overhead for turning the processors to the dormant mode. Then, for those processors with idle time at the critical speed, task re-assignment is performed by applying bin-packing algorithms (such as the first-fit algorithm) with the assumption that the critical speed cannot be violated.

However, is it really so critical by taking the critical speed as the limitation for task re-assignment? The following example for $m$ identical tasks shows the tightness of analysis of the algorithms in [1], which implies that, in worst cases, the derived solutions might consume almost $1.667$ times of (or twice when $s_{\min} > 0$) energy of the corresponding optimal solutions. Suppose that there are $m$ identical tasks in $\mathbf{T}$ arriving at time $0$, in which $m$ is an even number, $c_j = c_k = c$, $p_j = p_k = p$, and $c_j/p_j = 0.5s^* + \epsilon$ for some $\epsilon > 0$ and $\tau_j, \tau_k$ in $\mathbf{T}$. The critical speed in this example is




(a) Energy curves (b) Optimal energy curves

**Figure 4: An example of the energy consumption curves when the total amount of computation cycles ($\sum_{\tau_j \in \mathbf{T}} c_j$) is between $0$ and $0.05s^*$ on two processors.**

equal to $\sqrt[3]{\frac{\beta}{2\alpha}}$. The algorithms in [1] would assign each of these $m$ tasks on one processor. If the break-even time $t_\theta$ is equal to $0.5p$, the resulting solution executes tasks at critical speed $s^*$ and keeps idle at the minimum available speed, which leads to energy consumption $m(1.5\beta + \beta + \epsilon\beta) = m(2.5 + \epsilon)\beta$ when $s_{\min} = 0$ (or $3m\beta$ when $s_{\min} = s^*$) in the time interval $(0, p]$.

Instead, if we assign two tasks on each of the first $m/2$ processors and do not use any of the last $m/2$ processors, executing these tasks at speed $(1 + 2\epsilon)s^*$ results in a solution with energy consumption $1.5m(1 + 2\epsilon)^3\beta$ in the time interval $(0, p]$. As a result, the ratio of the energy consumption of the algorithms in [1] to the optimal solution could be close to $1.667$ when $s_{\min} = 0$ (or $2$ when $s_{\min} = s^*$).

# 4. OUR ALGORITHM

This section presents our proposed algorithm for the LAMS problem. We will first explore frame-based real-time tasks, and then show how to extend to systems with periodic real-time tasks. Moreover, at the end of this section, we will present how to deal with the cases that $s_{\min} > 0$ or $s_{\max}$ is violated in the derived solution.

Algorithm Leakage-Aware Largest-Task-First (LALTF) [1] is used to verify whether it is necessary to consider the speed-independent power consumption. It assigns the largest unassigned tasks (defined as the task with the largest ratio of the amount of the required computation cycles to its period) to the processor with the smallest workload. If all the processors have to execute at high speeds to complete all the tasks in time, speed-independent energy consumption, in such a case, is marginal, and can be ignored. When speed-independent power consumption is negligible, applying Algorithm LALTF has a 1.13-approximation factor [1]. Algorithm LALTF does not perform well when the system is not highly loaded at the critical speed such as the example in Section 3.2.

For notational brevity, let tasks be indexed from the largest to the smallest by breaking ties arbitrarily. Clearly, Algorithm LALTF will assign the first $m$ tasks to $m$ different processors. Suppose that after assigning task $\tau_k$, the following conditions are satisfied: $\frac{c_k}{p_k} \geq s^*$, $\frac{c_{k+1}}{p_{k+1}} < s^*$, and $\sum_{j=k+1}^{|\mathbf{T}|} \frac{c_j}{p_j s^*} < (m - k)$. It is not difficult to prove that there exists a lower-bounded solution which executes task $\tau_j$ for $j = 1, 2, \cdots, k$ at speed $\frac{c_j}{p_j}$. As a result, what we have to do is to decide how to schedule tasks $\{\tau_{k+1}, \tau_{k+2}, \ldots, \tau_{|\mathbf{T}|}\}$ on $m - k$ processors. If there does exist such a task $\tau_k$, applying Algorithm LALTF can yield to a 1.13-approximation solution.

As a result, we only discuss the other case that such a task $\tau_k$ exists or $k = 0$ (i.e., $\frac{c_1}{p_1} \leq s^*$ and $\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{p_j s^*} < m$). To simplify the presentation, we will abuse and override the symbol $m$ and $\mathbf{T}$ by setting $m$ as $m - k$ and $\mathbf{T}$ as $\mathbf{T} \setminus \{\tau_j \mid j = 1, 2, \ldots, k\}$. Moreover, the tasks in $\mathbf{T}$ are assumed to be re-indexed from the largest

**Algorithm 1** : RSLTF

**Input:** $\mathbf{T}, m, s^*$ with $\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{Ds^*} < m$ and $\frac{c_j}{D} < s^*$;

1: sort all tasks in $\mathbf{T}$ in a non-increasing order $c_j$ for $\tau_j \in \mathbf{T}$;
2: $m^* \leftarrow \left\lfloor \sum_{\tau_j \in \mathbf{T}} \frac{c_j}{s^* D} \right\rfloor$;
3: $C_{\delta,j} \leftarrow 0, \mathbf{T}_{\delta,j} \leftarrow \emptyset$ for all $\delta = 1, 2$ and $j = 1, 2, \ldots, m^*+1$;
4: **for** $j = 1$ to $|\mathbf{T}|$ **do**
5: find index $1 \leq \delta_1 \leq m^*$ ($1 \leq \delta_2 \leq m^* + 1$, respectively) with the smallest $C_{1,\delta_1}$ (smallest $C_{2,\delta_2}$, respectively);
6: $\mathbf{T}_{1,\delta_1} \leftarrow \mathbf{T}_{1,\delta_1} \cup \{\tau_j\}, \mathbf{T}_{2,\delta_2} \leftarrow \mathbf{T}_{2,\delta_2} \cup \{\tau_j\}$;
7: $C_{1,\delta_1} \leftarrow C_{1,\delta_1} + c_j, C_{2,\delta_2} \leftarrow C_{2,\delta_2} + c_j$;
8: return the better task partition between $(\mathbf{T}_{1,1}, \mathbf{T}_{1,2}, \ldots, \mathbf{T}_{1,m^*})$ and $(\mathbf{T}_{2,1}, \mathbf{T}_{2,2}, \ldots, \mathbf{T}_{2,m^*+1})$;

to the smallest, and the workload can be done by using all these $m$ processors at the critical speed $s^*$.

## 4.1 Algorithm for frame-based tasks

For notational brevity, let $D$ be the common deadline of the frame-based real-time tasks. Our approach is to apply the largest-task-first algorithm again, but not on $m$ processors. The idea is to use either $\left\lfloor \sum_{\tau_j \in \mathbf{T}} \frac{c_j}{s^* D} \right\rfloor + 1$ processors or only $\left\lfloor \sum_{\tau_j \in \mathbf{T}} \frac{c_j}{s^* D} \right\rfloor$ processors. As $\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{Ds^*} < m$, the above assignment does not violate the number of available processors.

For brevity, let $m^*$ be $\left\lfloor \sum_{\tau_j \in \mathbf{T}} \frac{c_j}{s^* D} \right\rfloor$. As a result, we will only focus on two cases: one is to use $m^*$ processors for scheduling tasks in $\mathbf{T}$, and the other is to use $m^* + 1$ processors. For these two cases, we greedily apply the largest-task-first strategy to assign tasks onto those processors. Then, each processor individually minimizes its energy consumption by (1) executing at the critical speed and then turning to the dormant mode[2], or (2) executing at a constant speed without idling or turning to the dormant mode. Between these two schedules, the better one (with the less energy consumption) is pickup as the solution.

Algorithm 1 illustrates the pseudo-code of the above algorithm, denoted by Algorithm RSLTF. The time complexity of Algorithm RSLTF is $O(|\mathbf{T}| \log(m|\mathbf{T}|))$, since it takes $O(|\mathbf{T}| \log |\mathbf{T}|)$ for the sorting of task set $\mathbf{T}$ and $O(|\mathbf{T}| \log m)$ for assigning tasks in the loop between Step 4 and Step 7 in Algorithm 1.

Note that Algorithm RSLTF is different from Algorithm LALTF developed in [1]. The difference is as follows: Algorithm RSLTF tries to assign tasks on either $m^*$ or $m^* + 1$ processors by applying the largest-task-first strategy without setting any workload constraints on these processors. Algorithm LALTF uses the largest-task-first strategy to assign tasks on $m$ processors and then performs task re-assignment by setting $s^*$ as the workload constraints on these processors. Take Figure 3 in Section 3 for two tasks for example. Algorithm RSLTF allocates two processors for task execution if $\frac{c_2}{p} \geq 1.285 s^*$; otherwise, only one processor is used for executing the tasks.

We now present the performance analysis of Algorithm RSLTF in terms of approximation factors. If $m^*$ is 0, the LAMS problem is equivalent to a uniprocessor scheduling problem, and, hence, the derived solution is optimal for frame-based real-time tasks.

For the case that $m^*$ is more than 0, multiprocessor platforms might be used in optimal solutions of the LAMS problem. We will analyze the approximation factor of Algorithm RSLTF for two cases: $m^* = 1$ and $2 \leq m^*$. The following lemma of the property of the largest-task-first strategy will be widely used for the analysis of approximation factors.

LEMMA 1. *Applying the largest-task-first strategy for task partitioning of $\mathbf{T}$ on $m'$ processors with $m' \leq |\mathbf{T}|$ and $\frac{c_j}{D} < s^*$ for all tasks $\tau_j$ in $\mathbf{T}$, the largest workload of these $m'$ processors is at most twice of the smallest workload of these $m'$ processors if the largest workload is no less than $s^*$, where the workload on a processor is defined as the summation of the computation requirements of the assigned tasks divided by the common deadline $D$.*

PROOF. As the processor with the largest workload must be no less than $s^*$, the difference between the maximum workload and the minimum workload is at most the workload of the task assigned to the processor with the maximum workload. As a result, this lemma is proved. $\square$

### 4.1.1 Analysis when $m^* = 1$

When $m^*$ is 1, Algorithm RSLTF allocates either one processor or two processors. For brevity, let $z$ be $\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{Ds^*}$. The energy consumption for executing all the tasks at the critical speed is the lower bound of the optimal solution for this case. As a result, the lower bound of the optimal solution is $1.5\beta z D$, where $1.5\beta$ is the power consumption at the critical speed.

For the case that only one processor is allocated, the energy consumption is $D(\alpha(zs^*)^3 + \beta) = D(0.5\beta z^3 + \beta)$, since $s^*$ is $\sqrt[3]{\frac{\beta}{2\alpha}}$. We now focus on the case that the derived solution of Algorithm RSLTF uses two processors. Suppose that $z_1 s^*$ ($z_2 s^*$, respectively) is the workload on the first (second, respectively) allocated processor for execution. Without loss of generality, we assume that $z_1 \leq z_2$. Here are two cases. If $z_2 > 1$, we know that $1 \leq z_2 \leq 2z_1$ based on Lemma 1. Executing at speed $z_1 s^*$ on the first processor completes all the tasks on this processor in time with energy consumption $D\beta(0.5z_1^3 + 1)$. Similarly, the energy consumption on the second processor in this case is $D\beta(0.5z_2^3 + 1)$. If $z_2 \leq 1$, the energy consumption is at most $1.5D\beta z + (2 - z)\beta D$ by executing at the critical speed without turning the processors to the dormant mode.

As there are at most two processors allocated for task execution, we also know that $z_1 + z_2 = z$. Therefore, we know that the approximation factor $\rho$ for this case is

$$\rho \leq \max_{1 \leq z < 2} \min \left\{ \frac{z^3 + 2}{3z}, \max \left\{ \frac{4 + z}{3z}, \frac{(z_1^3 + z_2^3) + 4}{3z} \right\} \right\} \quad (1)$$

Clearly, function $\frac{z^3+2}{3z}$ is a monotonically increasing function of variable $z$ when $z \geq 1$, while $\frac{4+z}{3z}$ is a monotonically decreasing function. However, the exact value of $\frac{(z_1^3+z_2^3)+4}{3z}$ depends on the value of $z$, $z_1$, and $z_2$, and is difficult to evaluate. Fortunately, it is possible to have an upper bound by applying the convexity of function $z^3$ and the relationship of $z_1$ and $z_2$ based on Lemma 1. Because of the convexity, it is known that $x_1^3 + x_2^3 \geq z_1^3 + z_2^3$ if $x_1 \leq z_1 \leq z_2 \leq x_2$ and $z_1 + z_2 = x_1 + x_2$ for any $x_1, x_2 \geq 0$. As $z_2 \leq 2z_1$, we know that $x_1^3 + (2x_1)^3 \geq z_1^3 + z_2^3$ for $x_1$ with $3x_1 = z$. As a result, we have

$$\frac{(z_1^3 + z_2^3) + 4}{3z} \leq \frac{(\frac{z}{3})^3 + (\frac{2z}{3})^3 + 4}{3z} = \frac{\frac{1}{3}z^3 + 4}{3z}. \quad (2)$$

LEMMA 2. *When $m^* = 1$, the approximation factor of Algorithm RSLTF is $1.21$.*

PROOF. The approximation factor $\rho$ in this case is bounded by either $\rho_1(z) = \frac{z^3+2}{3z}$ or $\max\{\rho_2(z) = \frac{\frac{1}{3}z^3+4}{3z}, \rho_3(z) = \frac{4+z}{3z}\}$ when $1 \leq z \leq 2$. Clearly, under the condition $1 \leq z \leq 2$,

[2]One can apply procrastination here for frame-based real-time tasks. As the optimal procrastination solution can be derived easily by applying the concept in [8]. This does not affect our results.

we know that $\rho_1(z)$ is an increasing function of $z$, while $\rho_2(z)$ and $\rho_3(z)$ are decreasing functions of $z$. Therefore, the value $\rho$ satisfies $\rho_1(z^*) = \rho_3(z^*) = \rho$ for some $z^*$. By solving

$$\frac{z^3+2}{3z} = \frac{4+z}{3z},$$

we know that $z^*$ is $\frac{1}{3}\sqrt[3]{27+3\sqrt{78}} + \sqrt[3]{\frac{1}{27+3\sqrt{78}}}$ and $\rho$ is close to 1.21. $\square$

### 4.1.2 *Analysis when $m^* \geq 2$*

When $m^*$ is no less than 2, Algorithm RSLTF allocates at least two processors. Again, for notational brevity, let $z$ be $\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{s^* D}$, and the lower bound of the optimal solution is $1.5\beta z D$. Similar to the analysis in Section 4.1.1, what we have to do is to find an upper bound for task execution when allocating $m^*$ or $m^*+1$ processors for task execution.

We will first show how to find an upper bound $E_{m^*}$ of energy consumption for the task partition, i.e., $(\mathbf{T}_{1,1}, \mathbf{T}_{1,2}, \ldots, \mathbf{T}_{1,m^*})$, when $m^*$ processors are allocated. Again, let $z_\ell$ be $\sum_{\tau_j \in \mathbf{T}_{1,\ell}} \frac{c_j}{s^* D}$. As executing all the tasks in $\mathbf{T}_{1,\ell}$ at speed $z_\ell s^*$ on a processor is an upper bound of the derived solution, we know that $E_{m^*} = D(m^*\beta + \sum_{\ell=1}^{m^*} \alpha(s^*)^3 z_\ell^3) = D\beta(m^* + 0.5\sum_{\ell=1}^{m^*} z_\ell^3)$ is an upper bound. The following lemma shows the upper bound of $\sum_{\ell=1}^{m^*} z_\ell^3$ based on Lemma 1.

LEMMA 3. *Let $m^\dagger$ be an integer, and $Z = \sum_{\ell=1}^{m^\dagger} z_\ell$. For any $\ell = 1, 2, \ldots, m^\dagger$, $z_\ell \geq 0$ and $z_\ell \leq 2z_{\ell'}$ for any $\ell' \neq \ell$. Then,*

$$\sum_{\ell=1}^{m^\dagger} z_\ell^3 \leq 1.412 m^\dagger \left(\frac{Z}{m^\dagger}\right)^3.$$

PROOF. The proof is in the technical report [3]. $\square$

Therefore, we know that the approximation factor $\rho$ for this case is bounded by

$$\rho_4(m^*, z) = \frac{2m^* + 1.412m^*\left(\frac{z}{m^*}\right)^3}{3z}, \tag{3}$$

where $m^* \leq z < m^*+1$. Note that, when $m^*$ is 2, even though the above equation stands, we will use Equation (2) for tighter analysis.

The other case by allocating $m^*+1$ processors can be done by two cases. If the maximum workload among these $m^*+1$ processors is more than $s^*$ after task assignment, with a simple corollary, the approximation factor $\rho$ is bounded by

$$\rho_5(m^*, z) = \frac{2(m^*+1) + 1.412(m^*+1)\left(\frac{z}{m^*+1}\right)^3}{3z}. \tag{4}$$

where $m^* \leq z < m^*+1$. Otherwise, the approximation factor $\rho$ is bounded by

$$\rho_6(m^*, z) = \frac{1.5\beta D z + \beta D(m^*+1-z)}{1.5\beta D z} = \frac{2(m^*+1)+z}{3z}, \tag{5}$$

while the denominator is the energy consumption by executing at the critical speed without turning the processor to the dormant mode.

Therefore, similar to Equation (1), when $m^* \geq 2$, the approximation factor $\rho$ is

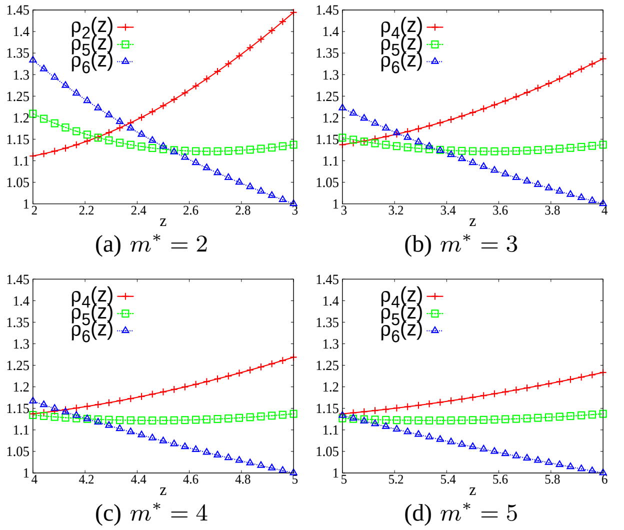
$$\rho = \max_{2 \leq m^* \leq z < m^*+1} \min\{\rho_4(m^*, z), \max\{\rho_5(m^*, z), \rho_6(m^*, z)\}\}, \tag{6}$$

where, more precisely, $\rho_4(m^*, z)$ can be replaced by $\rho_2(z) = \frac{\frac{1}{3}z^3+4}{3z}$ when $m^* = 2$. When $m^* \geq 6$, it is not difficult to see that

$$\rho_6(m^*, z) < \rho_5(m^*, z) < \rho_4(m^*, z),$$

for any $6 \leq m^* \leq z < m^*+1$. Hence, $\rho_5(m^*, z)$ dominates the approximation factor when $m^* \geq 6$. Because $\rho_5(m^*, z)$ is a convex function of $z$ for a fixed $m^*$, we know that either $\rho_5(m^*, m^*)$ or $\rho_5(m^*, m^*+1)$ is the maximal value. Moreover, when $m \geq 6$, we have

$$\rho_5(m^*, z) \leq \rho_5(m^*, m^*+1) \leq 1.138. \tag{7}$$




(a) $m^* = 2$ (b) $m^* = 3$

(c) $m^* = 4$ (d) $m^* = 5$

**Figure 5: The functions which bound the approximation factor when $2 \leq m^* \leq 5$, where $\rho_2()$ is $\rho_2(z) = \frac{\frac{1}{3}z^3+4}{3z}$, $\rho_4()$ is defined in Equation (3), $\rho_5()$ is defined in Equation (4), and $\rho_6()$ is in Equation (5).**

Figure 5 illustrates the corresponding functions $\rho_2(), \rho_4(), \rho_5()$, and $\rho_6()$ when $m^* = 2, 3, 4, 5$. As shown in Figure 5(a), the intersection of $\rho_2(2, z)$ and $\rho_6(2, z)$ ($z \approx 2.3553$) is the bound when $m^* = 2$. For the case that $m^* = 3$ ($m^* = 4$, respectively), the intersection of $\rho_4(m^*, z)$ and $\rho_6(m^*, z)$ when $z \approx 3.2154$ ($z \approx 4.1051$, respectively) bounds the approximation factor. Moreover, when $m^* = 5$, the approximation factor is bounded by $\rho_5(6, 6) = 1.138$.

By concluding all the above cases when $m^* = 0, 1, 2, 3, 4, 5$, and $m^* \geq 6$, we can reach the following concluding theorem.

THEOREM 1. *When $s_{\min} = 0$, the approximation factor of Algorithm RSLTF is 1.21.*

PROOF. By Lemma 2 and the above discussions, the approximation factor $\rho$ is 1 when $m^* = 0$, and is no more than 1.21 when $m^* = 1$, 1.183 when $m^* = 2$, 1.163 when $m^* = 3$, 1.146 when $m^* = 4$, and 1.138 when $m^* \geq 5$. Therefore, we reach the conclusion of the approximation factor $\rho \leq 1.21$. $\square$

## 4.2 Extensions to periodic real-time tasks

For periodic real-time tasks, the extension of Algorithm RSLTF is simple by changing all the operations in execution cycles $c_j$ to the cycle utilization $\frac{c_j}{p_j}$. That is, Step 1 in Algorithm 1 is revised to a non-increasing order of $\frac{c_j}{p_j}$, Step 2 in Algorithm 1 is revised as $m^* \leftarrow \left\lfloor \sum_{\tau_j \in \mathbf{T}} \frac{c_j}{s^* p_j} \right\rfloor$, and Step 7 in Algorithm 1 is revised as $C_{1,\delta_1} \leftarrow C_{1,\delta_1} + \frac{c_j}{p_j}, C_{2,\delta_2} \leftarrow C_{2,\delta_2} + \frac{c_j}{p_j}$. The only problem is the revision of Step 8 in Algorithm 1, since how to efficiently determine the *optimal* energy consumption of a set of periodic real-time tasks on a processor is unknown. Instead of finding optimal solutions, we greedily calculate the energy consumption by executing all the tasks in $\mathbf{T}_\ell$ assigned on processor $M_\ell$ at speed $\sum_{\tau_j \in \mathbf{T}_\ell} \frac{c_j}{p_j}$. Then, the revised algorithm returns the better task partition. We denote the above algorithm as Algorithm P-RSLTF.

*Without turning the processors off when $m^* \geq 1$, the energy consumption of the derived solution of Algorithm* P-RSLTF *in time interval length $L$ is determined after task partitioning, even though it might not be optimal for the resulting task partition.* However, because the analysis in Section 4.1.1 and Section 4.1.2 takes the worst-case analysis by executing the tasks on a processor at the critical speed without turning the processor off, the approximation factor of Algorithm P-RSLTF is also $1.21$ when $m^* \geq 1$.

THEOREM 2. *When $m^* \geq 1$ and $s_{\min} = 0$, the approximation factor of Algorithm* P-RSLTF *is* $1.21$. *When $m^* = 0$ and $s_{\min} = 0$, the proposed algorithm in [5] has a* $3$*-approximation factor.*

PROOF. The proof is the same as the proof in Theorem 1 by changing $D$ to the hyper-period of real-time tasks in $\mathbf{T}$ or evaluating the energy consumption in a time interval with length $L$ that is large enough. The second statement comes from [5] directly. □

## 4.3 Remarks for speed limitations

If $s_{\max} \neq \infty$ and the derived solution of Algorithm RSLTF violates the speed constraint, one can simply activate more processors for task assignment with some resource augmentation (or constraint violation), which is the same as in [1]. We focus the discussions on $s_{\min} > 0$ hereafter. With similar analysis to Lemma 3, when $s_{\min} > 0$, we need the following inequality for proving the approximation factor:

$$\sum_{\ell=1}^{m^\dagger} \max\left\{z_\ell^3, (\frac{Z}{m^\dagger})^3\right\} \leq 1.8 m^\dagger \left(\frac{Z}{m^\dagger}\right)^3,$$

where $m^\dagger, z_\ell$, and $Z$ are defined in Lemma 3. We can revise the definitions of $\rho_4$ and $\rho_5$ in Equations (3) and (4), respectively, to

$$\rho_4^\dagger(m^*, z) = \frac{2m^* + 1.8m^* \left(\frac{z}{m^*}\right)^3}{3z},$$

$$\rho_5^\dagger(m^*, z) = \frac{2(m^*+1) + 1.8(m^*+1)\left(\frac{z}{m^*+1}\right)^3}{3z}.$$

With similar analysis in Section 4.1, the following corollary holds:

COROLLARY 1. *When $s_{\min} > 0$, the approximation factor of Algorithm* RSLTF *is* $1.43$.

# 5. PERFORMANCE EVALUATION

In this section, we provide performance evaluation on the energy consumption of the proposed algorithm. For comparison, we also evaluate the proposed algorithms proposed in [1]. Moreover, the following algorithms are evaluated for comparisons:

- *Algorithm* RSLTF-CRITICAL (or P-RSLTF-CRITICAL for periodic real-time tasks) applies the task partition algorithm in this paper by taking the critical speed as the lower bound for task execution.
- *Algorithm* LALTF-FIRST-FIT is Algorithm LA+LTF in [1] by applying the first-fit strategy for task remapping.
- *Algorithm* LALTF-WORST-FIT is Algorithm LA+LTF in [1] by applying the worst-fit strategy for task remapping.

For Algorithm LALTF, we also evaluate the best-fit and the last-fit strategies for task remapping, but, their results are either close to those of the first-fit or the worst-fit strategy. For the clarity of figures, we will not include their results. For all evaluated algorithms, we also apply the existing procrastination approach [8].

*Workload Parameters and Performance Metrics.* We evaluate the energy consumption of the solutions by using the power consumption function of Intel XScale, in which $P(s)$ is modeled by $1.52(\frac{s}{1GHz})^3 + 0.08$ Watt [2, 12]. The available speeds are in the range of [$s_{\min}$, 1 GHz], where $s_{\min}$ is a specified input in the range of $[0, 0.25]$ GHz in our settings. The energy overhead $E_{sw}$ of switching is in the range of $[0.2, 1.5]$ mJoule, while the timing overhead $t_{sw}$ of switching is assumed to be $0.1$ msec.

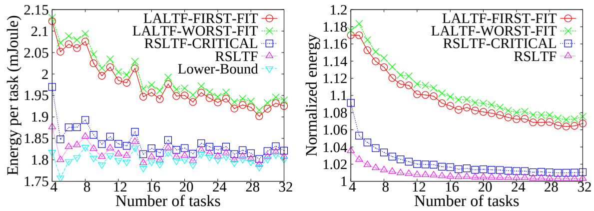
We use synthetic real-time task sets for performance evaluation. For each task $\tau_j$, the execution time at 1 GHz is a random variable uniformly distributed in the range of $[0.01, 0.297]p_j$, where $p_j$ is 30 msec for the setting of frame-based real-time tasks, and is a random variable in the range of $[10, 100]$ msec for the setting of periodic real-time tasks. Because Algorithm RSLTF applies Algorithm LALTF as a subroutine when $\frac{\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{p_j}}{m}$ is more than the critical speed, to show the effectiveness of Algorithm RSLTF, the number of processors $m$ is set as $2\left\lfloor \sum_{\tau_j \in \mathbf{T}} \frac{c_j}{p_j s^*} \right\rfloor$ for a given task set, where $\sum_{\tau_j \in \mathbf{T}} \frac{c_j}{p_j s^*} > 1$.

We evaluate three different settings by (1) changing the number of tasks from $4$ to $32$ with $s_{\min} = 0$ and $E_{sw} = 1$ mJoule, (2) changing the energy overhead $E_{sw}$ of switching from $0.2$ mJoule to $1.5$ mJoule for $20$ tasks with $s_{\min} = 0$, and (3) changing the minimum available speed $s_{\min}$ from $0$ GHz to $0.25$GHz for $20$ tasks with $E_{sw} = 1$ mJoule. Because procrastination might be performed, for an input instance, we evaluate the energy consumption of frame-based real-time tasks for $3$ sec and report the average energy consumption in $30$ msec as the result. For periodic real-time tasks, the energy consumption is evaluated in $3600$ sec.

For each configuration, we perform $512$ independent simulations, and report the average energy consumption. Moreover, to show the derived solutions of Algorithm RSLTF are quite close to the optimal solutions, we also report the energy consumption by executing all the tasks at the critical speed as the *Lower Bound* of the optimal energy consumption by ignoring the energy overhead $E_{sw}$. The *normalized energy* of an algorithm for an input instance is the energy consumption of the derived solution divided by the above lower bound. For each configuration, the average normalized energy is reported.

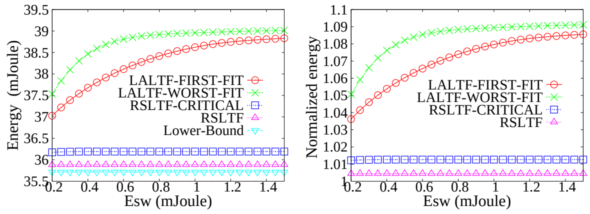
*Evaluation Results.* Figure 6 shows the results for frame-based real-time tasks with $s_{\min} = 0$ and $E_{sw} = 1$ mJoule, where Figure 6(a) is the average energy consumption per 30 msec, and Figure 6(b) is the average normalized energy consumption. Clearly, when the number of tasks increases, the resulting energy consumption increases. However, in terms of normalized energy consumption, all the evaluated algorithms are worse when the number of tasks is small. Similar to the argument of non-optimality of critical speeds in Section 3, the comparison of Algorithms RSLTF and RSLTF-CRITICAL in Figure 6(b) shows that taking the critical speed as the lower bound for task execution could be worse. This is because most processors assigned with tasks by applying Algorithm RSLTF for partitioning tasks are almost fully-utilized for task execution. Procrastination does not help too much, and, most of time, making the resulting schedules worse. The improvement of Algorithm RSLTF, compared to Algorithm LALTF, is about $8 \sim 13.5\%$.

Figure 7 illustrates the average energy consumption in 30 msec



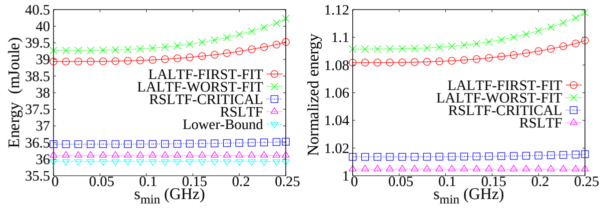

(a) Average energy consumption (b) Normalized energy consumption

**Figure 6: Experimental results for frame-based tasks with different numbers of tasks.**



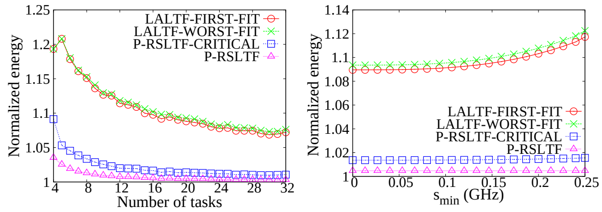

(a) Average energy consumption (b) Normalized energy consumption

**Figure 7: Experimental results for frame-based tasks with different energy switching overheads.**




(a) Average energy consumption (b) Normalized energy consumption

**Figure 8: Experimental results for frame-based tasks with different minimum available speeds.**




(a) Variant number of tasks (b) Variant $s_{\min}$

**Figure 9: Experimental results for periodic tasks.**

for 20 frame-based real-time tasks with $s_{\min} = 0$ for different energy overheads $E_{sw}$ of switching. Again, Algorithm RSLTF-CRITICAL does not help reduce the energy with the same reason in Figure 6. Because, most of time, Algorithm LALTF-FIRST-FIT and Algorithm LALTF-WORST-FIT would use more processors for task execution, when the energy overhead $E_{sw}$ of switching is lower, they have better results. The improvement of Algorithm RSLTF, compared to Algorithm LALTF, is about $3 \sim 8\%$. Figure 8 presents the results for 20 frame-based real-time tasks with $E_{sw} = 1$ mJoule, in which Algorithm RSLTF is better when $s_{\min}$ is higher.

Figure 9 shows the normalized energy consumption for periodic real-time tasks. The results are similar to frame-based real-time tasks. Since the optimal uniprocessor schedules can be determined easily for frame-based real-time tasks, in general, the normalized energy consumption for periodic real-time tasks is worse than that for frame-based real-time tasks. As shown in the simulations, Algorithm RSLTF outperforms the family of Algorithm LALTF in both average cases and worst cases, while the improvement in average cases could be at most $15\%$.

## 6. CONCLUSION

This paper explores energy-efficient scheduling in leakage-aware DVS systems. Even though the critical speed is optimal for job/task execution, we show that it might not be good enough to stick with the critical speed by taking it as the lower-bounded speed for execution or by treating it as the upper-bounded speed for task mapping in multiprocessor systems. Moreover, we also show that load balancing might not be good since assigning tasks on a light-loaded processor might waste energy to turn on/off the processor. The above observation motivates the research in this paper to improve existing scheduling algorithms for minimizing the energy consumption in homogeneous multiprocessor systems. This paper presents an efficient algorithm, which allocates the processors based on the critical speed and performs task assignment for load balancing without considering the critical speed. When multiple processors are necessary for energy minimization, the proposed algorithm is proved to derive solutions with at most $1.21$ ($1.43$, respectively) times of the energy consumption of the corresponding optimal solutions when the minimum available speed is $0$ (not $0$, respectively). Experimental results reveal that the proposed algorithm is more effective than existing algorithms also in average cases.

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